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Robustly optimal monetary policy in a behavioral environment*

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Abstract

This paper studies robustly optimal monetary policy in a behavioral New Keynesian model, where the private sector has myopia, while the central bank has Knightian uncertainty about the degree of myopia of the private sector and the degree of price stickiness. In such a setup the central bank solves an optimal robust monetary policy problem. We show that under uncertainty in myopia the Brainard's attenuation principle holds, while under uncertainty on price stickiness, alone or in addition to myopia, monetary policy becomes more aggressive.

Keywords: Optimal monetary policy, bounded rationality, min-max, parameter uncertainty.

JEL Classification: E31, E52.

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1 Introduction

There is a wide consensus that uncertainty affects the conduct of monetary policy, but there is less agreement on its impact on interest rate decisions. The current pandemic emergency can be seen as an example of Knightian uncertainty that makes policy responses highly difficult, as recently highlighted by the former Bank of Canada's governor, Stephen Poloz, who declared "*The pandemic is an example of Knightian uncertainty that will also force us to reconsider many fundamental ideas about how our economy can and should function*" (Lecture, 2020). The first ever contribution to this question, (Brainard, 1967), has established what is called Brainard's attenuation principle according to which the presence of uncertainty implies an attenuated policy response compared to settings where uncertainty is not taken into account.[1] A more recent literature contested this result showing that uncertainty may lead to aggressive policy actions (Giannoni, 2002; Pellegrino et al., 2020). To rationalize 'aggressive responses' to uncertainty, Barlevy (2011) considers in the case of uncertainty around persistence and policy trade-off parameters, arguing that *aggressiveness is not an inherent feature of robustness but is specific to the models explored*.

This paper studies robustly optimal monetary policy in a behavioral New Keynesian model (NK) entailing cognitive discounting. We assess the robustly optimal policy considering uncertainty around the degree of cognitive discounting and the degree of price stickiness. Introducing uncertainty in the behavioral parameter seems natural, as we lack solid empirical evidence on its estimate. Further, as this parameter pertains to psychological underpinnings of individual decision-making, it could be subject to the Lucas (1976) critique as the policy action itself could alter the way households discount the future. More interestingly, this kind of uncertainty falls in the characterization of Barlevy (2011), with parameter uncertainty affecting the persistence of macroeconomic variables, through the expectations' channel.

One of the key challenges in central banking is how to conduct mon-

[1] Friedman (1953) is another example where the uncertainty around one's model specification translates into caution in using the model altogether.

etary policy in the presence of uncertainty. Academic research and empirical evidence suggest that there can be different dimensions of uncertainty, ranging from unobservable variables in real time, as in Orphanides (2001), Orphanides (2003a) and Orphanides (2003b), to the perceptions of key relationships describing the economy as in Hansen and Sargent (2015). Giannoni (2002), Tillmann (2009) and Traficante (2013) show that incomplete information about some key elements of the reference model for the economy leads to disagreement about the effects of monetary policy. As a consequence, for monetary policy to be robust, the appropriate interest rate setting must take into account the risk that the policymaker does not know the structure of the economy accurately.

More recently, the baseline new Keynesian model has been enriched with behavioral elements to take into account deviations from rational expectations equilibrium. A notable example is ”cognitive discounting”, or myopia, as in Gabaix (2014) and Gabaix (2020). When agents are myopic, they form expectations with *perceived* laws of motion. Compared to the rational expectations setup, behavioral agents have a lower discount factor due to the fact that they pay limited attention to events that are more distant into the future. The introduction of myopia allows to overcome some puzzles that usually emerge in standard models with rational expectations.[2] With myopic agents monetary policy decision-making becomes highly difficult, and stabilization outcomes could deviate substantially from expectations in a context where the central bank has incomplete information over the degree of this myopia.

To overcome incomplete information challenges, we assess the robustly optimal monetary policy under uncertainty around (*i*) Phillips' curve slope, through the price stickiness parameter, and (*ii*) agents' myopia, or cognitive discounting parameter. Previous studies have documented uncertainty about Phillips' curve slope. Bils and Klenow (2004) use US Bureau of Labor Statistics data and show the frequency of price adjustment for 350 categories of consumer goods and services which cover 70% of total consumers expenditure. They show that the median firm in their dataset

[2] For example, compared to the rational model, there is no forward guidance puzzle and the zero lower bound is much less costly.

changes prices every 4.3 months. This result is very different from what documented by Galı and Gertler (1999), who find an average price stickiness ranging from 1 year and a half to 2 years. Nakamura and Steinsson (2013) discuss several empirical evidence on price frequency changes. All these papers point to different estimates, making it plausible to assume uncertainty about this parameter. Uncertainty about the degree of price stickiness affects the central bank's perception about the slope of the aggregate supply and the relative weights assigned to the objectives in the loss function.

As to uncertainty about agents' myopia, Ilabaca et al. (2020) provide estimates and confidence intervals for the degree of myopia that can be used in our analysis of robust optimal monetary policy, showing that the extent of inattention is substantial, particularly with respect to inflation.

Using the interval of estimates for both cognitive discounting and price rigidity, we assess optimal robust monetary policy under discretion and commitment. Under discretion, robustness implies that the policy maker should set the policy rate considering a higher value of the cognitive discounting and an attenuated interest rate reaction to a cost-push shock. The same result proves to be valid also when assessing robust optimal monetary policy under commitment. Since the robustly optimal policy is based on higher value of cognitive discounting, the transmission from the shock to inflation and output is much stronger compared to the optimal equilibrium. The same mechanism applies to the feedback from interest rate to macro variables, which is much stronger under the robustly optimal policy equilibrium. As a result, interest rate reaction under this equilibrium is attenuated. In case of uncertainty with respect only to price rigidity, the robust policy is stronger compared to the full-information case. This result is not different from the conventional rational-expectations wisdom of the NK literature. However, when uncertainty concerns both cognitive discounting and price rigidity, policy is set more aggressively both under discretion and under commitment. We also show that Knightian uncertainty on myopia and price rigidity increases the area of equilibrium determinacy with respect to the rational expectations case. Our interpretation is that this result comes from the fact that, under Knightian uncertainty on myopia and price rigidity, output gap is less reactive to inflation. Therefore, it is not only bounded rationality, as in Gabaix (2020), but also parameter uncertainty that implies a unique equilibrium without the Taylor principle.

The remainder of the paper is organized as follows. Section 2 presents a brief overview of the model and the robustness approach. In Section 3, we assess the optimal robust monetary policy under the discretionary case with the presence of uncertainty. Section 4 tackles the same question under the case of commitment. Section 5 concludes.

2 A behavioral New Keynesian model

We draw on recent work by Gabaix (2020) on the NK model augmented by behavioral elements to reflect households and firms bounded rationality. The model features cognitive myopia ($\bar{m}$), which is common to households and firms. The behavioral mechanism is the cognitive discounting: the world is not fully understood by agents, especially events that are far into the future. To capture this mathematically, we make the following key assumption: agents, as they simulate the future, shrink their simulations toward a simple benchmark, namely, the steady state of the economy. This follows from a microfoundation in which agents receive noisy signals about the economy. As a result, an innovation happening in k periods has a direct impact on agents' expectations that is shrunk by a factor $\bar{m}^k$ relative to the rational expectations, where $\bar{m} \in [0, 1]$ is a parameter capturing cognitive discounting. Hence, innovations that are deep in the future get heavily discounted relatively to a rational expectations benchmark, which corresponds to the case $\bar{m} = 1$.

2.1 Households

The representative household maximizes the utility function depending on real consumption c_t and on hours worked N_t

$$U\left(c_t, N_t\right) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \tag{1}$$

where γ is the coefficient of the household's relative risk aversion, i.e., the inverse of the intertemporal elasticity of substitution, ϕ is the inverse of the Frisch elasticity of labor supply, i.e., the inverse of the elasticity of work effort with respect to the real wage.

Utility maximization is subject to the budget constraint

$$k_{t+1} = \left(1 + r_t\right)\left(k_t - c_t + y_t\right) \tag{2}$$

that expresses the law of motion for real financial wealth k_t as a function of the real interest rate r_t, consumption c_t and real income y_t. The key assumption in Gabaix (2014) and Gabaix (2020) is that the law of motion of the state vector X_t (possibly including shocks, monetary and fiscal policy) is perceived as

$$X_{t+1} = \overline{m} G^X\left(X_t, \epsilon_{t+1}\right) \tag{3}$$

where $\overline{m} \in [0,1]$ represents agents' myopia on the state of the economy, for some equilibrium transition function G^X and zero mean innovations ϵ_{t+1}. When $\overline{m} = 1$, the law of motion under rational expectations is obtained.

Linearizing equation (3) leads to the following relationship between objective and subjective expectations

$$\mathbb{E}_t^{BR} X_{t+k} = \bar{m}^k \mathbb{E}_t X_{t+k} \tag{4}$$

where $\mathbb{E}^{BR}$ stands for subjective behavioral expectation operator, which depends on rational expectations $\mathbb{E}$. By solving the household's problem with these assumptions, we obtain the following IS equation

$$x_t = M\mathbb{E}_t x_{t+1} - \sigma\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^e\right) \tag{5}$$

where x_t and π_t are output gap and inflation expressed as deviations from efficient levels, i_t is the nominal interest, r_t^e is the efficient level of the real interest rate, and M is equal to the cognitive discounting parameter $\bar{m}$.

2.2 Firms

The technology of production for firm i is expressed as follows

$$Y_t(i) = A_t N_t(i) \tag{6}$$

where A_t is the technological factor and N_t denotes labor inputs. These intermediate goods are combined to obtain the final good $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon}{\varepsilon-1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}$

The problem of the behavioral firm is to maximize its profit

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t^{BR} \left[\Lambda_{t,t+k}\left(P_t^* Y_{t+k|t} - \Psi_{t+k}\left(Y_{t+k|t}\right)\right)\right] \tag{7}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k} \tag{8}$$

where $\Lambda_{t,t+k} = \beta^k \left(c_{t+k}/c_t\right)^{-\gamma} \left(P_{t+k}/P_t\right)$ is the stochastic discount factor in nominal terms, $\Psi_{t+k}(.)$ is the cost function, $Y_{t+k|t}$ is the output in period $t+k$ for a firm that last reset its price in period t, P_t^* is the optimal price the behavioral firm seeks to determine and P_t is the price level of the overall economy.

Solving this problem and log-linearizing we obtain the Phillips curve

$$\pi_t = \beta M^f \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa x_t + u_t \tag{9}$$

where β is the static discount factor, $M^f = \bar{m}\left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right)$, $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\gamma+\phi)$ and u_t is a cost-push shock.

Summing up, the "behavioral" model differs from the standard New

Keynesian model because of the terms M, $M^f \in [0,1]$ that appear in the IS curve and in the supply curve respectively. These two coefficients are derived by Gabaix (2020) in a setup where agents use cognitive discounting in their intertemporal choices. The idea behind this is that agents do not have full knowledge of the true structure of the economy, especially for events that take place far in the future. Therefore, in making their forecasts, agents consider a simple benchmark consistent with noisy signals about the economy that they receive. These boundedly-rational consumers and firms discount more the variables that they believe to happen more in the future; to that extent, M and M^f are interpreted as degree of myopia.

2.3 Monetary policy

Central bank policy is evaluated in terms of a welfare loss depending on output gap and inflation. To achieve the equilibrium inflation and output gap, monetary policymaker sets the interest rate. The micro-founded welfare loss measure is derived as the second order approximation for households' utility, and it is expressed as follows

$$\mathbb{W} = \frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) \qquad \vartheta = \frac{\kappa}{\epsilon} \tag{10}$$

where the weight attached to output gap stabilization depends on Phillips' curve slope κ and price elasticity of demand for the individual goods ϵ.

At the time of decision-making, the policy maker does not have a perfect knowledge about some parameter vector, defined with v, with Θ denoting the set of all possible values of v. The central bank, in this model, is playing a zero sum game against a fictitious evil agent who sets v in such a way to maximize the welfare loss. Optimal robust policy sets interest rate, or alternatively output and inflation[3], in a way to minimize the welfare

[3] In our setup it is equivalent to consider inflation and output gap as control variables and then find the interest rate consistent with the optimal policy. See for example Clarida et al. (1999).

loss resulting from the worst case scenario

$$\min_{p} \max_{v \in \Theta} \mathbb{E}W(p(v)) \tag{11}$$

In the analysis we conduct below, the policy variable p corresponds to the interest rate set by the central bank, or equivalently inflation and output gap levels. The vector v of uncertain parameters will include cognitive discounting $\bar{m}$ and price rigidity θ. We first assume that there is Knightian uncertainty either on $\bar{m}$ or on θ, and then we consider the case of uncertainty on both parameters.

3 Robustness under discretion

In this section we study robust optimal monetary policy under discretion. We consider uncertainty regarding both cognitive discounting and price stickiness.

3.1 Cognitive discounting uncertainty

The central bank minimizes the welfare loss related to the decision period, taking into account that expectations are given, which yields to the following first-order condition

$$\pi_t = -\frac{\vartheta}{\kappa} x_t \tag{12}$$

If the central bank takes into account the "endogenous" nature of the final objectives in the loss function, along the lines of Walsh (2005) and Traficante (2013), the optimal trade–off under discretion (12) can be rewritten in terms of structural parameters as

$$x_t = -\epsilon \pi_t \tag{13}$$

Inserting (13) into the Phillips curve (9) and rearranging, we can get the equilibrium solution for inflation and output gap

$$\pi_t = \frac{1}{1 - \beta M^f \rho_u + \kappa\epsilon} u_t \tag{14}$$

$$x_t = -\frac{\epsilon}{1 - \beta M^f \rho_u + \kappa\epsilon} u_t \tag{15}$$

Therefore, the unconditional welfare–based loss function will be

$$L = \frac{1 + \kappa\epsilon}{\left(1 - \beta M^f \rho_u + \kappa\epsilon\right)^2} \sigma_u^2 \tag{16}$$

Given (14)-(15), the policymaker can use the IS equation (5) to set the desirable interest rate and to achieve the optimal inflation and output gap:

$$i_t = r_t^e + \frac{\frac{\epsilon}{\sigma}\left(M\rho_u - 1\right) + \rho_u}{1 - \beta M^f \rho_u + \kappa\epsilon} \tag{17}$$

However, when the central bank has uncertainty about $\bar{m}$, which influences inflation and output gap in equilibrium, it is not clear how to set the policy rate to achieve an optimal outcome in face of uncertainty.

To take into account the uncertainty facing the central bank, the policy maker can conjecture the worst parameter constellation, that delivers the maximum welfare loss, which solves the problem

$$\max_{\bar{m}} \frac{1 + \kappa\epsilon}{\left(1 + \kappa\epsilon - \beta M^f \rho_u\right)^2} \sigma_u^2 \tag{18}$$

By solving this unconstrained optimization problem, we obtain the $\bar{m}$ value leading to the worst-case scenario regarding welfare loss.

Proposition 1 *If the policy maker is uncertain about $\bar{m}$, a robust policy should be based on $\bar{m} = \bar{m}^{max}$.*

Proof. See Appendix A.1. ■

We solve the model numerically, setting $\bar{m} \in [0.49, 0.92]$ as in (Ilabaca et al., 2020). Accordingly, the worst case belief of the central bank about bounded rationality is materialized with the highest value for $\bar{m} = \bar{m}^{max} = 0.92$. From this proposition it is clear that the cognitive discounting value, obtained for the robust policy, tends to be close to the value under rational expectations ($\bar{m} = 1$). Consequently, a central bank should have only little concerns about cognitive discounting uncertainty when setting its monetary policy.

We compare the equilibrium outcomes derived under information uncertainty with those with full information, that arise with $\bar{m} = 1$, where the apex r denotes the equilibrium under full information and rational expectations:

$$\pi_t^r = \frac{1}{1 - \beta\rho_u + \kappa\epsilon} u_t \tag{19}$$

$$x_t^r = -\frac{\epsilon}{1 - \beta\rho_u + \kappa\epsilon} u_t \tag{20}$$

$$i_t^r = r_t^e + \frac{\frac{\epsilon}{\sigma}(\rho_u - 1) + \rho_u}{1 - \beta\rho_u + \kappa\epsilon} \tag{21}$$

It is possible to observe that the amplitude of inflation reaction to cost-push shock under the behavioral model (equation 14) is lower than under the rational expectations model (equation 19), while the response of output gap is larger under the behavioral model, as can be seen by comparing (15) with (20). Computing the difference of i_t and i_t^r will produce cumbersome results. For this reason we resolve to drawing the impulse response function of the optimal policy.

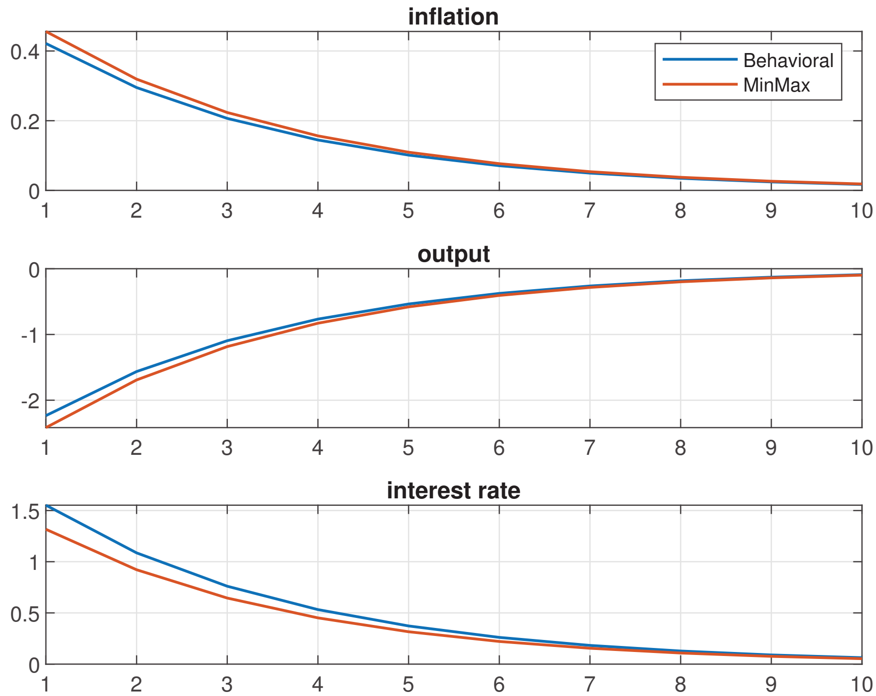
Table 1: Calibration.

Parameter	Calibration	Description
$\bar{m}$	0.7059	Cognitive discounting parameter
β	0.99	Static discount factor
γ	1	Household's relative risk aversion
ε	6	Elasticity of substitution between goods
ϕ	1	Frisch elasticity of labor supply
θ	2/3	Probability of firms not adjusting prices
ρ_u	0.7	Persistence of the cost-push shock

Source: Galí (2015), Ilabaca et al. (2020).

We simulate the model using standard values in the literature and previous work such as Traficante (2013), Gabaix (2020) and Benchimol and Bounader (2019). In particular, as shown in Table 1 we set discount factor $\beta = 0.99$, the demand elasticity $\epsilon = 5.3$, while the coefficient of risk aversion γ and the inverse of Frisch elasticity are both equal to one. As to attention parameters, cognitive discounting $\bar{m} = 0.7059$, which is the value estimated by Ilabaca et al. (2020) for the period 1982–2007 that guarantees determinacy of equilibria. This estimate oscillates in the interval $[0.49, 0.92]$. We now consider the calibration of Calvo parameter θ which affects several other coefficients in the model, such as Phillips' curve slope, relative welfare weight on output and cognitive discounting of firms M^f. We assume that survival rates of prices $\theta = \frac{2}{3}$, while we will introduce uncertainty also about this parameter in the next sections. Finally, we set the value for the autocorrelation of cost–push shock $\rho_u = 0.7$; its standard deviation is assumed, without loss of generality, equal to one. Figure 1 shows the dynamic response to a cost-push shock under the optimal policy in the behavioral New Keynesian model (blue line), and the robust optimal policy with uncertainty around cognitive discounting parameter (red line). The figure shows that, compared to the full information case, in the minmax case inflation increases more and output decreases more: this overall larger volatility is consistent with a milder interest rate response to the cost-push shock. In the minmax case, expected inflation is discounted more and this amplifies the effect of the shock. To the extent that $\bar{m}$ is higher under the min-max equilibrium, the transmission from the shock to inflation and output is much stronger compared to the optimal equilibrium. As a result, deviations of inflation and output under the min-max are larger. The interest rate reaction to these deviations is weaker under the minmax policy, given the higher $\bar{m}$ that makes the transmission channel of monetary policy to macro variables much stronger. Therefore, these results are in line with Brainard (1967)'s attenuation principle: in face of uncertainty regarding the cognitive discounting parameter, the robust optimal policy implies an interest rate reaction which is lower than what is suggested in the baseline behavioral model. The lower interest rate response implies that the central bank stabilizes less inflation and output gap under Knightian uncertainty on cognitive discounting.

Figure 1: Impulse response function to a cost-push shock under (robust) optimal policy with uncertain cognitive myopia $\bar{m}$.



3.2 Price rigidity uncertainty

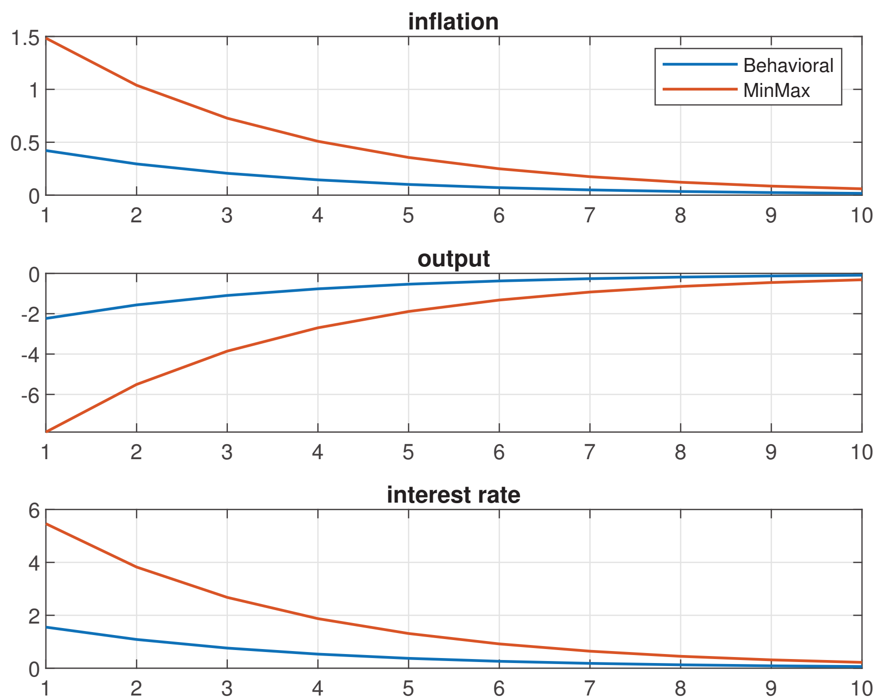
In this section we assume that the only source of uncertainty for the central bank is price rigidity. Therefore, as a second step of the min–max problem, evil agent will choose the value of θ that maximizes the loss function.

Proposition 2 *If the policy maker is uncertain about θ, a robust policy should be based on $\theta = \theta^{max}$.*

Proof. See Appendix A.2. ■

Figure 2 shows the dynamic response to a cost-push shock under the optimal policy in the behavioral NK model (blue line) and the robust optimal policy with uncertain price stickiness, that implies picking a value of

Figure 2: Impulse response function to a cost-push shock under (robust) optimal policy with uncertain price stickiness θ.



$\theta = \theta^{max}$ (red line). Knightian uncertainty about this parameter is modelled by assuming that $\theta \in [\theta_L, \theta_H]$, with $\theta_L = 0.3$ and $\theta_H = 0.9$ respectively, as in Traficante (2013), while the rest of calibration comes from Table 1. We observe that the high price rigidity in the min-max equilibrium induces a larger reaction of interest rates, inflation and the output gap. By comparing this simulation with that in Figure 1, it is possible to notice that the overall volatility increases much more when there is uncertainty in price rigidity. We interpret this finding by looking at the worsened trade-off that arises with Knightian uncertainty on the degree of price rigidity: in such a case the Phillips curve becomes less elastic and more persistent (because of the higher θ in minmax equilibrium). Differently from the case of uncertain $\bar{m}$, however, the Brainard principle does not hold here and the interest rate is set much more aggressively.

3.3 Joint uncertainty in cognitive discounting and price stickiness

Here we consider jointly the uncertainty about price stickiness and myopia to evaluate how the economy responds; in particular, we are interested to see if the interest rate becomes more aggressive (as in the case of uncertain θ) or less aggressive (as in the case of uncertain $\bar{m}$).

Under this case, the problem of the central bank becomes

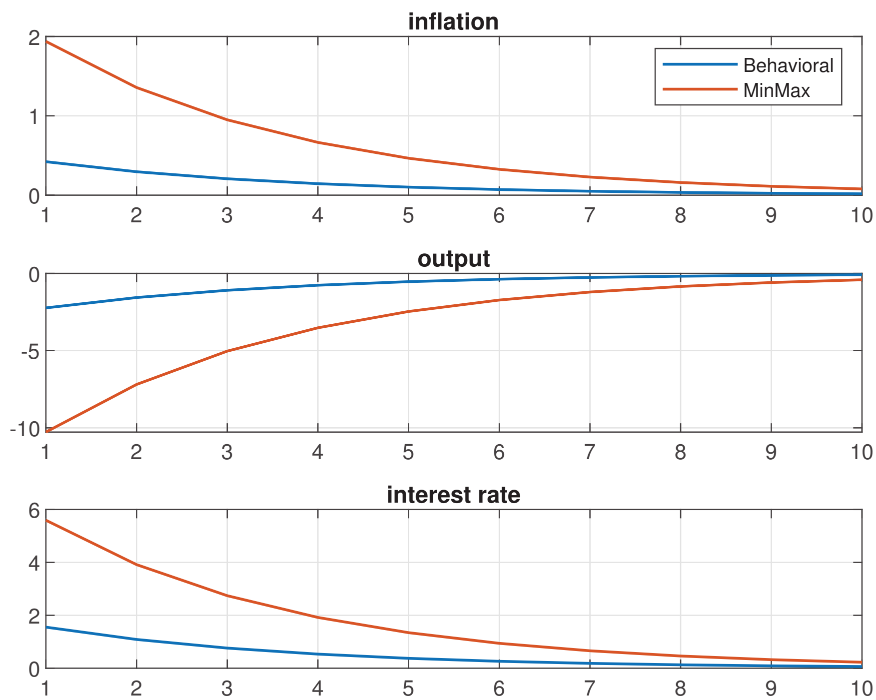
$$\max_{\bar{m},\theta} \frac{1 + \kappa\epsilon}{\left(1 + \kappa\epsilon - \beta M^f \rho_u\right)^2} \sigma_u^2 \tag{22}$$

Following the same steps as in the previous sections, we find, numerically, that optimal robust policy implies picking $\bar{m} = \bar{m}^{max}$ and $\theta = \theta^{max}$:

Proposition 3 *If the policy maker is uncertain about $\bar{m}$ and θ, jointly, a robust policy should be based on $\bar{m} = \bar{m}^{max}$ and $\theta = \theta^{max}$.*

The impulse responses presented in figure 3 show that the dynamics are very close to the case in which $\bar{m}$ is known with certainty. Quantitatively, uncertainty on theta weighs more and implies a more aggressive response to cost-push shock, as highlighted in figure 2.

Figure 3: Impulse response function under full information and uncertainty on private sector's myopia and price rigidity



4 Robustness under commitment

In this section we study robustness of monetary policy to parameter uncertainty related to myopia under the case where the central bank is operating under commitment. We first consider the commitment to a simple rule and then the optimal commitment.

4.1 Commitment to non-inertial rules

We follow the procedure developed by Giannoni (2002) and we restrict our attention to the class of rules of the form

$$i_t = \psi_\pi \pi_t + \psi_x x_t \tag{23}$$

This rule is of the form of Taylor (1993), in which the interest rate is expressed in deviation to a steady-state. We use (23) to substitute for the interest rate in the IS equation (5) and we can write the equation system as follows

$$E_t z_{t+1} = \Lambda z_t + \tau \gamma_t \quad z_t = [\pi_t, x_t]' \quad \gamma_t = [u_t, r_t^e]' \tag{24}$$

where

$$\Lambda = \begin{pmatrix} \frac{1}{\beta M^f} & \frac{-\kappa}{\beta M^f} \\ \frac{\sigma(\beta M^f \psi_\pi - 1)}{\beta M M^f} & \frac{\beta M^f (1+\rho\psi_x)+\sigma\kappa}{\beta M M^f} \end{pmatrix}$$

For the system (24) to be determinate, it is sufficient to check that the eigenvalues of the matrix Λ are inside the unit circle. As shown in Gabaix (2020), this occurs when the following condition holds

$$\psi_\pi + \frac{1-\beta M^f}{\kappa}\psi_x + \frac{\left(1-\beta M^f\right)(1-\beta M)}{\kappa\sigma} > 1 \tag{25}$$

Based on (25), Gabaix (2020) shows that, when monetary policy is passive (i.e., when $\psi_\pi = \psi_x = 0$), we have a determinate equilibrium if and only if bounded rationality is strong enough, meaning

$$\frac{\left(1-\beta M^f\right)(1-\beta M)}{\kappa\sigma} > 1 \tag{26}$$

Compared to models with fully rational agents, where $M = M^f = 1$, the presence of bounded rationality reduces the occurrence of multiple equilibria because agents are less reactive to their future decisions. Here we discuss what occurs in the case of optimal robust equilibrium, in the case in which the central bank has Knightian uncertainty on θ and $\bar{m}$. We start with the case in which the central bank is uncertain only about $\bar{m}$. In the latter case, we have shown that the optimal robust policy implies that the central bank considers that the subjective discount factor hinges the largest value. For a larger value of $\bar{m}$, it is easy to show that the threshold (26) shrinks with respect to Gabaix (2020), so that when the central bank is uncertain about how agents discount the future, they are less succesful in attaining a unique equilibrium. Interestingly, the Knightian uncertainty with respect to $\bar{m}$ implies that we are closer to the benchmark case of rational expectations, where a less than proportional response to the inflation rate delivers indeterminacy.

Now we consider the case in which uncertainty is about both θ and $\bar{m}$. While we have just shown that the condition (26) shrinks for Knightian uncertainty on myopia, the evaluation is more difficult for the case in which price stickiness is uncertain, since this parameter influences the slope of IS and Phillips curve. By differentiating (26) with respect to θ we have

$$-\frac{1-\beta\bar{m}}{\kappa^2\sigma}\left(\beta\kappa\underbrace{\frac{\partial M^f}{\partial\theta}}_{>0}+\left(1-\beta M^f\right)\underbrace{\frac{\partial\kappa}{\partial\theta}}_{<0}\right) \tag{27}$$

Taking into account that in the optimal robust equilibrium both θ and $\bar{m}$ take their maximum values, ex ante it is not clear if the joint uncertainty of these two parameters implies that the threshold under no response to inflation increases or decreases. In particular, if the effect of θ on the slope of Phillips curve dominates, then the threshold increases. Using the calibration described above we have that this is the case, so that uncertainty on both price rigidity and myopia increases the determinacy area for any policy that does not respond at all to inflation. Notice that this is in line with what found by Gabaix (2020), even if the mechanism here is different. In Gabaix (2020), the presence of myopia implies that agents are less reactive to the future. As a consequence, we have a reduction in complementarity between agents' consumption choices that dampens the possibility of multiple equilibria. Differently, in the model with Knightian uncertainty on myopia, the latter effect is less strong, and what explains the increase in the determinacy area is the fact that under uncertain price rigidity implies that output gap is less reactive to inflation.

4.2 Optimal commitment

Here we derive the solution for output and inflation consistent with optimal commitment, ie when the central bank minimizes the loss function assuming that it canmanipulate private sector expectations of the future. This implies that the optimal choices of π_t and x_t do not depend only on the contemporaneous value of the shock u_t, but they dependon the entire history of shocks.

$$\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right) \tag{28}$$

subject to the Phillips curve Eq. (9). The FOCs of this problem are

$$\pi_t + \phi_t - M^f \phi_{t-1} = 0 \tag{29}$$

$$\vartheta x_t - \kappa \phi_t = 0 \tag{30}$$

where ϕ_t is the Lagrange multiplier associated with the Phillips curve equation. Solving for ϕ_t in Eq. (30), replacing it in Eq. (29) we obtain the following targeting rule

$$\pi_t = -\frac{\vartheta}{\kappa} x_t + \frac{\vartheta M^f}{\kappa} x_{t-1} \tag{31}$$

Proposition 4 *The interest rate rule that implements the commitment policy is* *of the form*

$$i_t = r_t^e + \psi\pi_t + \frac{1}{\sigma}(M\psi - 1)\, x_t + \frac{\psi}{1-\beta\psi\rho_u}\left(1 - \rho_u + \frac{M\kappa\rho_u}{\vartheta}\right) u_t \qquad (32)$$

where $\psi = \frac{1-\sqrt{1-4\beta\alpha^2}}{2\alpha\beta}$ *and* $\alpha = -\frac{M^f}{\vartheta^2\left(1+\beta(M^f)^2\right)+\vartheta\kappa^2}$

Proof. See Appendix A.3 ■

To assess the minmax solution, we revert in this section to a quantitative exercise. We use the three-equation model so far developed; IS, PC, and interest rate rule equations (Eqs. 5, 9, and 32).

Uncertainty in cognitive discounting

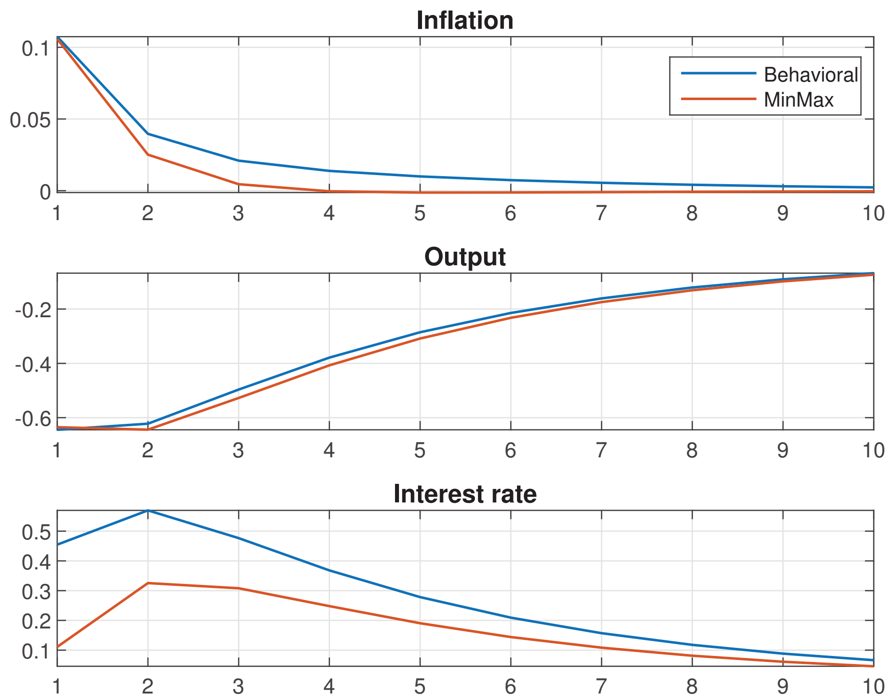
To derive the worst-case scenario regarding $\bar{m}$, we compute welfare losses numerically rather than analytically. Regarding the welfare loss in each of these parametric constellations $\bar{m} \in \{0.49, 0.7, 0.92\}$, table 2 below shows clearly that the case of $\bar{m} = \bar{m}^{max} = 0.92$ is delivering the highest welfare loss, which is consistent with the result under discretion.

Table 2: Welfare loss under different myopia parameters

	Higher myopia	Behavioral-Baseline-	Lower myopia
Myopia values	**0.49**	**0.7**	**0.92**
Welfare loss	0.154729	0.14594	0.15949

Figure 4 reports the impulse response function of the commitment case under the baseline and the minmax calibrations.

Figure 4: Impulse response function to a cost-push shock under uncertainty relative to myopia $\bar{m}$.



While output reactions are almost identical under baseline and minmax equilibria, inflation response looks quite attenuated under the minmax equilibrium after a couple of periods following the shock. This is a result of the dynamics of the interest rate, which reacts less under the minmax compared to the baseline. One explanation of this result lies in the amplitude of the feedback from expectations to contemporaneous variables. Since the minmax equilibrium has a higher $\bar{m}$, the effects of expectations become stronger and, hence, interest rate needs to react less. Therefore, the interest rate reaction is lower compared to the baseline case suggesting that, under this kind of uncertainty, the Brainard principle is well and alive .

Uncertainty in price rigidity

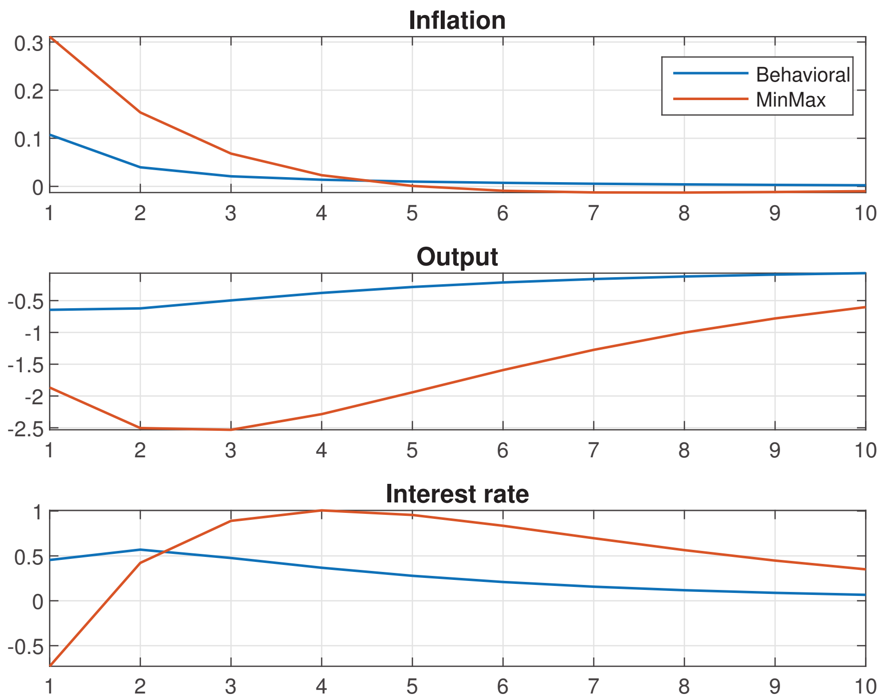
We follow the same quantitative approach as in the case of cognitive discounting uncertainty. Here we compute the welfare loss varying the degree of price rigidity, showing the results in the table below

Table 3: Welfare loss under different price rigidity parameters

	Higher θ	Baseline θ	Lower θ
θ **values**	**0.9**	**3/4**	**0.3**
Welfare loss	0.57	0.15	0.01

As in the case of discretion, the worst-case scenario seems to be achieved under $\theta = \theta^{max}$ according to the table 3. Figure 5 reports the impulse response function of the commitment case under the baseline and the min-max equilibria with $\theta \in \{3/4; 0.9\}$. Because of higher price rigidity under Knightian uncertainty, interest rate reactions to cost-push shocks are more aggressive not only compared to the case of uncertainty, but also with respect to uncertain cognitive discounting. This result goes against the Brainard principle.

Figure 5: Impulse response function to a cost-push shock under uncertainty relative to price rigidity.



Joint uncertainty in cognitive discounting and price rigidity

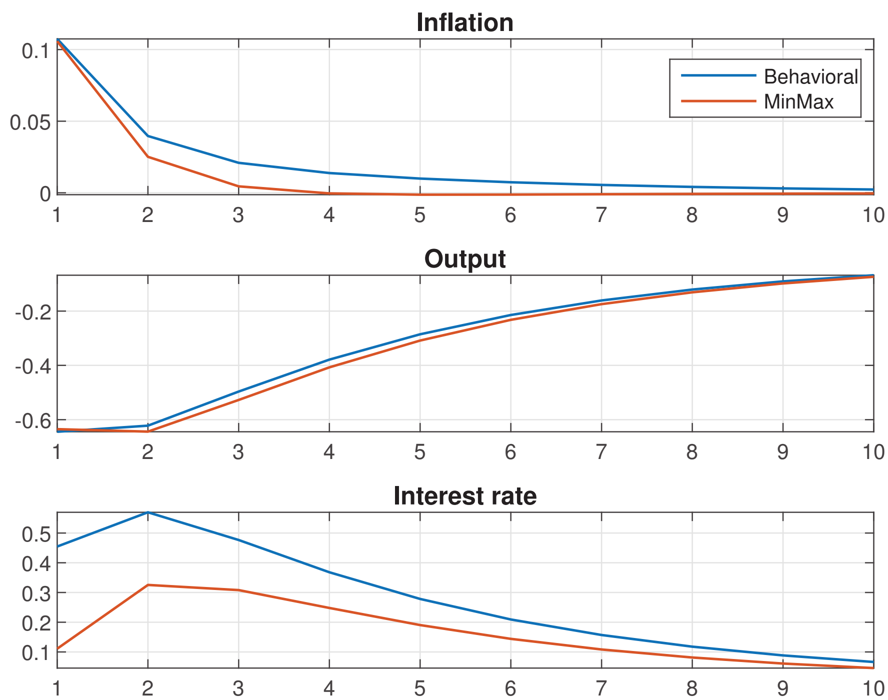
The welfare loss computations of different parameter combinations, in table 4, indicate that the worse-case scenario is obtained when $\bar{m} = \bar{m}^{max}$ and $\theta = \theta^{max}$.

Table 4: Welfare loss under different parameter combinations

	Low θ	**High θ**
Low $\bar{m}$	0.01	0.41
High $\bar{m}$	0.01	0.67

Figure 6 shows the impulse response function in case of Knightian uncertainty surrounding both price rigidity and cognitive discounting. We observe that uncertainty on price rigidity dominates the effect implied by uncertain cognitive discounting: except for the on impact response, interest rates are moved more aggressively, in contrast with the Brainard principle. Interestingly, in the aftermath of the positive cost-push shock, the central bank lowers the interest rate and probably this implies that afterwards policy must be set more aggressively. The joint uncertainty on $\bar{m}$ and θ induces lower stabilization of output and inflation, despite the more aggressive policy stance.

Figure 6: Impulse response function to a cost-push shock under uncertainty relative to cognitive myopia $\bar{m}$.



5 Conclusion

This paper analyzes optimal robust monetary policy in a behavioral new Keynesian model where agents have bounded rationality, expressed in terms of myopia towards future events. Central bank has Knightian uncertainty about the Phillips' curve slope – because it does not know how often they can set their prices optimally – and how agents discount future variables. The central bank tries to minimize the impact of this strong uncertainty by following minmax policies, with the objective to minimize the impact of the worst-case scenario.

Our findings show that robustness implies that the central bank should set its policy considering that cognitive discounting takes on its highest value in the range of uncertainty. This implies that central bank should assume a lower myopia to the future, which brings about an attenuated interest rate reaction to a cost-push shock. Following the notation commonly used in this literature, in such a case, the Brainard principle holds. On the other hand, uncertainty on price rigidity induces an optimal robust policy that is more aggressive compared to the full-information case. In the case in which uncertainty concerns both cognitive discounting and price rigidity, policy becomes more aggressive, in contrast with the Brainard's principle. These results hold also in the case of optimal policy in a commitment regime.

When the central bank follows a simple rule, we show that the joint uncertainty on myopia and price rigidity reduces the occurrence of multiple equilibria. In a setup in which the degree of myopia is known by the central bank, Gabaix (2020) has a similar finding because bounded rationality reduces complementarity between agents' consumption choices. In our paper the mechanism is more related to the fact that, under Knightian uncertainty on myopia and price rigidity, output gap is less reactive to inflation. Therefore, it is not only bounded rationality but also parameter uncertainty that imply determinate and stable equilibria without the the Taylor principle.

Summing up, this paper is a theoretical exercise to analyze how the central bank should act in presence of Knightian uncertainty on bounded rationality and price rigidity. Of course, there are several theoretical and empirical questions that can be explored starting from our study. Considering how the central bank learns over time the uncertain parameters is just one of the possible future extensions for future work.

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A Proofs

A.1 Proposition 1

The evil agent's problem is the following

$$\max_{\bar{m}} \frac{1+\kappa\epsilon}{\left(1+\kappa\epsilon-\beta M^f\rho_u\right)^2}\sigma_u^2 \tag{33}$$

We compute the FOC with respect to $\bar{m}$ obtaining:

$$\frac{\partial L}{\partial \bar{m}} = \frac{-2(1+\kappa\epsilon)\left(-\beta\rho_u\frac{\partial M^f}{\partial \bar{m}}\right)}{\left(1+\kappa\epsilon-\beta M^f\rho_u\right)^3}\sigma_u^2 \tag{34}$$

where

$$\frac{\partial M^f}{\partial \bar{m}} = \theta + \frac{(1-\beta\theta)(1-\theta)}{1-\beta\theta\bar{m}} + \frac{\beta\theta\bar{m}(1-\beta\theta)(1-\theta)}{(1-\beta\theta\bar{m})^2} \tag{35}$$

It is easy to verify that $\frac{\partial M^f}{\partial \bar{m}} > 0$. Moreover, since the denominator is surely positive ($\beta M^f\rho_u$ is the product of three positive parameters less than one) we can conclude $\frac{\partial L}{\partial \bar{m}} > 0$.

A.2 Proposition 2

The evil agent's problem in this case is the following

$$\max_{\theta} \frac{1+\kappa\epsilon}{\left(1+\kappa\epsilon-\beta M^f\rho_u\right)^2}\sigma_u^2 \tag{36}$$

The parameter θ enters in coefficient κ, hence the derivative of the loss function (16) with respect to θ will be

$$\frac{\partial L}{\partial \theta} = \left\{-\frac{2\left(1+\kappa\epsilon\right)\left(\epsilon\frac{\partial\kappa}{\partial\theta}-\beta\rho_u\frac{\partial M^f}{\partial\theta}\right)}{\left(1-\beta M^f\rho_u+\kappa\epsilon\right)^3} + \frac{1+\epsilon\frac{\partial\kappa}{\partial\theta}}{\left(1-\beta M^f\rho_u+\kappa\epsilon\right)^2}\right\}\sigma_u^2 \tag{37}$$

which can be simplified as follows

$$\frac{\partial L}{\partial \theta} = \left\{ \frac{-\left(1+\beta M^f \rho_u + \kappa\epsilon\right)\epsilon\frac{\partial \kappa}{\partial \theta} + 2\left(1+\kappa\epsilon\right)\beta\rho_u\frac{\partial M^f}{\partial \theta} + 1 - \beta M^f \rho_u + \kappa\epsilon}{\left(1-\beta M^f\rho_u + \kappa\epsilon\right)^3} \right\} \sigma_u^2 \tag{38}$$

Now we write down the expression for derivatives inside

$$\frac{\partial \kappa}{\partial \theta} = \frac{\beta\theta^2 - 1}{\theta^2}(\gamma + \phi) < 0 \tag{39}$$

and,

$$\frac{\partial M^f}{\partial \theta} = 1 + \bar{m}\frac{\beta\bar{m} - (\beta\theta)^2\bar{m} + 2\beta\theta - \beta - 1}{(1-\beta\theta\bar{m})^2} \tag{40}$$

By working on (40), we can show that the derivative is positive if

$$\beta\theta^2\bar{m} - 2\theta + 1 > 0$$

which occurs for values of $\theta < \frac{2-\sqrt{4-4\beta\bar{m}}}{2\beta\bar{m}}$, while the other analytical condition that implies a positive sign for the inequation, ie $\theta > \frac{2+\sqrt{4-4\beta\bar{m}}}{2\beta\bar{m}}$ is ruled out because this second root is always larger than one.

Therefore, in this case, the sign of the loss function derivative is unclear. The whole denominator should be evaluated. By replacing (39) and (40) in (38) and simplifying, we obtain

$$\begin{aligned}\frac{\partial L}{\partial \theta} \propto & -\left(1+\beta M^f\rho_u + \kappa\epsilon\right)\epsilon\left(\beta\theta^2 - 1\right)(\gamma+\phi)(1-\beta\theta\bar{m})^2 \\ & + 2\left(1+\kappa\epsilon\right)\beta\rho_u\bar{m}\theta^2\left(\beta\bar{m} - (\beta\theta)^2\bar{m} + 2\beta\theta - \beta - 1\right) \\ & + \left(1-\beta M^f\rho_u + \kappa\epsilon\right)\theta^2(1-\beta\theta\bar{m})^2\end{aligned}$$

Evaluating each term of the above derivative reveals that $\frac{\partial L}{\partial \theta} > 0$. Thus, we obtain the proposition.

A.3 Proposition 4

The policy problem of the central bank in this setup is the following

$$\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right) \tag{41}$$

subject to the Phillips curve Eq. (9).

As in the main text, the FOCs of this problem could be expressed as a targeting rule in a single equation

$$\pi_t = -\frac{\vartheta}{\kappa} x_t + \frac{\vartheta M^f}{\kappa} x_{t-1} \tag{42}$$

We replace π_t in the Phillips curve Eq. (9), which yields the following difference equation

$$-\left(\frac{\vartheta}{\kappa} + \beta\frac{\vartheta}{\kappa}(M^f)^2 + \kappa\right) x_t = -\frac{\vartheta}{\kappa} M^f x_{t-1} - \beta M^f \frac{\vartheta}{\kappa} \mathbb{E}_t x_{t+1} + u_t \tag{43}$$

By changing the notation to $\alpha = -\frac{M^f}{\vartheta^2\left(1+\beta(M^f)^2\right)+\vartheta\kappa^2}$ and rearranging, this can be written as

$$x_t = \alpha x_{t-1} + \alpha\beta\mathbb{E}_t x_{t+1} - \frac{\alpha\kappa}{\vartheta} u_t \tag{44}$$

The stationary solution of this equation is

$$x_t = \psi x_{t-1} - \frac{\kappa\psi}{\vartheta(1-\beta\psi\rho_u)} u_t \tag{45}$$

where $\psi = \frac{1-\sqrt{1-4\beta\alpha^2}}{2\alpha\beta}$.

Plaguing this expression to the Phillips curve yields

$$\pi_t = \psi\pi_{t-1} + \frac{\psi}{1-\beta\psi\rho_u}(u_t - u_{t-1}) \tag{46}$$

Solving the IS Eq. (5) for the interest rate, we obtain

$$i_t = -\frac{1}{\sigma}(x_t - M\mathbb{E}_t x_{t+1}) + \mathbb{E}_t \pi_{t+1} + r_t^e$$

We now replace the expressions for x and π Eqs. (45)-(46) in the interest rate equation above, which completes the proof of this proposition.