INFER WORKING PAPER SERIES

INTERNATIONAL NETWORK FOR ECONOMIC RESEARCH

No. 11 | 2021

A Theoretical Foundation for Prudential Authorities Decision Making

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A Theoretical Foundation for Prudential Authorities Decision Making

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October, 26, 2021

Abstract

In the aftermath of the Global Financial Crisis, financial regulation uses micro- and macro-prudential rules, most of the time motivated by empirical studies. This article suggests a theoretical explanation for countercyclical and progressive capital requirements that incorporate micro- and macro-prudential stabilization objectives. The Capital Adequacy Ratio (CAR) imposed to individual banks by a Prudential Authority (PA) would thus represent an optimal regulation whose aim is to avoid individual and systemic risk accumulation by imposing minimal constraints to financial institutions. This corresponds to the implementation of optimal time-varying prudential capital requirements to banks, with non-linear structure, that allows PA to take progressive countercyclical actions in order to insure financial stability. We also test the mechanism in a DSGE model and show that it would be more suitable for the financial and real stability compared to the existing fixed prudential ratios.

Keywords: prudential regulation model, optimal CAR, time-varying capital requirements, DSGE model

JEL Code : E44, G21, G28

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1 Introduction

The Global Financial Crisis of 2008 shed light on important issues from the financial regulation perspective. Microprudential policies are not sufficient to manage financial risk (Galatti and Moessner, 2013). The risk-taking behaviour of banks generates systemic risk accumulation and the occurrence of financial crisis (Pontell at al., 2014). The lack of macroprudential regulation facilitates such unsuitable accumulation of systemic risk. To deal with these aspects, Basel committee has decided to implement specific macroprudential tools in Basel III agreements. We note among them additional capital buffers , as the capital conservation buffer and the countercyclical capital buffer, in particular.[1] As defined by the European System of Financial Supervision, the capital conservation buffer represents additional 2.5% capital requirements whose objective is to conserve banks' capital and thus limit financial instability. When a bank breaches the buffer, automatic mechanisms apply to limit the amount of dividend and bonus payment it can make. The countercyclical capital buffer is an additional flexible buffer that depends on the financial cycle, designed to counter pro-cyclicality in the financial system. Capital is supposed to accumulate when cyclical systemic risk is increasing and creates buffers to increase the resilience of the banking sector during periods when losses materialise. This could help to maintain the credit supply in periods of stress and dampen excessive credit growth during financial booms.

The implementation of these two additional buffers adjusts the original microprudential capital requirements or capital adequacy ratio (CAR) by taking into account macroeconomic issues and the financial cycle.[2] Cosimano and Hakura (2011) or Lim et al. (2011) provide an empirical evaluation of these macroprudential tools and show that their efficiency differs across countries and depends on the financial boom or bust periods. Kashyap et al. (2014), Agénor and Da Silva. (2017) or Hassine and Rebei (2019) also analyse the benefits of such policy tools for financial stability. However, as for the previous microprudential ratios, the quantitative levels fixed by prudential authorities for macroprudential capital requirements can be questioned.

To explain the level of capital requirements for instance, prudential regulators try to take up the challenge with empirical exercises such as stress tests for macroprudential requirements (Acharya et al., 2014) or statistical assumptions for microprudential requirements (Risk Weighted Asset follows a Gaussian law). But what are the theoretical

[1]Additional capital requirements may also be defined for Systemically Important Financial Institutions, depending on their risk for the financial system stability, from a national or international perspective.

[2]Thus, in Basel III agreement the fixed capital requirements ratio(including the conservation buffer) represents 10.5% of the Risk Weighted Assets (RWA) instead of 8% previously and additional discretionary countercyclical capital buffer between 0 and 2.5% may be added depending on the national financial cycle.

reasons to explain Basel prudential ratios ? Is the PA right to discretionary apply positive capital buffers over the existing fixed prudential ratios instead of defining time-varying capital requirements, such as suggested by Shleifer and Vishny (2010), for instance? What are the criteria to be used for optimizing the prudential regulation?

In our article, we address such questions and propose to compare the stabilization performances of two different prudential regulations. The first one is a prudential rule that uses fixed capital buffers (discretionary defined by the PA), while the second one is a time-varying rule for capital requirements implemented by the PA. In order to compare the stabilization performance of the two regulations, we choose the dynamic stochastic general equilibrium (DSGE) framework previously proposed by Gerali et al. (2010)). This choice is motivated by the structure of this model, which includes financial intermediaries and a prudential regulation based on fixed capital ratios. It is, indeed, the benchmark model for the analysis of a macroprudential regulation that uses ”capital requirements”. The prudential constraint simply introduces, in the banks' profit function, a quadratic cost which depends on the spread between the current Capital Adequacy Ratio (CAR) of the bank and the expected 10.5% level defined by Basel III (including the conservation buffer). Each deviation of banks' capital from the fixed CAR imposed by the regulator is costly and these costs constrain banks to follow the regulation. The model of Gerali et al. (2010) is estimated on European data and show how a prudential regulation based on fixed capital buffer would have contributed to regulate the financial system during the 2007-2009 financial crisis.

However, the very simple representation of Gerali et al. (2010) for a macroprudential policy based on the minimum level of capital requirements corresponds only to the consideration of the capital conservation buffer and does not take into account the countercyclical capital buffer. This is far from representing the real decision-making process of prudential supervisors. This would ask for the use of time-varying capital requirements that depends on the financial cycle and would converge towards the reference level of 8% or 10.5%. It is what Poutineau and Vermandel (2017) propose, by supposing that expected CAR depends on the dynamics of the credit/GDP ratio in the economy and converges to a constant 10% level in the long-term. But nor Gerali et al. (2010) neither Poutineau and Vermandel (2017) provide theoretical foundations for this suitable long-term level.

Instead of making capital requirements depend on an external financial stability indicator such as Credits to GDP ratio after a shock, we suggest an endogenous countercyclical prudential regulation given by a progressive and non-linear convergent process for capital requirements towards the 10.5% long-run optimal level (steady-state level). For us, this theoretical long-run optimal level have an economic justification. It would correspond to the minimal constraints imposed on the activity of financial institutions, in the absence of any tension on the financial market. Since the level of risk in the financial sphere is constantly building up, as suggested by Minsky (1986), the regulation must deviates in the short-run from the steady-state level to fight against instability. The constraints imposed to the activity of financial institutions would thus temporarily increase and the higher is the level of risk taken by banks, the stronger are the regulatory constraints. The prudential regulation thus endogenously evolves with the financial risk until the steady state is reached.

Our contribution to the previous literature is thus twofold. First, we propose a theoretical mechanism for the determination of an optimal time-varying prudential regulation. Second, we implement the mechanism in Gerali et al. (2010) and compare the performances of the two types of regulation face to a negative shock on the banks capital (seen as the starting point for the 2007-2009 crisis). We particularly show that the rule we propose performs better in terms of financial and economic stability, being able to simultaneously respond to micro- and macro-prudential stabilization objectives. It presents an implicit counter-cyclical dimension and is moreover based in theoretical foundations and easy to implement in DSGE models designed for the study of the prudential regulation.

The rest of the paper is structured as follows. Section 2 is dedicated to the set-up of the prudential mechanism. It notes the main theoretical assumptions and explains how they can be linked to the Basel financial regulation' principles. Section 3 describes the main features of the baseline DSGE model (Gerali et al., 2010), with a formal presentation only for the banking sector and banks' reaction to the prudential regulation, under the two alternative scenarios. We also remind the main transmission mechanisms from the financial sector to the real activity. Section 4 comparatively simulates the transmission of a negative bank capital shock in the economy under a prudential regulation based on a time varying capital requirement rule and the fixed capital requirement rule, respectively. Finally, Section 5 concludes and presents some perspective for future research.

2 Set-up of the prudential mechanism

As an example of transmission of the financial regulation towards the real economy, let's assume an economy in which financial institutions are banks whose activity is regulated by a PA. Banks accumulate capital subject to a capital adequacy requirement. As in Gerali et al. (2010), the banking sector is monopolistically competitive. Banks provide loans to households and firms, financed by deposits and bank capital. The following balance-sheet identity holds: *loans=deposits+capital*. Banks set interest rates on deposits and on loans in order to maximize their profits. Bank capital may adjust thanks to the accumulation of retained earnings, but it is almost fixed in the short run. PA defines capital requirements (CAR) that banks must respect. Prudential CAR thus becomes an exogenous target for the banks' capital-to-assets ratio. Quadratic costs are imposed to banks if they deviate from this CAR target (inverse of the bank leverage). Given these assumptions, the bank capital becomes a key variable for the determination of the credit supply and for the transmission of shocks from the financial sphere to the real economy and inversely. If a negative shock hits the bank capital, the banks' leverage increases and their capital-to-assets ratio decreases compared to the exogenous target. To avoid costs, banks reduce lending and this has a negative impact on investments and output. This mechanism is also active when macroeconomic conditions deteriorates, affecting banks profits and capital. As noted by Gerali et al.(2010), this transmission channel of a distress from the financial side to the real side of the economy might account for the 'credit cycle' observed in the 2008 recession. The initial reduction of banks profits and capital translated into credit restrictions and a further weakening of the real economy. If the prudential regulation is based on a fixed CAR rule, the deviation of bank capital-to-assets from the target is intuitively larger than in the case of a countercyclical time varying CAR rule. So, such a countercyclical rule should perform better in stabilizing the financial and real cycles.

We propose, in what follows, a prudential time varying countercyclical CAR rule. In the previous literature and according to Basel III, a counter-cyclical rule implicitly calls for higher capital requirements during periods of sustained growth in order to avoid over-accumulation of risk on financial markets. This allows to constitute capital buffers that can be released in times of crisis and promote economic recovery. We suggest another possible interpretation for the counter-cyclical prudential actions. We assume that in the absence of tensions in the financial markets, the constraints on banks should be minimal. Otherwise, stronger constraints on the activity of banks must be imposed and the greater the tensions induced by an individual bank on the market, the stronger the constraints imposed to it. We therefore base our prudential mechanism on these theoretical principles and show, in section 5, its countercyclical power within the meaning of the previous literature (i.e. stabilize the financial cycle, see the Credit-to-GDP ratio, as suggested by Poutineau and Vermandel, 2017).

The CAR (Bank Capital/Risk-Weighted assets ratio) is the main instrument of the PA and it is simultaneously used for micro- and macro-prudential purposes. From a *micro-prudential* perspective, the PA can "limit the risk of episodes of financial distress at individual institutions, regardless of their impact on the overall economy", as suggested by Borio (2003). From a *macro-prudential* perspective, the PA conducts a countercyclical policy so as to avoid systemic risk accumulation and thus "limit the risk of episodes of financial distress with significant losses in terms of the real output for the economy as a whole" (Borio, 2003). [3]

[3] Galati and Moessner (2013), De Nicolo, Favara and Ratnovski (2012) or Borio (2011), Claessens (2015) distinguish two aspects of systemic risk: the times series aspect (related to a progressive accumulation of risk on a specific market) and the cross-sectional aspect (related to markets and banks interconnectedness). In this article, we only focus on the time series aspect of the macro-prudential regu-

Two features of the prudential mechanism seem important to us, given these purposes: the *progressive* and *non-linear* adjustment of the prudential capital requirements when tensions occur on the financial market.
From a macro-prudential point of view, this progressive and non-linear adjustment towards a baseline fixed CAR (as previously proposed in the Basel agreements) means that the regulation has the ability to adapt to the real and financial situation of the economy. The higher the deviation of financial/real conditions is from stability, the more constraint would be the banks' activity, forcing them to progressively adjust their credit supply and stabilize the situation. The non-linearity suggests that such constraints increase more than proportional with the deviation from stability, which is a suitable feature for a better financial and real stabilization. The aim is the same as for the CCYB (Countercyclical Capital Buffers) and perfectly in line with the Basel III willingness of reform. Moreover, applied to the regulation of an individual bank and supposing that Global systemically important banks (G-SIB) are the most likely to induce high instability, the non-linearity of the mechanism allows to implicitly impose higher constraints to these institutions compared to the others. This is a second willingness of Basel III agreement, i.e. to put stronger restrictions on the activity of riskier institutions.(BIS, 2017).
From a micro-prudential point of view, the progressive and non-linear adjustment of the CAR presents two advantages. The first one is that the capital requirements take into account the specific situation of each bank. The constraints imposed to less disciplined institutions are higher than for disciplined ones. Second if a deviation from the stable trend is observed, the bank is forced to adjust its activity and avoid instability. This is done progressively, as suggested by the Basel Consultative Group. If an individual bank do not temporarily respect regulatory constraints, it must commit to progressive adjustments towards the required CAR within a given time interval (ECB, 2019). Speaking about progressiveness, we also note a quite long period of implementation for the Basel III reforms that assumed a transition period for the implementation of Basel III reforms that assumed a tightening of capital requirements - i.e. from January 2013 to January 2019 for an increase in the CAR (based on Minimum total capital plus conservation buffer) from 8% to 10.5% for all banks (BIS, 2011). The same idea of progressive adjustment applies to the new 2017 Basel reform designed to create a more robust, risk-sensitive output floor in order to limit the amount of capital benefit a bank can obtain from its use of internal risk evaluation models, relative to using the standardised approaches (from January 2022 to January 2027, as noted by BIS, 2017).

Based on these assumptions, our theoretical prudential mechanism is introduced as follows. The PA defines a minimal CAR or a maximum level of risk exposure (Risk-weight

lation which is more intuitively associated to the micro-prudential original instrument, the CAR. Further research will focus on the integration of the cross-sectional aspect of the prudential regulation in the baseline mechanism proposed here.

assets over Bank capital ratio) up to which a standard regulatory regime applies. For a higher risk exposure than this maximum (critical) level, banks are considered too risky and unsuitable for the system. The standard regime based on capital requirements is no more sufficient to regulate their activity and it is to be completed by additional pecuniary or non-pecuniary measures.[4] We are aware of the importance of such additional punishments definition, that would take the form of stopping the payment of dividends and bonuses (European Council, 2015) or in worse situations, loosening banking licence (ECB, 2019). However, the current paper focuses on the design of the theoretical prudential rule in the *standard regulatory regime*.

Taking into account its measure of individual risk, the PA first computes and communicates to banks a long-run optimal CAR. As shown below, this corresponds to the minimal constraints that can be imposed to banks' activity, when the situation on the credit market is optimal and there is no sign of instability. If any tensions are detected, additional constraints are imposed to the banks' activity, in order to avoid instability. The short-run optimal CAR temporarily deviates from the long-run level and then progressively converges to it.

In order to introduce the mathematical form of the mechanism, let us assume that the PA focuses on the inverse of CAR, i.e. on the asset to capital ratio (ACR) to take into account the risk exposure of banks. The current ACR of a bank i that operates on the credit market is $ACR(i)_t$. By definition, $ACR(i)_t$ is given by:

$$ACR(i)_t = \frac{A(i)_t}{K(i)_t},$$

where $A(i)_t$ corresponds to the level of risk weighted assets of bank *i* granted at period t and $K(i)_t$ gives the bank capital taken into account by the PA in the definition of the capital requirements ratio. Each period t can be seen as a quarter, in line with the DSGE model of Gerali et al. (2010) DSGE in which the present theoretical mechanism is implemented in the next section.

At each period t, the PA defines an intermediate ACR which is applied to the bank *i* in the next period. We denote these values $\widetilde{ACR(i)}_{t+1}$. As noted before, the PA also defines a maximum ACR from which bank i is not allowed to carry on its activity. We denote this value by $ACR_t^{\max}$. We normalize the prudential regulation by reporting it to $ACR_t^{\max}$. Thus, the main idea for the PA in what follows is to define individual prudential ratios $\widetilde{ACR(i)}_{t+1}$ in percentage of $ACR_t^{\max}$ so as to regulate the financial system.

[4]Since big banks are the most susceptible to take excessive risk, this maximum individual risk exposure can also be put in relation with the willingness of a PA to avoid banks to become ”Too big to fail”, for instance (Masciandaro (2009)).

We suggest the following definition of the prudential ratio :

$$\widetilde{ACR(i)}_{t+1} = \gamma ACR_t^{\max} e^{-\left(X(i)_t \frac{ACR_t^{\max} - ACR(i)_t}{ACR(i)_t}\right)^2} \tag{1}$$

In equation (1), $X(i)_t$ is a global measure of the risk induced in the financial system by the bank *i*, that can include default probability, exposure risk and systemic risk dimensions. The γ component is used to guarantee stability of long-term prudential objective.

The following propositions resume the main features of this capital requirements rule.

Proposition 1. *For a given ACR_t^{max} previously defined by the PA, the ACR constraint imposed to a bank i negatively depends on the risk associated to this bank (its capacity to destabilize the financial system): $\frac{\partial \widetilde{ACR(i)}_{t+1}}{\partial X(i)_t} < 0$. Told differently, the capital constraint on CAR and imposed to the bank i exponentially increases with its risk.*

Proof Proposition 1:

Starting from equation (1) and knowing that the risk measure $X(i)_t \geq 0$, the ACR imposed by PA to a bank i is tighter when the risk $X(i)_t$ increases. The first derivative of the regulatory constraint with respect to $X(i)_t$ prooves this idea:

$$\frac{\partial \widetilde{ACR(i)}_{t+1}}{\partial X(i)_t} = -2X(i)_t \left(\frac{ACR_t^{\max} - ACR(i)_t}{ACR(i)_t}\right)^2 \gamma ACR_t^{\max} e^{-\left(X(i)_t \frac{ACR_t^{\max} - ACR(i)_t}{ACR(i)_t}\right)^2}$$

$$\frac{\partial \widetilde{ACR(i)}_{t+1}}{\partial X(i)_t} = -2X(i)_t \left(\frac{ACR_t^{\max} - ACR(i)_t}{ACR(i)_t}\right)^2 \widetilde{ACR(i)}_{t+1}$$

Thanks to the positive sign of the risk measure $X(i)_t$, there is an inverse relationship between this risk and prudential regulation:

$$\frac{\partial \widetilde{ACR(i)}_{t+1}}{\partial X(i)_t} < 0$$

For a reasonable measure of risk defined by the PA, we can show that the equation (1) translates into a prudential rule that progressively conducts individual banks towards an optimal long-run ACR. This rule implicitly defends micro-prudential objectives (soundness of individual banks) and the macro-prudential objectives (fight against episodes of financial distress with significant losses in terms of the real output for the economy) as will be discussed in Section 4.

We consider in what follows the simplest risk measure and assume that both the individual and systemic risk associated to a financial institution are proportional to its ACR to $ACR_t^{\max}$ ratio. This measure is in line with the Basel Committee principles. If a bank is not sufficiently capitalized, it is risky and dangerous for the financial stability. The more the individual ACR is close to $ACR_t^{\max}$, the more dangerous the institution is for the financial stability.

For $X(i) = \frac{ACR(i)_t}{ACR_t^{\max}}$, equation (1) becomes:

$$\widetilde{ACR(i)}_{t+1} = \gamma ACR_t^{\max} e^{-\left(1-\frac{ACR(i)_t}{ACR_t^{\max}}\right)^2} \tag{2}$$

Based on equation (2), the PA first determines the long-run optimal level of capital constraints, see the level of $\widetilde{ACR(i)}$, when there is no instability on the credit market. Because there are no opportunities induced by instability, banks has no incentive to deviate from the prudential capital requirements and the restrictions imposed to their activity are minimal. It is a long-run optimal situation when PA fixes minimal constraints and the financial stability is achieved.

Proposition 2. *If the risk associated to a financial institution is proportional to* $\frac{ACR(i)_t}{ACR_t^{max}}$*, the long-run optimal level of exposure is given by* $\frac{ACR_t^{max}}{2}$*. Put in terms of CAR, that means an optimal long-run CAR equal to* $2CAR_t^{min}$*. Thus, an optimal CAR of 8% (as defined by the Basel regulation), would simply correspond to the PA belief that a bank covering less than 4%* [5] *of assets with own capital is unsuitable for the system, because too risky to be regulated in the standard supervision regime.*

Proof Proposition 2:

In order to obtain this optimal solution, we assume that, if banks deviates from the optimal solution, this is a sign of increasing risk of instability. The PA implicitly reacts to the risk and adjusts capital requirements to stabilize the financial conditions in the economy. For a representative bank, we compute the elasticity $\varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t}$ of $\widetilde{ACR(i)}_{t+1}$ in respect to $ACR(i)_t$ starting from a long-run situation in which the bank *i* would entirely satisfy regulatory capital requirements $(\widetilde{ACR(i)} = ACR(i))$. The lower the elasticity is, the stronger are the constraints imposed to the bank activity. We choose the long-run equilibrium as the one where regulatory constraints imposed are minimal,[6] i.e. the elasticity $\varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t}$ is maximal. Details on the computation of this elasticity are reported in Appendix 1 and Figure 1 resumes our results.

[5] see 5.25% if we include the capital conservation buffer

[6] Note that the γ component does not influence the determination of the optimal value. See Appendix 1 for the proof.

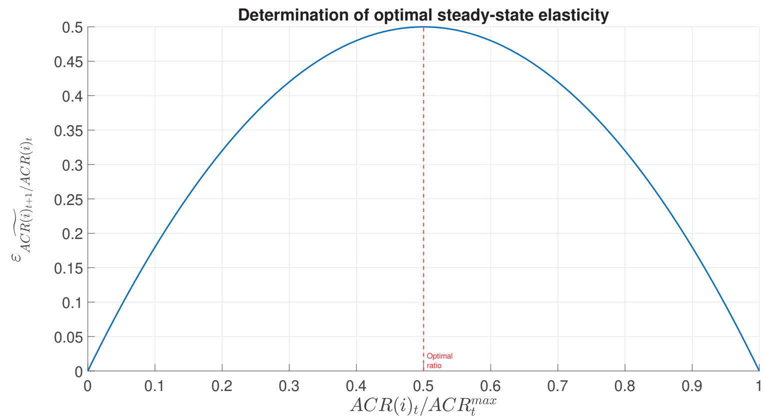

Figure 1: Steady-state (optimal long-run value) of capital requirements

We can easily observe in Figure 1 that the level of $ACR(i)$ corresponding to a maximal elasticity is equal to 50% of $ACR^{\max}$, i.e. to a ratio $\frac{ACR(i)}{ACR^{\max}} = 0.5$. For such level of banks capital ratios, the regulatory constraints are minimal for banks, and subsequently the pressure on the real economy as well. It is what we call the long-run optimal situation.

Out of this optimal long-run situation, the prudential policy rule described by equation (2) implies additional temporary constraints on the activity of banks. Given the quadratic cost that banks must support for a deviation from the regulation, this insures a progressive convergence to the optimum after each suboptimal situation. Figure 2 explicitly shows this ability of the prudential regulation to adapt to the situation in the short run and to encourage financial institutions to target the optimal financial situation after a shock.

Thanks to the bell-shaped elasticity curve (see figure 1), if an individual bank ACR deviates from the equilibrium, the PA automatically reacts to limit this deviation [7]. For example, if the bank faces an ACR increases (or decrease) of 10% after a shock compared to the optimum level, the PA asks to significantly limit it in the next periods. Moreover, the more important the deviation of ACR is from the optimum, the stronger the constraints imposed by the PA to the bank activity will be. Both individual and systemic risk are thus managed by the PA in a simple, progressive and transparent way, as suggested by the Basel Committee.

It is the message represented in figure 2. It depicts the 10% deviation of the elasticity $e_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t}$ from the steady-state level. A negative deviation of the elasticity implies that the PA tightens its policy in order to avoid the deviation of $ACR(i)_t$ from

[7] To guarantee stability condition of long term optimal regulation, $\gamma = 0.5e^{0.25}$.

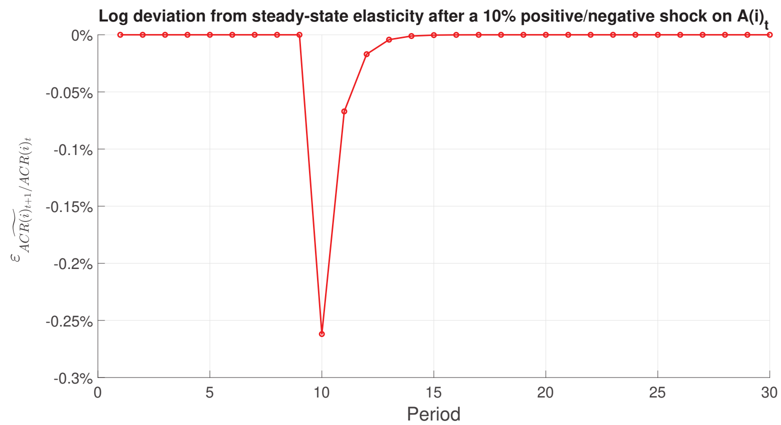

Figure 2: Dynamics of the regulatory capital requirements out of the long-run equilibrium

the steady state.[8] A higher absolute value of elasticity deviation represented in figure 2 corresponds to stronger constraints imposed by the PA on the activity of a bank.

We note that the target for such prudential regulation is the financial stability. If banks detain insufficient CAR, there would be risk accumulation in the system and the PA needs to impose stronger constraints to facilitate the return to stability. Otherwise, if banks detain more capital than necessary, financial conditions in the economy are too strict, to the detriment of the real activity. Thus, the role of a PA is to constrain under-capitalized banks to progressively increase their CAR up to the optimal level and to encourage in the same time over-capitalized banks to provide more credits and reduce their CAR to the optimal level.

This double role of prudential regulation to stabilize the financial cycle and support the real economy is also evoked in the literature dedicated to the CCyB, an additional positive buffer that could depend on the deviation of the Credits-to-GDP ratio compared to the steady-state (as suggested by Basel III agreement). If the credits-to-GDP ratio increases relative to its equilibrium level, higher capital requirements are expected in order to avoid a credit boom and create buffers for downward periods. In such cases, the credit-to-GDP ratio is under the steady-state level, the economic situation is not good and the prudential regulation should temporarily use buffers in order to support the economic recovery. From empirical works, the implementation of CCyB in 2010 in some economies provides better stabilization effect on the financial and economic cycle than fixed target (Drehman & Gambacorta, 2012 or Edge and Liang, 2020). A grow-

[8]It must be noted here that the tightening of the prudential policy defined the stronger intervention of the PA on banks and, depending on the shock, it could take the form of an required limited increase or decrease of banks ACR over time.

ing body of theoretical literature also underline financial stability benefits of a *time-varying* CCyB: Aikman et al. (2019) in a partial equilibrium model, or Angelini et al. (2010), Brzoza-Brzezina et al. (2013), Rubio and Carrasco-Gallego (2016) or Poutineau and Vermandel (2017) and Garcia-Revelo & Levieuge (2020) in a general equilibrium framework. Some doubts exists however on the measure of financial stability. Is it the Credits-to-GDP-ratio the best financial stability indicator?

For us, the prudential policy targets the same objectives, but the restrictions are defined differently, without calling an external indicator of financial stability. We do not provide in this paper a comparison of the stabilization performance of our theoretical rule relative to the countercyclical CCyB. Our aim is to understand how this theoretical prudential rule performs compared to the fixed prudential regulation defined in Gerali et al. (2010) and to show that it integrates an implicit counter-cyclical component that may be interesting for prudential regulators.

The next sections propose to integrate our prudential mechanism in the previous model developed by Gerali et al. (2010) which is the baseline model in the literature for the study of the prudential policy.

3 The DSGE framework

To get a better understanding of our prudential mechanism efficiency, we implement it in a Dynamic Stochastic General Equilibrium model (henceforth DSGE model) which integrates a banking system with credit supply regulation. To do so, we use the DSGE model of Gerali et al. (2010) because it seems adequate to study the consistency of our mechanism in managing bank credit supply compared to the traditional regulation based on fixed capital adequacy ratio. Since the Gerali et al. (2010) model encompasses many characteristics, this section will focus on the description of the banking environment in the model and how our prudential mechanism could be implemented inside it. The rest of the model will be briefly explained with a focus on the banking part.

The banks customers

The model is composed of patient and impatient households and entrepreneurs. Impatient households are characterized by a high consumption profile and they borrow to finance their new housing activity, while patient households get a lower consumption profile. They invest in housing, hold bank deposits and are the banks' owners. Both types of households consume, work and accumulate housing stock. Entrepreneurs use capital and labour (from households) to produce homogeneous goods. Impatient households and entrepreneurs borrow from retail banks to finance new housing stock and new investment project respectively. However, these two kinds of agents face a borrowing constraint that depends on the future value of their collaterals : impatient households use the value of their stock housing as collateral, while entrepreneurs use their physical capital.

The two-stages banking system

In Gerali et al. (2010), there is a two stages banking system. The first stage is composed of competitive wholesale banks that provide loans to the second stage of banks which are monopolistic bank retailers.

Wholesale banks

Wholesale banks use patient households deposits and bank capital to finance loans. The role of these banks is to finance credit supply of bank retailers. However, this credit supply has an influence on the leverage of the wholesale banks. To regulate credit supply, supervisor imposes to these banks a quadratic cost related to their capital ratio deviation from the prudential capital requirements. Therefore, the profit function of the wholesale banks depends positively on the remuneration of loans and negatively on their costs: financing costs (remuneration of households deposits) and costs related to prudential regulation. The wholesale banks thus choose their amount of loans B_t and deposits D_t to maximize the following profit function :

$$\max_{\{B_t, D_t\}} R_t^b B_t - R_t^d D_t - \frac{\kappa_{K_b}}{2} \left(\frac{K_t^b}{B_t} - \upsilon^b \right)^2 K_t^b$$

R_t^b and R_t^d are the loan rate and deposit rate respectively, while υ^b is a parameter associated to the capital adequacy ratio imposed by the regulation to limit risk in banks balance sheet. We observe that this cost is weighted by another parameter κ_{K_b} which reflects the degree of capital regulatory constraint (or cost) on wholesale banks profit. The first order conditions on B_t and D_t provide the following optimal loan rate of wholesale banks :

$$R_t^b = R_t^d - \kappa_{K_b} \left(\frac{K_t^b}{B_t} - \upsilon^b \right) \left(\frac{K_t^b}{B_t} \right)^2 \tag{3}$$

We can easily observe on (3) that the deviation of the bank capital ratio from the capital requirements have a negative impact on the loans rate. Thus, a negative shock on the bank capital for example translates into a negative deviation of the capital ratio from the regulator requirements and higher cost of loans. This is so because banks seek to reduce the regulation costs, by reducing loans supply and increasing the capital ratio.
We can also observe that the spread Sp_t between wholesale rate on loans and deposits only depends on the regulatory constraint cost :

$$Sp_t = R_t^b - R_t^d = -\kappa_{K_b}\left(\frac{K_t^b}{B_t} - \upsilon^b\right)\left(\frac{K_t^b}{B_t}\right)^2$$

This means that wholesale banks are able to make a positive interest margin if their capital position is weaker. In other words, it is costly to these banks to get additional capital or to reduce their loans.
Furthermore, it is assumed that bank capital accumulation process only depends on previous net capital level and profit :

$$K_t^b = (1-\delta^b)K_{t-1}^b + j_{t-1}$$

Where j_{t-1} is the level of profit in the previous period.

Retailer banks

The second stage of the banking system is composed of retail banks which act in a monopolistic competitive market. These banks use wholesale loans B_t to provide loans to impatient households (b_t^I) and entrepreneurs (b_t^E). Since retail banks are not competitive, they are able to practice a mark-up on loan rates to impatient households and entrepreneurs. However, these mark-up rates are not perfectly flexible. Indeed, some frictions such as inelastic demand in short run or the wish of banks to preserve customer relationship have an impact on loan rates decisions. To display these features, changes in loan rates of impatient households and entrepreneurs (mark-up variations) correspond to a quadratic cost in the profit function of retail banks. Therefore, these banks choose loan rates of households and entrepreneurs (r_t^{bH} and r_t^{bE} respectively) to maximize the following expected profit program:

$$\max_{\{r_t^{bH}, r_t^{bE}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \Bigg[r_t^{bH} b_t^I + r_t^{bE} b_t^E - R_t^b B_t \\ - \frac{\kappa_{bH}}{2}\left(\frac{r_t^{bH}}{r_{t-1}^{bH}} - 1\right)^2 r_t^{bH} b_t^I - \frac{\kappa_{bE}}{2}\left(\frac{r_t^{bE}}{r_{t-1}^{bE}} - 1\right)^2 r_t^{bE} b_t^E \Bigg]$$

With $\mathbb{E}_0$ the expected operator at time $t = 0$ and $\Lambda_{0,t}^P$ the stochastic discount factor of patient households. Since these latter are the owners of retail banks, this factor reflects their shareholder behavior in the profit of banking activity.
We note that the capital regulatory cost nested in the rate of the wholesale banks R_t^b is directly absorbed in the cost of the retail banks. This means that the higher is the capital constraint on wholesale banks, the higher is the cost of credit supply for retail banks.

Introduction of the time-varying capital regulation

As explained above, the Gerali et al. (2010) model provide a fixed prudential constraint through the parameter v^b inside the quadratic cost in the profit function of wholesale bank. This parameter reflects expectations of prudential supervisors regarding the adequate banks' capital ratio. In real world, this parameter can be associated to the optimal capital adequacy ratio decided by Basel Committee to manage micro and macro financial risk emerging from credit supply activities of banks. However, we can ask ourselves whether a fixed target regulation is more efficient than a time-varying regulation. Indeed, since credit cycle has a close relationship with level of prudential constraints, it would be consistent to study the effect of a time varying capital requirement to dampen periods of excess credit growth and support financial market during recession. To do so, we implement our prudential mechanism in the Gerali et al model through the target capital regulation v^b imposed to wholesale banks activities. Instead of getting a fixed target as in the traditional model, our mechanism provides an endogenous capital adequacy ratio. Starting from equation (1), v_t^b, which is equal to $\frac{1}{\widetilde{ACR(i)_t}}$ takes the following form :

$$v_t^b = \frac{e^{\left(1 - \frac{ACR(i)_t}{ACR(max)}\right)^2}}{\gamma} \tag{4}$$

$ACR(i)_t$ corresponds to the asset to capital ratio of bank i at time t and $ACR(max)$ to the maximum ratio allowed by the prudential supervisor (equivalent to the minimal capital ratio acceptable for banks). As noted in the description of equation (1), the parameter γ is a stabilization parameter of the prudential regulation used to guaranty the convergence of our mechanism towards the steady-state (long-run optimum level).
The optimal interest rate on loans to retailer banks, given the regulation constraint imposed by PA, are obtained from the profit maximization program of wholesale banks. Therefore, it can expressed by :

$$R_t^b = R_t^d - \kappa_{K_b} \left(\frac{K_t^b}{B_t} - v_t^b \right) \left(\frac{K_t^b}{B_t} \right)^2 \tag{5}$$

It is to be noted that in (5), the CAR imposed by the regulation is now time-varying and given by equation (4). After a negative bank capital shock, for instance, the regulation moves downward its capital requirements. The regulation costs for banks is lower than in the fixed regulation scenario summarized in (3) and the loans rate would increase less.

The rest of the model

The rest of our model is similar to the one of Gerali et al. (2010)
Entrepreneurs use capital and labour to produce output in a constant scale return way (Cobb-Douglas production function). Capital is bought from capital producer. The labour market is composed of labor unions which sold differentiated labor types of households to labor packers. Packers assemble these labor types in a CES aggregator to produce a homogeneous labor to entrepreneurs. The role of the unions is to take account of sticky wages in the labor market which will be transmitted to entrepreneurs labor cost via the price of labor supply from labor packer.
On the goods market, there are competitive firms which buy last-period undepreciated capital from entrepreneurs and final goods from monopolistic retailer companies. By using these two inputs, competitive firms are able to produce specific output which is sold to entrepreneurs. The role of competitive firms is to take account of frictions in goods price evolution coming from retailers companies. These frictions correspond to sticky prices of price indexation on inflation and to monopolistic mark-up of retailers companies. Hence, competitive firms capture these frictions to the price of new capital goods they sold back to entrepreneurs.
In banking market, there is a deposit branch which collects deposits from patient household and consolidate them to sell deposit package D_t to wholesale banks at the price R_t^d. The role of the deposit branch is to take account of the mark-down effect on the deposit rate which is observed empirically. There is also a central bank which implements a monetary policy rate which depends on past rate, inflation and output dynamics (standard Taylor rule).

4 Simulation results

In order to test the financial and economic implications of our theoretical prudential mechanism, we integrate it the model Gerali et al.(2010). We calibrate the model by following the median values of the estimation provided by Gerali et al. (2010) for the euro area and we simulate an exogenous and unexpected 5% destruction of bank capital as a starting point for a financial crisis. In a first scenario, the PA fixes a CAR of 10.5%, while, in the second scenario, it chooses to follow the policy rule proposed in our paper with time-varying capital requirements. We do not propose a quantitative experiments, but only focus on the qualitative results of the experiment in order to understand how the transmission of the shock depends on the prudential policy scenario.
Simulations are represented in Figure 3, in dotted line for the baseline fixed prudential regulation scenario and in black when our countercyclical prudential rule is implemented. The baseline scenario is very useful to understand how a financial shock (a negative bank capital shock) can affect the real economy.

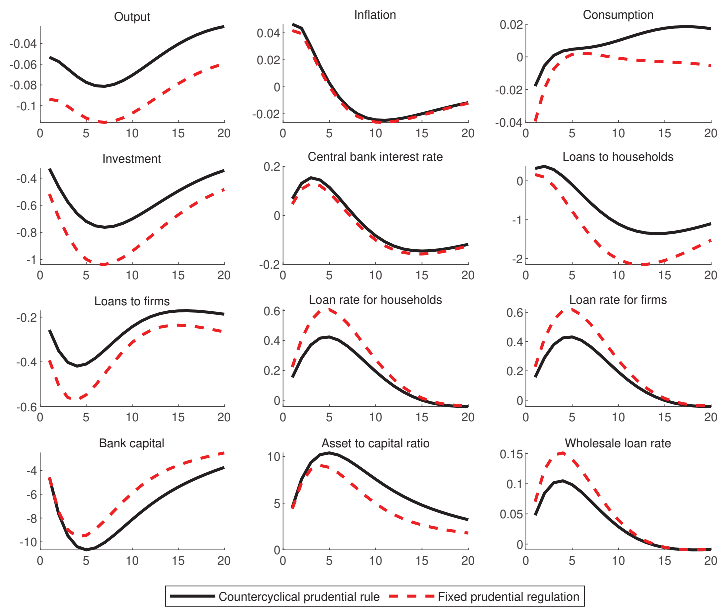

Figure 3: Dynamics of the economy after a negative shock on bank capital

The transmission channel is the same as in Gerali et al. (2010). After the shock, the Assets-to-capital ratio of banks goes up. Their capital-to-assets ratio decreases. This implies a deviation from the 10.5% level of PA requirements, which is costly for banks. In order to rebalance their assets and liabilities, banks increase lending rates, but this reduces the demand for credit and therefore investment. The demand for capital weakens and the use of capital increases. The price of capital falls and it becomes less useful as collateral. Financial conditions are becoming even more restrictive for firms, with negative impact on investment and output. The financial shock finally affect the aggregate supply. To produce, firms increase labor demand, wages increase also and allow for higher consumption that limit the output fall. Given the wages dynamics and the higher financing costs, inflation increases. The central bank slightly increases the policy rate to stabilize inflation. Comparatively to this baseline situation (fixed prudential regulation), we can easily observe that the negative impact of the shock on the real economy is much more limited when the prudential regulation follows our countercyclical prudential rule. This is so because the PA reacts to the shock by temporarily reducing capital requirements. The countercyclical nature of the theoretical rule that we propose in more than evident if we look to the dynamics of the Credit-to-GDP ratio in Figure 4. Following a shock that negatively affect the Credit-to-GDP ratio, the temporary release of the CAR in the theoretical rule allows for a better stabilization of the financial cycle. This will

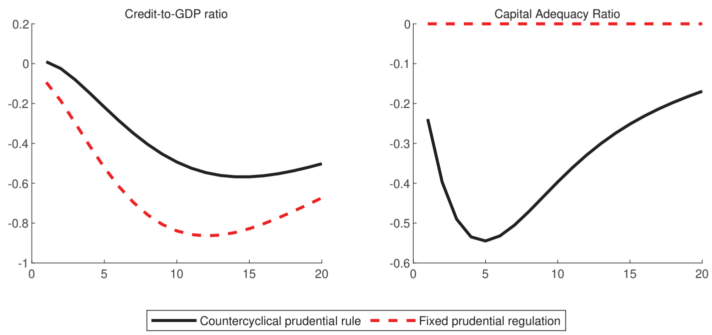

Figure 4: Countercyclical time-varying regulation

also conduct to a better stabilization of the real business cycle. Indeed, the deviation of the banks capital ratio from the new lower capital requirements is less important than in the baseline scenario and the costs imposed by this deviation for banks is lower. The rebalancing of their assets and liabilities implies lower increase of loan rates for all economic agents (households and entrepreneurs). The financial market provide more loans to the real economy compared to the baseline scenario. Firms investment decreases less, as well as the housing investment of households. Housing and capital prices also decrease less, they keep more value and are relatively more useful as collateral. Stimulated by the higher demand, labor demand is even higher, wages increase more and the dynamics of consumption limit even more the output loss. The central bank interest rate increases more to stabilize inflation, but the costs of credits in the economy is lower thanks to the prudential regulation.

5 Conclusion and policy implications

Starting from the experience of the 2007-2009 financial crisis and taking into account the interaction between financial and real variables, we suggest a common theoretical mechanism for the design of both micro and macroprudential regulation. This regulation is based on capital requirement and incorporates a countercyclical dimension, as well as progressive adjustments of individual constraints allowing to take account of the specific situation of each bank.

One of our contributions corresponds to the determination of an optimal long-run CAR defined by the minimal constraints to be imposed on the banks activity when there is stability on the credit market. Another contribution consists to define a short-run regulation path when there are signs of instability on the credit market. To get a regulation approach consistent with the Basel main principles, we built the optimal regulation path in a non-linear fashion that allows supervisor to take efficient progressive countercyclical actions and insure financial stability.

The paper insists on the definition of the new prudential mechanism and shows its countercyclical power in a standard DSGE framework. Further research can concern the evaluation of the mechanism relative to other alternative specifications of countercyclical macro-prudential regulation, such as the CCyB. Concerning the macro-prudential component of the mechanism, the focus is, in this paper, on the time-series aspect of the regulation, but we ignore the cross-sectional aspect in the definition of systemic risk. It is an important aspect and we note that it is possible to adapt our baseline mechanism and take into account this second dimension of the macro-prudential regulation. Further research will indeed be dedicated to this question.

Speaking about the policy implication of our research, we propose a prudential rule that respect the main directions defined by the Basel regulation. Its implementation is not more complicated than for a monetary policy rule and it would be perfectly in line with the definition and the transparency principle for banking regulation and supervision. Indeed, as noted by Frait and Tomsik (2014), the banking regulation and supervision are designed to support the stability of the financial system and to increase the resilience of credit institutions. Thanks to its double micro- and macro-prudential dimension, the rule that we propose is able to fulfil these criteria. Furthermore, the main objective of supervisory authorities to "ensure robustness, reliability, and transparency of prudential outcomes from the adoption of Basel standards". As for the prudential reforms' implementation, the Basel Consultative Group recommends to establish priorities for progressive movement to more sophisticated regulation within the Basel III framework. The implementation of a time-varying non-linear CAR rule such as suggested in our paper may be a solution to simplify the prudential regulation implementation and improve its transparency.

The last section of the article gives an example of application of the new suggested regulation to stabilize a purely financial shock. However, since real and financial sectors are highly interconnected, the prudential rule would support in any situation the financial and economic stability. During the recent Covid-19 health crisis, for instance, the lockdown decisions taken all over the world during this crisis have emphasized one more time the main role of banks in supporting a distressed real economy. Among the consequences of lockdowns, the drop of revenues and profits (despite the large amounts of transfers, subsidies and other state aid schemes announced by the governments), the degradation of net financial position of economic agents and the increased uncertainty are all synonyms of more difficult access to external finance. However, banks were invited to continue to provide financial support to distressed agents even if their activity had implicitly become riskier. From this point of view, the financial shock induced by the crisis for banks would have materialized in a significant increase of the Risk-weighted assets and of the key variable (ACR) of the theoretical prudential mechanism that we propose.[9] Since the regulation endogenously takes into account the instability risk, the PA would temporarily relax its requirements compared to the optimal long-run level, stimulating banks to continue supporting the real activity with credits. The return to optimum would also have been progressive, sign of the ability of the supervisor to take into account of the overall economic and financial conditions when taking prudential regulation decisions.

[9] Some instruments (like state guaranties for corporate loans) were also deployed in order to limit the impact of the crisis on banks and on their main prudential ratios. These measures confirm that the crisis has induced a degradation of the quality of banks' corporate loans portfolio that implicitly deteriorate their capital adequacy ratios.

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Appendix 1

By assuming that the risk measure is $X(i)_t = \frac{ACR(i)_t}{ACR_t^{\max}}$, it is possible to rewrite the prudential objective as following :

$$\widetilde{ACR(i)}_{t+1} = \gamma ACR_t^{\max} e^{-\left(1-\frac{ACR(i)_t}{ACR_t^{\max}}\right)^2}$$

It is possible to analyze the PA reaction to a change of the bank i risk by comouting the elacticity. To do so, one has to consider the elasticity formula :

$$\begin{aligned}
\varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t} &= \frac{\partial \widetilde{ACR(i)}_{t+1}}{\partial ACR(i)_t} \frac{ACR(i)_t}{\widetilde{ACR(i)}_{t+1}} \\
\varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t} &= 2\left(1-\frac{ACR(i)_t}{ACR_t^{\max}}\right)\gamma ACR_t^{\max} e^{-\left(1-\frac{ACR(i)_t}{ACR_t^{\max}}\right)^2} \frac{ACR(i)_t}{\widetilde{ACR(i)}_{t+1}} \\
\varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t} &= 2\left(1-\frac{ACR(i)_t}{ACR_t^{\max}}\right)\widetilde{ACR(i)}_{t+1} \frac{ACR(i)_t}{\widetilde{ACR(i)}_{t+1}} \\
\varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t} &= 2\left(1-\frac{ACR(i)_t}{ACR_t^{\max}}\right)\frac{ACR(i)_t}{ACR_t^{\max}}
\end{aligned}$$

To get the optimal ratio $\frac{ACR(i)_t}{ACR_t^{\max}}$ which minimizes capital constraint in the long run, it is possible to use the first order condition :

If $x = \frac{ACR(i)_t}{ACR_t^{\max}}$, then this condition becomes :

$$\begin{aligned}
\frac{\partial \varepsilon_{\widetilde{ACR(i)}_{t+1}/ACR(i)_t}}{\partial x} &= 0 \\
\frac{\partial (1-x)x}{\partial x} &= 0 \\
\frac{\partial \left(x-x^2\right)}{\partial x} &= 0 \\
1-2x &= 0 \\
x &= 0.5
\end{aligned}$$

As a result :

$$\frac{ACR(i)_t}{ACR_t^{\max}} = 0.5$$