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Assessing Debt Stationarity and Sustainability in the Longer Run with Fourier DF Unit Root Tests and Time-Varying Fiscal Reaction Functions*

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Abstract

Thanks to various Fourier DF unit root tests, time-varying fiscal reaction functions and threshold regressions, this study examines the stationarity and the sustainability of public finance for six industrial countries over the period spanning from 1870 to 2017. Longer-run debt sustainability is not rejected for the UK, Sweden, and for the US. The evidence is more mixed for Canada, Italy and Portugal.

Keywords: Fourier DF unit root test, Debt sustainability, Primary balance
JEL: C22, E62, H62

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Highlights

- Assessing debt sustainability and stationarity in the longer run.
- Smooth structural breaks, time-varying and threshold effects are considered.
- Debt-to-GDP ratio is stationary for a sample of industrialized countries.
- Debt sustainability is almost certainly ensured in Sweden.
- Evidence is more mixed for Italy and Portugal.

1. Introduction

After the beginning of the Great Recession and during the pandemic crisis, we have observed impressive surges in the public debt-to-GDP ratio. In the US, the total public debt moved from around 108% in the first quarter of 2020 to above 122% in the third quarter of 2021.[1] In several industrialized countries, similar evolutions have raised concerns about the sustainability of public finance in future years. Following Afonso (2005), Afonso and Rault (2010) and Afonso and Jalles (2014), we can derive the so-called present value of the budget constraint:

$$B_{t-1} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^{s+1}} (R_{t+s} - E_{t+s}) + \lim_{s\to\infty} \frac{B_{t+s}}{(1+r)^{s+1}} \tag{1}$$

In absence of Ponzi games, the second term of the right-hand side of equation (1) converges to zero and the intertemporal budget constraint is fulfilled. Thus, the fiscal policy ensures that deficits are not used to repay the interests on the debt stock. The fiscal policy is considered as sustainable. From equation (1), we can derive a complementary definition of sustainability suitable for empirical testing:

$$\lim_{s\to\infty} \frac{B_{t+s}}{(1+r)^{s+1}} = 0 \tag{2}$$

Therefore, testing the stationarity of the first difference of the stock of public debt amounts to testing the absence of Ponzi games. When the null of unit root is rejected, the fiscal policy can be regarded as sustainable.[2]

The length of the timespan and the consideration of structural breaks are two important questions in the empirical testing of debt sustainability. Some studies have used samples with more than one hundred years of observations (see e.g., Yoon (2012), for a recent illustration).

Our empirical investigation is the first attempt that simultaneously considers the issues of smooth structural breaks and long span time series in the study of debt sustainability. Together with fiscal reaction functions (hereafter FRF) this will allow us to assess debt stationarity and sustainability in the longer run and consider various types of structural breaks.

2. Data and Research Methodology

2.1. Data Descriptions

In order to analyze debt sustainability in the longer run, we use the version 5 of the Jordà-Schularick-Taylor Macrohistory Database (Jordà et al., 2017).[3] The following series are selected: nominal GDP in local currency, nominal government revenues in local currency, nominal government expenditures in local currency, and the public debt-to-GDP ratio. In order to have complete

[1] U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis.

[2] According to Trehan and Walsh (1991), it is a sufficient condition for sustainability. A complementary approach would be to estimate a reaction function where the primary balance reacts to the public debt, as suggested by Bohn (2007). A positive (negative) coefficient in the reaction function amounts to sustainability (unsustainability).

[3] https://www.macrohistory.net/database/

series for the primary balance and for the debt, we select 6 countries out of 18, namely Canada, UK, Italy, Portugal, Sweden and the US. In the appendix, we show the descriptive statistics (Table A.1).

2.2. *Research Methodology*

The double Frequency Fourier Dickey-Fuller test proposed by Cai and Omay (2022) has better power properties than previous alternatives and is presented as follows,

$$y_t = c + a\sin\left(\frac{2\pi k^s t}{T}\right) + b\cos\left(\frac{2\pi k^c t}{T}\right) + \beta y_{t-1} + \varepsilon_t \tag{3}$$

where k^s and k^c are frequencies to be determined by an updated-driven method.[4] To minimize the sum of squared residuals, they find the optimal frequencies over a pre-determined range, that is $k^s, k^c \in [0, k_{\max}]$ where $k_{\max}$ is the upper bound.[5] We can rewrite equation (3) as the following form,

$$\Delta y_t = c + a\sin\left(\frac{2\pi k^s t}{T}\right) + b\cos\left(\frac{2\pi k^c t}{T}\right) + \gamma y_{t-1} + \varepsilon_t \tag{4}$$

where $\gamma = \beta - 1$. Accordingly, the modified test statistic can be expressed as follows,

$$\tau = \frac{\hat{\gamma}}{\sigma_{\hat{\gamma}}} \tag{5}$$

We generate the critical values with 10,000 replications by stochastic simulations. Augmenting equation (4) with lags of Δy_t is used to solve the autocorrelation issue. A data-driven method is adapted to solve the so-called Davies problem (Davies, 1987). By minimizing the sum of squared residuals in equation (3), one can select the optimal frequency pairs. We follow Cai and Omay (2022) by searching both integer and fractional frequencies over the range $[0, k_{\max}]$. Their simulations suggest more power gains than in both Enders and Lee (2012) and Omay (2015).

3. Empirical Results

Our interests are to investigate the debt stationarity with unit root tests. According to previous studies, there are three popular Fourier DF unit root tests proposed by Enders and Lee (2012), Omay (2015) and Cai and Omay (2022), respectively. The only difference among these three papers is the selection of frequency in trig functions (e.g., single against double and integer against fraction). Table 1 shows the results obtained by using Enders and Lee (2012). Regarding the first difference of the debt ratio, all the countries reject the unit root hypothesis. Omay (2015) suggested that using fractional frequencies could gain additional testing power compared to the test proposed by Enders and Lee (2012). After taking fractional frequency into account, the unit root hypothesis is rejected for the first difference of the debt ratio in all the countries.

[4] Enders and Lee (2012) suggest using single frequency in Fourier Dickey-Fuller unit root tests.

[5] According to Enders and Lee (2012), minimizing the sum of squared residuals is equal to maximizing the F test by imposing the restrictions $a = b = 0$ on equation (4). For more details, please refer to Enders and Lee (2012).

Table 1: Debt-to-GDP ratio in first difference

	Enders and Lee (2012)			Omay (2015)			Cai and Omay (2022)				Cai and Omay (2022)			
	k	Lags	t stat.	k	Lags	t stat.	k^s	k^c	Lags	t stat.	k^s	k^c	Lags	t stat.
Canada	2	3	-7.015***	2.5	3	-7.181***	2	3	3	-7.018***	1.2	2.6	3	-7.342***
UK	2	1	-5.773***	1.6	1	-5.855***	2	5	1	-6.017***	2	4.9	1	-6.104***
Italy	1	1	-8.036***	1.5	1	-8.046***	4	1	1	-8.043***	1.5	2.9	1	-8.319***
Portugal	1	1	-7.427***	0.3	1	-7.581***	5	1	1	-7.761***	0.7	4.7	1	-8.294***
Sweden	3	4	-7.522***	3.1	4	-7.514***	3	4	4	-8.075***	3	4	4	-8.075***
US	2	1	-6.830***	1.9	1	-6.829***	2	4	1	-7.028***	2	4.1	1	-7.101***

Note: this procedure only selects optimal single integer frequency over the range from 0 to 5. The critical values are generated by using stochastic simulation with 10,000 replications. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Table 2: Primary balance in level

	Enders and Lee (2012)			Omay (2015)			Cai and Omay (2022)				Cai and Omay (2022)			
	k	Lags	t stat.	k	Lags	t stat.	k^s	k^c	Lags	t stat.	k^s	k^c	Lags	t stat.
Canada	4	2	-6.293***	4	2	-6.293***	3	4	1	-5.970***	2.9	3.9	2	-6.865***
UK	5	1	-5.279***	1.4	1	-6.011***	2	5	1	-5.744***	4.6	1.5	1	-6.182***
Italy	5	1	-3.444**	4.7	1	-3.430**	2	5	1	-3.885**	3.6	5	1	-4.254***
Portugal	5	2	-2.252	4.9	2	-2.238	1	2	1	-3.744*	2.2	4.8	2	-3.492***
Sweden	1	2	-5.402***	3.7	2	-5.433***	3	4	1	-5.096***	3.5	2.7	2	-6.239***
US	4	1	-6.063***	4	1	-6.063***	2	4	1	-6.298***	0.3	4.1	1	-6.466***

Note: this procedure only selects optimal single integer frequency over the range from 0 to 5. The critical values are generated by using stochastic simulation with 10,000 replications. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Finally, we employ the double Frequency Fourier Dickey-Fuller unit root test proposed by Cai and Omay (2022). We employ, first, double integer frequencies and, then, double fractional frequencies in trig functions. According to Table 1, all countries reject the null hypothesis for the first difference of the debt ratio. The frequencies in trig functions are different in all countries.

Turning to the primary balance in Table 2, the results are consistent with those on the debt ratio. Interestingly, all the time series reject the null hypothesis for the primary balance, but only at 10% at best for Portugal. This result contrasts with those of Tsong et al. (2016) where the government expenditures and revenues are cointegrated for Portugal, but their sample is shorter and they use a single frequency. After considering double fractional frequencies in Table 1 and 2, both the debt ratio and the primary balance are stationary among all countries. The frequencies in trig functions have an integer value only in Sweden. The results of the Fourier DF unit root tests are in line with the results of the time-varying FRF in Table 3 and Figure 1.

Longer-run fiscal sustainability is not rejected for the UK, Sweden, and the US. The evidence is more mixed for Canada, Italy and Portugal, in contrast with Afonso and Jalles (2014). We can underline two polar cases thanks to our empirical analyses. Firstly, the debt sustainability is almost certainly ensured in Sweden, as the Fourier DF unit root tests always reject the presence of a unit root and the initial debt coefficient in the time-varying FRF is positive and statistically significant. Secondly, the presence of a unit root in the primary balance and the non-significant debt coefficient in the time-varying FRF may imply that debt sustainability is not ensured for the Portuguese economy.

In the threshold regressions in Table 4 to 6, we found two thresholds for Italy and Portugal.

Table 3: Time-varying fiscal reaction function estimated with OLS

	Canada	UK	Italy	Portugal	Sweden	US
Variables	PB	PB	PB	PB	PB	PB
L.PB	0.8077***	0.8778***	0.8516***	0.8386***	0.8594***	0.7872***
	(0.0585)	(0.0398)	(0.0442)	(0.0505)	(0.0523)	(0.0575)
L.debt	0.0116	0.0158**	0.0076	0.0046	0.0587***	0.0275**
	(0.0093)	(0.0064)	(0.0098)	(0.0041)	(0.0163)	(0.0114)
L.lgdp	-0.0466	0.1423	0.0112	-0.0590*	-0.2655***	-0.2993**
	(0.0855)	(0.153)	(0.0661)	(0.0327)	(0.0884)	(0.1248)
Constant	-0.9750*	-1.9156**	-1.5453	0.0347	0.8958	0.1526
	(0.5053)	(0.884)	(1.125)	(0.3832)	(0.6751)	(0.5513)
Observations	147	147	147	147	147	147
R-squared	0.6056	0.7777	0.7277	0.7895	0.6684	0.5995

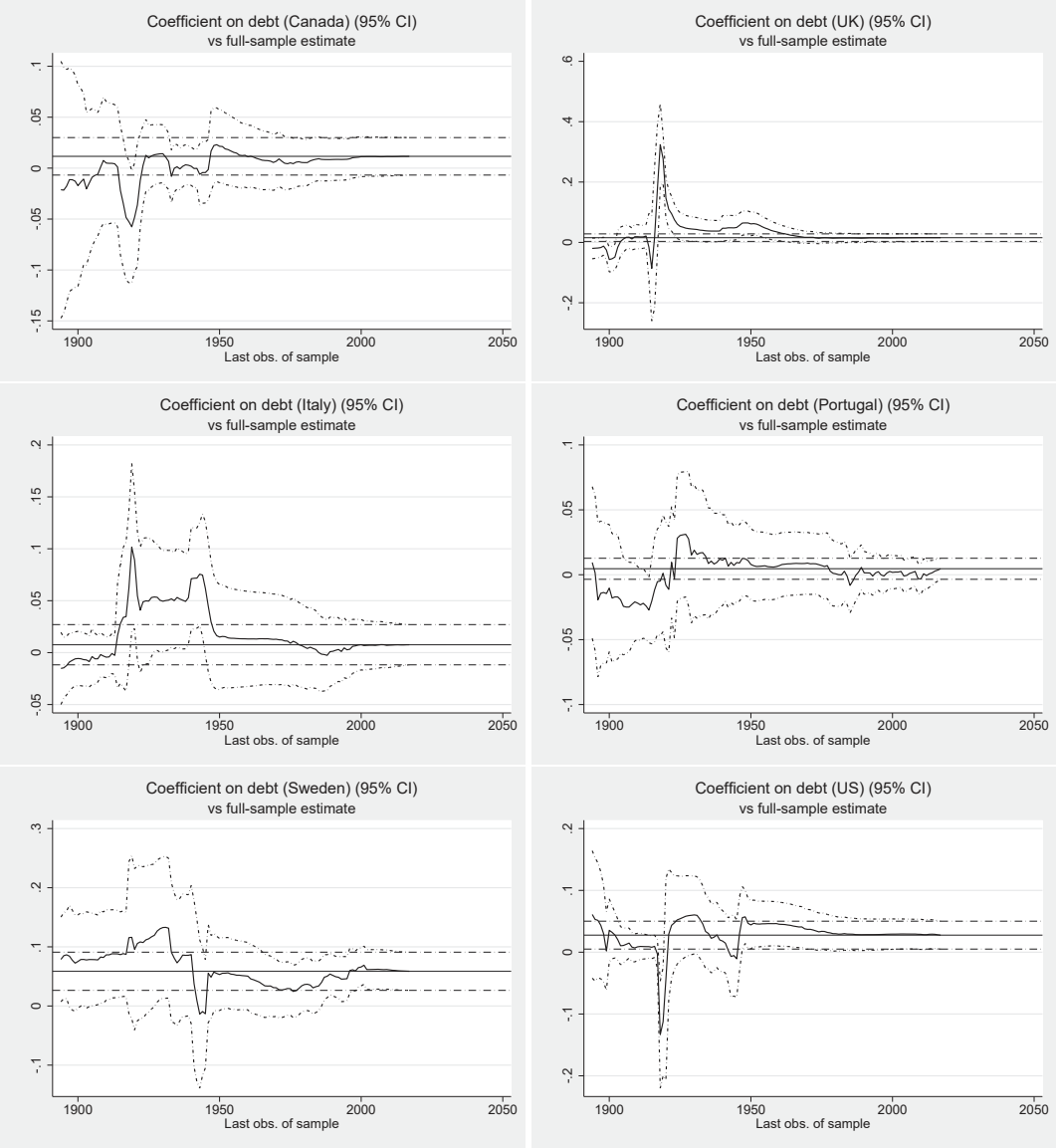
Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. We follow D'Erasmo et al. (2016) by adding an AR(1) term in the fiscal reaction functions. PB is the primary balance computed as the ratio between the nominal revenues minus nominal expenditures and nominal GDP, debt is the debt-to-GDP and lgdp is the natural logarithm of the nominal GDP. L stands for the lag operator.

Besides, we found one threshold for the US and Canada. For Sweden, the results are the same as those of the time-varying FRF, as there is no evidence of threshold effects. In Canada and in the US, the debt sustainability is ensured for levels of initial debt above 92% and 78%, respectively, as the debt coefficient becomes significantly positive. Overall, these three approaches (Fourier DF unit root tests, time-varying FRF and threshold regressions) are complementary and may help to reveal interesting pieces of evidence about debt stationarity and sustainability in the longer run.

4. Conclusion

Longer-run debt sustainability has been considered with econometric techniques well-suited for dealing with a long-time span and the presence of structural breaks. Longer-run debt sustainability is not rejected for the UK, Sweden, and for the US. However, our results cast doubts on the sustainability of public finance for Canada, Italy and Portugal in the longer run. These results may act as a useful warning for policy makers.

Figure 1: Time-varying fiscal reaction functions estimated with OLS



Note: Rolling regressions estimated with OLS.

Table 4: Threshold regressions estimated with OLS for Canada and the UK

	Canada		UK		
Var.	PB		PB		
L.PB	0.9590***		0.8757***		
	(0.0602)		(0.035)		
L.lgdp	0.0683		0.0153		
	(0.0885)		(0.1361)		
	L.debt ≤ 92%	L.debt > 92%	L.debt ≤ 36%	36% < L.debt ≤ 125%	L.debt > 125%
	-0.0019	0.1990***	1.6566***	0.0495***	0.0793***
	(0.0126)	(0.0330)	(0.3562)	(0.0181)	(0.0194)
Cons.	-0.2314	-22.0697***	-57.7954***	-2.6916**	-14.2658***
	(0.6003)	(3.6013)	(11.7636)	(1.2904)	(3.563)
Obs.	147		147		

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. We follow D'Erasmo et al. (2016) by adding an AR(1) term in the fiscal reaction functions. PB is the primary balance computed as the ratio between the nominal revenues minus nominal expenditures and nominal GDP, debt is the debt-to-GDP and lgdp is the natural logarithm of the nominal GDP. L stands for the lag operator. The maximum number of thresholds is set to 2.

Table 5: Threshold regressions estimated with OLS for Italy and Portugal

	Italy			Portugal		
Var.	PB			PB		
L.PB	0.7785***			0.8077***		
	(0.0427)			(0.0501)		
L.lgdp	-0.046			-0.1442***		
	(0.0639)			(0.0378)		
	L.debt ≤ 61%	61% < L.debt ≤ 81%	L.debt > 81%	L.debt ≤ 24%	24% < L.debt ≤ 56%	L.debt > 56%
	-0.0786	-0.8204***	0.0156	0.1608**	0.0450***	0.0323***
	(0.0531)	(0.2021)	(0.0217)	(0.0678)	(0.0151)	(0.0085)
Cons.	2.3131	55.6189***	-2.0203	-1.7947	-0.706	-1.3970***
	(2.1116)	(15.0514)	(2.298)	(1.4542)	(0.7623)	(0.5282)
Obs.	147			147		

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. We follow D'Erasmo et al. (2016) by adding an AR(1) term in the fiscal reaction functions. PB is the primary balance computed as the ratio between the nominal revenues minus nominal expenditures and nominal GDP, debt is the debt-to-GDP and lgdp is the natural logarithm of the nominal GDP. L stands for the lag operator. The maximum number of thresholds is set to 2.

Table 6: Threshold regressions estimated with OLS for Sweden and the US

	Sweden	US	
Var.	PB	PB	
L.PB	0.8594***	0.7830***	
	(0.0519)	(0.0557)	
L.lgdp	-0.2655***	-0.1715	
	(0.0877)	(0.1344)	
	L.debt	L.debt ≤ 78%	L.debt > 78%
	0.0587***	0.0089	0.2739***
	(0.0162)	(0.0161)	(0.0728)
Cons.	0.8958	-25.1670***	
	(0.6704)	(0.5355)	
Obs.	147	147	

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. We follow D'Erasmo et al. (2016) by adding an AR(1) term in the fiscal reaction functions. PB is the primary balance computed as the ratio between the nominal revenues minus nominal expenditures and nominal GDP, debt is the debt-to-GDP and lgdp is the natural logarithm of the nominal GDP. L stands for the lag operator. The maximum number of thresholds is set to 2.

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Online appendix for "Assessing Debt Stationarity and Sustainability in the Longer Run with Fourier DF Unit Root Tests and Time-Varying Fiscal Reaction Functions"

Appendix A.1. Descriptive statistics

Table A.1: Descriptive statistics

Variables	N	Mean	SD	Min	Max
Primary balance (Canada)	148	-2.000	3.843	-21.58	4.253
Primary balance (UK)	148	-0.479	9.479	-44.66	13.04
Primary balance (Italy)	148	-5.325	7.018	-36.33	1.757
Primary balance (Portugal)	148	-2.378	2.553	-9.171	1.433
Primary balance (Sweden)	148	-1.771	3.238	-14.08	6.635
Primary balance (US)	148	-1.869	4.213	-26.86	4.298
Debt-to-GDP ratio (Canada)	148	63.06	27.92	18.45	155.5
Debt-to-GDP ratio (UK)	148	89.92	60.07	27.27	269.8
Debt-to-GDP ratio (Italy)	148	85.98	32.19	24.57	154.1
Debt-to-GDP ratio (Portugal)	148	51.47	25.32	13.3	130.6
Debt-to-GDP ratio (Sweden)	148	28.45	15.74	9.245	73.06
Debt-to-GDP ratio (US)	148	40.13	28.96	2.445	118.9

Appendix A.2. Debt-to-GDP ratio in level

In the following Tables, the procedure only selects optimal single integer frequency over the range from 0 to 5. The critical values are generated by using stochastic simulation with 10,000 replications. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Table A.2: Debt-to-GDP ratio in level (Enders and Lee, 2012)

	k	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2	1	-3.819**	-2.938	-3.317	-4.005
UK	2	1	-2.387	-2.934	-3.31	-4.051
Italy	1	1	-2.809	-3.55	-3.862	-4.536
Portugal	1	1	-0.647	-3.554	-3.876	-4.487
Sweden	3	1	-2.566	-2.682	-3.028	-3.766
US	2	2	-1.945	-2.953	-3.319	-4.037

Table A.3: Debt-to-GDP ratio in level using single fractional frequency (Omay, 2015)

	k	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2.2	1	-3.591**	-2.864	-3.263	-3.987
UK	1.5	1	-4.617***	-3.174	-3.563	-4.26
Italy	1.4	1	-3.614*	-3.254	-3.627	-4.288
Portugal	0.1	1	-1.467	-3.65	-3.967	-4.608
Sweden	3.1	1	-2.565	-2.708	-3.077	-3.765
US	1.9	2	-1.94	-2.947	-3.317	-4.038

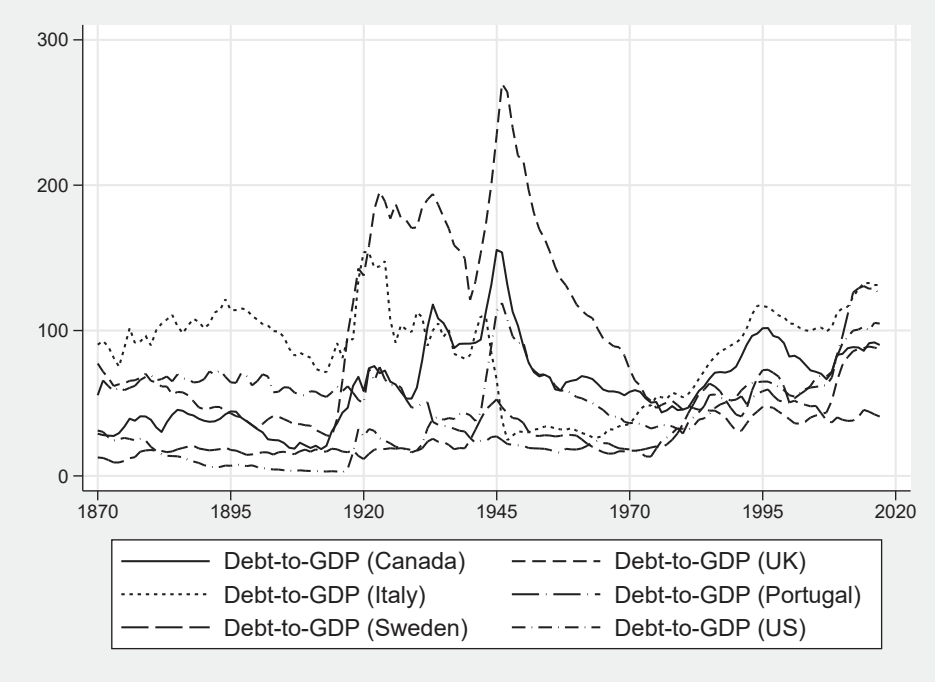
Table A.4: Debt-to-GDP ratio in level using double integer frequency (Cai and Omay, 2022)

	k^s	k^c	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2	2	1	-3.819**	-2.933	-3.328	-4.095
UK	2	5	1	-1.848	-2.84	-3.214	-3.979
Italy	2	1	1	-3.47	-3.487	-3.881	-4.636
Portugal	5	1	1	-2.185	-3.155	-3.533	-4.202
Sweden	3	4	5	-0.139	-2.738	-3.107	-3.818
US	2	4	2	-1.287	-2.859	-3.271	-4.005

Table A.5: Debt-to-GDP ratio in level using double fractional frequency (Cai and Omay, 2022)

	k^s	k^c	Lags	F stat.	t stat.	10% cv	5% cv	1% cv
Canada	0.4	2.3	1	6.517	-4.682***	-3.366	-3.756	-4.477
UK	1.7	4.7	1	13.722	-3.687**	-2.913	-3.298	-4.149
Italy	1.3	2.6	1	7.849	-4.144**	-3.236	-3.671	-4.435
Portugal	0.6	4.7	1	12.831	-1.56	-3.18	-3.532	-4.245
Sweden	2.9	4	5	9.685	-0.734	-2.713	-3.099	-3.934
US	1.9	4.1	2	10.319	-1.7	-2.871	-3.272	-4.067

Figure A.1: Debt-to-GDP ratio



Source: Jordà et al. (2017).

Appendix A.3. Critical Values

In the following Tables, the procedure only selects optimal single integer frequency over the range from 0 to 5. The critical values are generated by using stochastic simulation with 10,000 replications. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Table A.6: Debt-to-GDP ratio in first difference (Enders and Lee, 2012)

	k	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2	3	-7.015***	-2.905	-3.276	-3.98
UK	2	1	-5.773***	-2.909	-3.285	-4.021
Italy	1	1	-8.036***	-3.565	-3.9	-4.538
Portugal	1	1	-7.427***	-3.536	-3.841	-4.541
Sweden	3	4	-7.522***	-2.742	-3.081	-3.828
US	2	1	-6.830***	-2.977	-3.348	-3.99

Table A.7: Primary balance in level (Enders and Lee, 2012)

	k	Lags	t stat.	10% cv	5% cv	1% cv
Canada	4	2	-6.293***	-2.602	-2.941	-3.578
UK	5	1	-5.279***	-2.599	-2.911	-3.562
Italy	5	1	-3.444**	-2.619	-2.938	-3.579
Portugal	5	2	-2.252	-2.597	-2.92	-3.564
Sweden	1	2	-5.402***	-3.563	-3.886	-4.588
US	4	1	-6.063***	-2.624	-2.962	-3.628

Table A.8: Debt-to-GDP ratio in first difference using single fractional frequency (Omay, 2015)

	k	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2.5	3	-7.181***	-2.849	-3.183	-3.882
UK	1.6	1	-5.855***	-3.111	-3.486	-4.272
Italy	1.5	1	-8.046***	-3.184	-3.563	-4.236
Portugal	0.3	1	-7.581***	-3.655	-3.969	-4.625
Sweden	3.1	4	-7.514***	-2.71	-3.068	-3.761
US	1.9	1	-6.829***	-2.98	-3.335	-4.072

Table A.9: Primary balance in level using single fractional frequency (Omay, 2015)

	k	Lags	t stat.	10% cv	5% cv	1% cv
Canada	4	2	-6.293***	-2.654	-2.979	-3.653
UK	1.4	1	-6.011***	-3.279	-3.622	-4.33
Italy	4.7	1	-3.430**	-2.622	-2.951	-3.56
Portugal	4.9	2	-2.238	-2.599	-2.917	-3.599
Sweden	3.7	2	-5.433***	-2.681	-3.017	-3.761
US	4	1	-6.063***	-2.628	-3.003	-3.613

Table A.10: Debt-to-GDP ratio in first difference using double integer frequency (Cai and Omay, 2022)

	k^s	k^c	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2	3	3	-7.018***	-2.965	-3.388	-4.14
UK	2	5	1	-6.017***	-2.831	-3.228	-3.984
Italy	4	1	1	-8.043***	-3.197	-3.601	-4.322
Portugal	5	1	1	-7.761***	-3.139	-3.508	-4.36
Sweden	3	4	4	-8.075***	-2.746	-3.126	-3.85
US	2	4	1	-7.028***	-2.896	-3.289	-4.142

Table A.11: Primary balance in level using double integer frequency (Cai and Omay, 2022)

	k^s	k^c	Lags	t stat.	10% cv	5% cv	1% cv
Canada	3	4	1	-5.970***	-2.733	-3.111	-3.866
UK	2	5	1	-5.744***	-2.872	-3.246	-4.024
Italy	2	5	1	-3.885**	-2.849	-3.24	-4.031
Portugal	1	2	1	-3.744*	-3.505	-3.879	-4.633
Sweden	3	4	1	-5.096***	-2.732	-3.104	-3.904
US	2	4	1	-6.298***	-2.882	-3.279	-4.032

Table A.12: Debt-to-GDP ratio in first difference using double fractional frequency (Cai and Omay, 2022)

	k^s	k^c	Lags	t stat.	10% cv	5% cv	1% cv
Canada	1.2	2.6	3	-7.342***	-3.272	-3.643	-4.409
UK	2	4.9	1	-6.104***	-2.823	-3.239	-3.981
Italy	1.5	2.9	1	-8.319***	-3.147	-3.545	-4.335
Portugal	0.7	4.7	1	-8.294***	-3.237	-3.599	-4.245
Sweden	3	4	4	-8.075***	-2.768	-3.123	-3.86
US	2	4.1	1	-7.101***	-2.903	-3.292	-4.036

Table A.13: Primary balance in level using double integer frequency (Cai and Omay, 2022)

	k^s	k^c	Lags	t stat.	10% cv	5% cv	1% cv
Canada	2.9	3.9	2	-6.865***	-2.758	-3.148	-3.901
UK	4.6	1.5	1	-6.182***	-2.942	-3.316	-4.035
Italy	3.6	5	1	-4.254***	-2.67	-2.994	-3.654
Portugal	2.2	4.8	2	-3.492***	-2.802	-3.198	-3.89
Sweden	3.5	2.7	2	-6.239***	-2.788	-3.178	-3.84
US	0.3	4.1	1	-6.466***	-3.288	-3.645	-4.326