Endogenous Gender Power: The Two Facets of Empowerment

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Abstract

A large body of evidence suggests that women’s empowerment, both within the household and in politics, benefits to children and has the potential to promote economic development. Nevertheless, the existing interactions between these two facets of empowerment have not been considered thus far. The aim of the present paper is to fill this gap by proposing a theoretical framework in which women’s bargaining power within both the private sphere and the public sphere is endogenous. We show that the mutual interplay between the evolution of women’s voice in the family and in society may lead to the emergence of multiple equilibria and pathdependency phenomena. We also discuss policy interventions that are the most suitable to promote women’s empowerment when its multidimensional nature is taken into account.

Keywords: Female Empowerment, Intrahousehold Bargaining Power, Institutional Bargaining Power, Gender Inequality, Economic Development.

JEL Classification: J13, J16, O11, O43, P16

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1 Introduction

Women’s empowerment in developing countries has emerged as a key objective for international organizations and policy makers. As a striking illustration, “empower all women and girls” is identified as part of the fifth sustainable development goal by the United Nations (see United Nations 2018). This has been justified not only to achieve equity but also as a necessary step to promote economic development. A rising body of empirical work supports this view by reporting a positive impact of female empowerment on human capital formation. For instance, Lott and Kenny (1999), Aidt et al. (2006), Miller (2008) and Brollo and Troiano (2016) show that a rise in women’s political participation results in larger government spending targeted towards children, while Hoddinot and Haddad (1995), Lundberg et al (1997), Allendorf (2007), Schady and Rosero (2008), Rublacava et al. (2009) and Calvi et al. (2018) conclude that an improvement in the intrahousehold bargaining position of women changes household spending in a way that benefits to children. Those empirical findings have offered a rationale for public policies intended to promote female empowerment both in politics¹ and within the household² in developing countries. These policies, as well as their empirical basis, give rise to two views of women’s empowerment, each associated with a specific sphere: the intrahousehold empowerment associated with the private sphere (the family or the household) and the institutional empowerment associated with the public sphere (society or politics). These two views capture two different facets of the empowerment phenomenon (see Doepke et al. 2012 for an enlightening discussion of these two facets and how they both impact investments in children). Indeed, the economic emancipation of women, which is largely considered a determinant of women’s bargaining position within the household, does not necessarily improve the ability of women to have a voice in society and to influence policy³; in turn, improved participation by women in the political decision-making process does not mechanically translate into their empowerment within the household (see Beath et al. 2013).

To clarify the link between women’s empowerment in these two different spheres, Figure 1 plots a measure of women’s empowerment in politics against a measure of empowerment within the household for a set of developing countries. In brief, intrahousehold empowerment is measured by the proportion of households in which women participate in decisions, while political empowerment is measured by the gap between the percentage of women and the percentage of men who occupy positions allowing them to have a voice at the society level (as positions in parliament or ministerial posts).⁴ While these measures are clearly distinct, Figure 1 suggests that they are highly correlated

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¹ In 2012, quotas for women in policy-making positions were instituted in 87 countries (Duflo 2012).
² Microcredit schemes or conditional cash transfer programs are directed almost exclusively at women (Duflo 2012). A stated justification for this targeting strategy is that it might improve the bargaining power of women within the family (see Fiszbein and Schady 2009).
³ Duflo (2012) notes that, compared to economic opportunities, education and legal rights, the gender gap in political participation has narrowed the least between 1995 and 2005.
⁴ A precise definition of these two measures and a description of our data can be found in the Online Appendix. We
such that these two facets of empowerment are likely to be linked. In particular, in some countries, women have considerable voice within both society and the family (countries that are located near the top-right corner such as the Philippines, Namibia, Guyana, Nicaragua or Rwanda), while at the other extreme, in others countries, women have almost no influence on the decision-making process in neither the public nor private sphere (countries that are located near the bottom-left corner such as Chad, Mali, Nigeria, Ivory Coast or Niger). In order to understand these situations and, more broadly, the engines of empowerment, it may be crucial to disentangle the interplay between these two dimensions.

International organizations recognize the importance of considering the two spheres and the existing interactions between them as they may contribute to the emergence of gender inequality traps (World Bank 2006 and 2012). However, the existing theoretical literature focuses on one single dimension and, typically, associates women’s empowerment with an increase in their relative intrahousehold bargaining power. Hence, the feedbacks between different aspects of empowerment has not been considered thus far. The aim of the present article is to fill this gap. To that end, we provide a theoretical framework in which women’s bargaining power within both the private and the public spheres is endogenous. It allows us to figure out the joint evolution of these two facets of empowerment and its implications for the long-run evolution of gender inequality.

also show in this appendix that our two measures of empowerment are indeed correlated with an index of educational achievement.

5. An emergent empirical literature also emphasizes the multidimensional nature of women’s empowerment and the interactions between the different facets of this phenomenon (see Mabsout and van Stereven 2010, Gottlieb and Robinson 2016 or Brulé and Gaikwad 2018).
To do so, we propose a two-sex overlapping generations model in which individual utility is derived from private consumption and children’s human capital achievement and women place a higher weight on human capital than men. Then, we consider the simplest framework capturing this basic fact: human capital formation depends on choices made both by parents (in the private sphere) and by policy makers (in the public sphere). Formally, children’s human capital is produced by combining two substitutable inputs: public spending provided by the government and parents' child rearing time. Due to gender differences in preferences regarding children’s human capital, the provision of these two inputs depends on the ability of women to influence decisions at the society and at the household level. We consider that the relative bargaining power of women vs. men might be different in the private sphere than in the public sphere. Below, we elaborate on the ways in which choices are made within these two spheres and how the two facets of women’s bargaining power evolve over time.

In the private sphere, the time that parents devote to their children results from a collective decision-making process. Formally, this time is determined jointly by the two spouses to maximize a collective household utility function. The parameter measuring the relative weight of the husband (vs. wife’s) utility in this collective function is interpreted as the intrahousehold bargaining power of men. Moreover, we model this intrahousehold bargaining power as endogenous by assuming, as is often the case in the literature (see, among others, de la Croix and Vander Donckt 2010, Iyigun and Walsh 2007, Baudin et al 2015 and 2018 or Pretner and Strulik 2017), that it depends on the relative income of men compared to that of women. If the relative income of women increases, the wife’s bargaining position improves. Finally, an individual’s income depends on brains (the human capital endowment, which is equally shared between girls and boys) and brawn (a physical strength parameter that is higher for males than for females).

In the public sphere, political power is shared between different political groups with potentially conflicting interests. The two relevant groups for our purpose are males and females. Following Bisin and Verdier (2017), we assume that public policy is determined by a social planner that accounts for the preferences of the two groups and their relative political power. Formally, the planner chooses the amount of public spending to maximize a social welfare function – that is, a weighted average of the utility of males and females – taking as given the private decisions of the agents. The weight placed on the utility of males (vs. females) is interpreted as the relative political power – also called institutional bargaining power – of men. The lack of commitment on the part of the social planner leads to suboptimal public policy. However, in line with Bisin and Verdier (2017), we assume that institutional changes serve as a commitment device. In other words, the institutional bargaining power of males adjusts over time to move the equilibrium closer to a hypothetical configuration in which the social planner would be able to commit to a tax rate. In this setting, female empowerment is viewed as a change in the institutional arrangements of society that benefits women, which will arise if it helps society achieve a less inefficient situation.
In our model, the two facets of empowerment interact and influence one another. On the one hand, an increase in women’s institutional power will induce a rise in public spending devoted to human capital formation. More human capital allows for a reduction in the relative importance of brawn in the productivity of labor such that the gender wage gap decreases. This generates an improvement in women’s bargaining position in the private sphere. On the other hand, when the empowerment of women in the private sphere is low relative to their empowerment in the public sphere, the government must over-invest in human capital to compensate for the small amount of time that parents devote to their children. In this configuration, if the government were able to commit to a level of public spending, it would choose a lower level and let the households adjust by investing more time in human capital formation. Hence, to close the gap between the public policy arising in equilibrium and the one that would have prevailed in the absence of a commitment problem, the institutional bargaining power of women must decrease such that public spending will decrease. A symmetric reasoning applies when the empowerment of women is greater in the private than in the public sphere such that, in that case, the institutional bargaining power of women must improve.

These mutual interplays between the evolution of women’s voice in society and in the family may induce the emergence of multiple equilibria. In particular, a patriarchal steady state in which decisions are dictated by males both in the private and in the public sphere (countries located near the bottom-left corner in Figure 1) may co-exist with another locally stable equilibrium in which institutional and intrahousehold bargaining power are more balanced between genders (countries located near the top-right corner in Figure 1). Obviously, the patriarchal steady state is also characterized by a depressed amount of human capital and a large gender pay gap. The existence of multiple equilibria implies that two countries with different initial conditions – in terms of technology or institutional arrangements – in the distant past may ultimately have a substantially different balance of power between genders even if these discrepancies were eliminated later. This path-dependency result might be related to a recent set of studies that show that historical events may have persistent effects on several aspects of gender inequality and, in particular, on female empowerment (see Alesina et al. 2013, Xue 2018 or Teso 2019). In particular, we propose transmission channels allowing us to explain the persistence of an inegalitarian balance of power between genders even long after the original cause of gender inequality should have disappeared. Our results also allow us to shed new light on the impact of public policies that have been proposed to favor women’s empowerment in developing countries. We particularly consider two types of policy. The first, directed towards the private sphere, consists of targeting some public transfers at women. The second, directed towards the public sphere, consists of imposing quotas for women in policy-making positions. Our results offer a rationale for the mixed results concerning the effectiveness of these policies for empowering women. In particular, for a country trapped in the patriarchal equilibrium, their marginal impact is almost null if the country remains in the trap, while this impact can be
substantial if it allows the economy to escape the trap. For a country that has reached the interior steady state, these policies have a positive, albeit more limited, impact. Finally, we conclude that policies directed towards the private sphere and policies directed towards the public sphere exhibit complementarities and should be implemented conjointly.

Our article closely relates to the body of theoretical work that regards the bargaining power between men and women as endogenous (see Iyigun and Walsh 2007, Doepke and Tertilt 2009, de la Croix and Vander Donckt 2010, Bertocchi 2011, Fernandez 2014, Pretner and Strulik 2017). In most of this literature, this bargaining power is understood as intrahousehold bargaining power (in the private sphere, in our terminology). A notable exception is Bertocchi (2011), who considers empowerment as the extension of political rights of women (in the public sphere, in our terminology). Our paper is a first attempt to model the joint evolution of different facets of women’s empowerment: intrahousehold and institutional empowerment. Since these two facets are clearly identified in the empirical literature and by policy makers, we believe that a theoretical analysis of how they interact fills an important gap in the literature. In particular, we show that those interactions might give rise to multiple equilibria, preventing some countries from escaping from a patriarchal trap. When addressing the engine of empowerment, the majority of the abovementioned articles assume that gender bargaining power is shaped by the relative earnings of men and women. We make a similar assumption concerning empowerment in the private sphere. Doepke and Tertilt (2009), Bertocchi (2011) and Fernandez (2014) consider another driving force for empowerment. In their papers, men initially have all the power and may decide to grant women rights when it is in their self interest to do so. Hence, in those articles, the level of empowerment tomorrow is determined to maximize a welfare function in which the weight given to men and women utility is determined by those who have the power today. In a similar spirit, we assume that, in the public sphere, the institutions (i.e., the balance of power between men and women) in one period determine the institutions for the next period to achieve a higher degree of social efficiency. Specifically, we adopt the framework proposed by Bisin and Verdier (2017) in which the balance of power adjusts over time to indirectly internalize the lack of commitment that plagues social choice problems. Importantly, this framework is sufficiently tractable to allow for modeling institutional empowerment as a continuous variable, just like intrahousehold empowerment. In contrast, Doepke and Tertilt (2009), Bertocchi (2011) and Fernandez (2014) consider only two levels of power sharing: the patriarchy regime, in which men have all the decision power, and the empowerment regime, in which the power is equally shared between genders. Moreover, while in these papers the balance of power evolves in one direction (patriarchy to empowerment), it can also go in the opposite direction in our framework. This is a key feature that allows for the emergence of multiple equilibria.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3

6. Geddes and Lueck (2002) also make this point without a formal model.
explores the dynamics and long-run equilibria. Section 4 presents comparative statics results and discusses the effectiveness of selected public interventions intended to promote women’s empowerment. Finally, Section 5 concludes the paper.

2 The Model

We propose an overlapping-generations model of a developing economy. Time is discrete (indexed by \( t \)), and the economy is populated by households. One household consists of two parents (a male and a female spouse) and two children (a boy and a girl).

2.1 Preferences and constraints

At date \( t \), each spouse \( s \in \{m, f\} \) derives utility from joint household consumption \( (c_t) \) and the children’s human capital \( (h_{t+1}) \):

\[
u^s_t = \gamma^s \ln c_t + (1 - \gamma^s) \ln h_{t+1}
\]  

The parameter \( \gamma^s \) accounts for the relative weight placed on consumption vs. the investment in children in the preferences of spouse \( s \). We assume that women place more weight on children’s human capital than men (see Pretner and Strulik 2017 for a discussion of the empirical evidence as well as Filipiak et al. (2017) and references therein):

\textbf{Assumption 1 } \( \gamma^m > \gamma^f \)

Many theoretical papers rely on the assumption that men and women have different preferences, with women being more oriented towards the well-being of their family, in particular that of their children, or the provision of public goods, than men (see, for instance, Doepke and Tertilt 2009, Bertocchi 2011 or Pretner and Strulik 2017).\textsuperscript{7}

Human capital depends on the time that parents devote to their children and on public investment (such as educational spending or public health investment targeted towards children). Moreover, in line with Glomm and Kaganovich (2003), Kimura and Yasui (2009) or Azarnert (2010), we assume that private and public inputs are substitutes.\textsuperscript{8} Formally, the human capital of generation \( t + 1 \) is given by:

\[
h_{t+1} = \mu [a\tau_t + (1 - a)x_t]
\]  

\textsuperscript{7}Baudin and Hiller (2019) propose a theoretical explanation for the emergence and persistence of these gender differences in preferences.

\textsuperscript{8}Note that, the existence of a complementarity between private and public inputs in the production function of human capital would have strengthened our results by adding a mutually reinforcing link between the decisions taken in the private sphere and those taken in the public sphere.
with $\tau_t \in [0,1]$ being the tax levied by the government that is used to finance public spending devoted to children and $x_t \in [0,1]$ being the parental time devoted to children. We assume that the only input for the private provision of human capital is this parental time (a similar assumption may be found in Galor and Weil 2000 or Tamura 2006). For the sake of simplicity, we also assume that child rearing time is equally shared between the two spouses. Finally, the parameter $a \in (0,1)$ accounts for the relative weight of public spending vs. parental time in the children’s human capital, while $\mu > 0$ is a scale parameter. Each adult member of the household is endowed with one unit of time, and the household faces the following budget constraint:

$$c_t = (1 - x_t)(1 - \tau_t)(w^f_t + w^m_t)$$

(3)

where $w^f_t$ and $w^m_t$ are the wage rates per unit of labor for men and women, respectively.

### 2.2 Women’s empowerment

We consider two different dimensions of women’s empowerment:

i Intrahousehold bargaining power captures the capacity of women to influence her household’s choices.

ii Institutional power captures the capacity of women to have a voice in the political decision-making process at the society level.

These two dimensions both measure the relative say of women in economic decisions, but while the first dimension is limited to the private sphere, the second dimension concerns the public sphere. To more formally define these two dimensions of women’s empowerment, let us explain how decisions are made within these two spheres.

**The private sphere.** To model the decision-making process within the household, we adopt the collective approach proposed by Chiappori (1988, 1992). According to this approach, the household maximizes a collective utility function, which is a weighted average of the utilities of the two spouses, under income pooling. We denote by $\theta_t$ the relative weight associated with males’ utility in the household utility function. This variable measures the degree of women’s empowerment in the private sphere: When $\theta_t$ decreases, women gain influence in the household’s decisions. The collective household utility function is written as:

$$H_t = \theta_t u^m_t + (1 - \theta_t) u^f_t$$

(4)

Using the expression for the individual utility function (1), the household utility function (4) can be rewritten as:

$$H_t = \Gamma_t \ln c_t + (1 - \Gamma_t) \ln h_{t+1}$$

(5)
with
\[ \Gamma_t \equiv \gamma^f + \theta_t \Delta \gamma \quad \text{and} \quad \Delta \gamma \equiv \gamma^m - \gamma^f \]

The variable \( \Gamma_t \) accounts for the relative weight placed on consumption rather than children’s human capital within the household. Under Assumption 1, \( \Delta \gamma > 0 \), meaning that \( \Gamma_t \) is increasing in \( \theta_t \). When the relative bargaining power of women declines, human capital becomes relatively less valued – compared to consumption – in the household’s utility. Most of the existing literature models women’s empowerment as a change in intrahousehold bargaining power \( \theta_t \) (see de la Croix and Vander Donckt 2010, Iyigun and Walsh 2007, Doepke and Tertilt 2009, Rees and Riezman 2012, Baudin et al 2015 and 2018 or Pretner and Strulik 2017). For the sake of clarity, in the following, we will refer to \( \Gamma_t \) as our measure of the relative power of men in the private sphere.

**The public sphere.** We model the political decision-making process in a reduced-form way by adopting the model proposed by Bisin and Verdier (2017). In this model, public policy is decided by a social planner (the policy maker or the government) to maximize a utilitarian social welfare function. Moreover, institutions are defined as the Pareto weights associated with the utility of each social group in this social welfare function. Two social groups are relevant for our purposes: men and women. The weight associated with men is denoted \( \beta_t \), so \( 1 - \beta_t \) measures the institutional power of women. When \( \beta_t \) decreases, women exercise more voice in society, thereby having a larger influence on public policy. Hence, the social welfare function is written as:

\[ W_t = \beta_t u^m_t + (1 - \beta_t) u^f_t \]  

(6)

Using the expression for the individual utility function (1), the social welfare function (6) is rewritten as:

\[ W_t = \Psi_t \ln c_t + (1 - \Psi_t) \ln h_{t+1} \]  

(7)

with

\[ \Psi_t \equiv \gamma^f + \beta_t \Delta \gamma \quad \text{and} \quad \Delta \gamma \equiv \gamma^m - \gamma^f > 0 \]

The variable \( \Psi_t \) plays exactly the same role, at the society level, as \( \Gamma_t \) does at the household level. It is increasing in \( \beta_t \), so when the institutional power of males increases, the weight placed on human capital in the social welfare function decreases. Then, in the following, we will refer to \( \Psi_t \) as our measure of the relative power of men in the public sphere.

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9. This view of institutions as mechanisms allowing to aggregate the preferences of distinct social groups and to implement social choices is in line with a large literature (North 1981, 1990; David 1994; Greif 2006). Moreover, in the spirit of Greif and Laitin (2004), Bisin and Verdier (2017), Bisin et al (2018) or Iyigun et al (2018), institutions are considered exogenous when decisions are made (see Section 2.3), even if those decisions will be the driver of endogenous institutional changes (see Section 3.2).
2.3 The static equilibrium

As previously explained, consumption \((c_t)\) and child rearing time \((x_t)\) are chosen within the household, while public policy \((\tau_t)\) is decided by the social planner. Following Bisin and Verdier (2017), we assume that the planner cannot commit to a level of public spending. Hence, the tax rate is chosen without internalizing the effect on households’ decisions. Under this assumption, the static equilibrium may be defined as the Nash equilibrium of a game played between the social planner and the representative household.

**Definition 1** The static equilibrium is defined as the pair \(\{x_t^s, \tau_t^s\}\) such that:

\[
x_t^s \in \arg \max_{x_t} \{\Gamma_t \ln(1 - x_t) + (1 - \Gamma_t) \ln(a\tau_t + (1 - a)x_t)\}
\]

\[
\tau_t^s \in \arg \max_{\tau_t} \{\Psi_t \ln(1 - \tau_t) + (1 - \Psi_t) \ln(a\tau_t + (1 - a)x_t)\}
\]

Equations (8) and (9) are obtained by plugging the expression for human capital (2) and the household budget constraint (3) into the household’s utility function (5) and the social welfare function (7), respectively, using the fact that the policy maker regards \(x_t\) as given and the representative household regards \(\tau_t\) as given. The first-order conditions associated with the maximization of the household utility function (8) and the social welfare function (9) lead to:

\[
x_t = \max \left\{0, 1 - \Gamma_t \left[\frac{1 - a + a\tau_t}{1 - a}\right]\right\}
\]

\[
\tau_t = \max \left\{0, 1 - \Psi_t \left[\frac{a + (1 - a)x_t}{a}\right]\right\}
\]

Equation (10) describes a negative relationship between child rearing time and the tax rate. This is driven by a simple substitution effect. The same kind of effect explains why \(x_t\) negatively affects \(\tau_t\) in equation (11).

To avoid the special configurations in which a corner solution arises either for \(x_t\) or \(\tau_t\), let us assume a sufficiently balanced level of \(a\) such that the following condition applies:

**Assumption 2** \(a \in \left[\frac{\gamma^m(1 - \gamma')}{1 - \gamma m' \gamma'}, \frac{1 - \gamma m''}{1 - \gamma m' \gamma'}\right]\)

Under Assumption 2 and combining the two best response functions (10) and (11), we obtain the following expressions for child rearing time and the tax rate in the static equilibrium:

\[
x_t^s = 1 - \frac{\Gamma_t(1 - \Psi_t)}{(1 - a)(1 - \Psi_t \Gamma_t)} \equiv x^s(\Gamma_t, \Psi_t)
\]

\[
\tau_t^s = 1 - \frac{\Psi_t(1 - \Gamma_t)}{a(1 - \Psi_t \Gamma_t)} \equiv \tau^s(\Gamma_t, \Psi_t)
\]
Through a direct effect, child rearing time is decreasing in $\Gamma_t$, while the tax rate is decreasing in $\Psi_t$. Then, through a substitution effect, child rearing time is increasing in $\Psi_t$, while the tax rate is increasing in $\Gamma_t$. The positive impact of women’s institutional empowerment on government spending, in particular spending targeting children, is supported by several empirical findings. Lott and Kenny (1999) show that, in the US, the extension of the franchise to women had a positive impact on the size of the government. Miller (2008) reaches the same conclusion, particularly for local public health spending. Aïdt et al. (2006) confirm that in Europe, female suffrage increased spending on public goods such as health or education. In developing countries, Brollo and Troiano (2016) show that Brazilian municipalities ruled by female mayors have better health outcomes. Several empirical findings also confirm that a greater intrahousehold empowerment of wives changes household spending in a way benefiting children (see Hoddinot and Haddad 1995, Lundberg et al. 1997, Allendorf 2007, Schady and Rosero 2008, Rubalcava et al. 2009 or Calvi et al. 2018).

Plugging the values of $x^s_t$ and $\tau^s_t$ into the production of human capital (2) leads to the following value of $h_{t+1}$ in the static equilibrium:

$$h_{t+1}^s = \mu \frac{(1 - \Psi_t)(1 - \Gamma_t)}{1 - \Psi_t \Gamma_t} = h^s(\Gamma_t, \Psi_t) \quad (14)$$

Children’s human capital is decreasing in both $\Psi_t$ and $\Gamma_t$. The empowerment of women, in either the public or private sphere, leads to an increase in the overall investments targeting children.

## 3 The Dynamics

### 3.1 The evolution of intrahousehold empowerment

Following de la Croix and Vander Donckt (2010), Rees and Riezman (2012), Baudin et al (2015 and 2018) and Prettner and Strulik (2017), among others, we assume that the balance of power within a household is determined by the relative income of the husband and the wife. A rationale for this assumption is that higher earnings allow a spouse to have a better outside option, which improves her/his bargaining position. Another argument lies in the fact that a spouse who contributes more to the household budget is likely to have greater legitimacy to make decisions about this budget. These arguments are captured in the following stylized way.

When the two spouses have the same income ($w^m_t = w^f_t$), the decision power is equally shared ($\theta_t = 1/2$). Otherwise, the husband (resp. the wife) obtains more power as $w^m_t/w^f_t$ increases (resp. decreases). We choose the following convenient formulation that is close to that adopted in Prettner.

10. See also Attanasio and Lechene (2002) for empirical evidence.
and Strulik (2017):\textsuperscript{11}

\[ \theta_t = \min \left\{ \frac{1}{2} \left( \frac{w^m_t}{w^f_t} \right), 1 \right\} \]  \tag{15}  

The consumption good is produced by using two types of perfectly substitutable inputs: human capital and physical strength. We assume that the productivity (the wage) of an individual is the sum of her/his endowment of human capital and her/his endowment of physical strength.\textsuperscript{12} Moreover, we normalize the endowment of physical strength of women to zero, while it is assumed to be positive, equal to \( \chi \), for men. Thus, we have \( w^f_t = h_t \) and \( w^m_t = h_t + \chi \), and expression (15) can be rewritten as:

\[ \theta_{t+1} = \min \left\{ \frac{1}{2} \left( 1 + \frac{\chi}{h_{t+1}} \right), 1 \right\} \] \tag{16}

Combining this expression with the definition of \( \Gamma_t \) and equation (14), we obtain the following expression for \( \Gamma_{t+1} \):

\[ \Gamma_{t+1} = \min \{ g(\Gamma_t; \Psi_t), \gamma^m \} \] \tag{17}

with

\[ g(\Gamma_t; \Psi_t) = \frac{1}{2} \left[ \gamma^m + \gamma^f + \frac{\chi(\gamma^m - \gamma^f)(1 - \Psi t \Gamma_t)}{\mu(1 - \Psi t)(1 - \Gamma_t)} \right] \] \tag{18}

This equation describes how the intrahousehold empowerment parameter (\( \Gamma_t \)) evolves over time for a given value of the institutional empowerment parameter (\( \Psi_t \)). According to equations (17) and (18), \( \Gamma_{t+1} \) is increasing in \( \Gamma_t \). Indeed, at date \( t \), if women have less bargaining power within the household, the private provision of human capital is dampened such that \( h_{t+1} \) is low. Then, at the next date, the gender wage gap will be enhanced, and the intrahousehold bargaining position of women will be reduced. To ensure that (for any value of \( \Psi_t \)) the dynamical equation (17) exhibits a unique steady state, we assume that the parameter \( \mu \) is large enough:

**Assumption 3** \( \mu \geq \frac{\chi(\gamma^m - \gamma^f)}{2(1 - \gamma^m)^2} \equiv \hat{\mu} \)

Moreover, to focus our attention on the configurations that are the most interesting for our purposes, we define

\[ \tilde{\gamma}^m = \frac{5 + 3\gamma^f - (1 - \gamma^f) \sqrt{9 + 8\gamma^f}}{4(1 + \gamma^f)} \quad \text{and} \quad \hat{\gamma}^m = \frac{\sqrt{5 + 4\gamma^f} - 1}{2} \]

and we assume that

**Assumption 4** \( \gamma^m \in (\tilde{\gamma}^m, \hat{\gamma}^m) \)

Assumption 4 allows us to abstract from less interesting cases, for instance the case in which, whatever the value of \( \Psi_t \), in the long-run men have all the power (\( \Gamma_t \) always converges towards \( \gamma^m \)).

\textsuperscript{11}Since \( w^m_t > w^f_t \) (as explained below), \( \theta_t \) is always higher than \( 1/2 \). Moreover, the effects of an increase in \( w^m_t \) and an increase in \( w^f_t \) are not symmetric. In particular, when \( w^m_t \) becomes too large with respect to \( w^f_t \), males have all the bargaining power (\( \theta_t = 1 \)). This is an important feature for the emergence of patriarchal traps.

\textsuperscript{12}This assumption is in the spirit of Galor and Weil (1996), Bertocchi (2011) and Hiller (2014).
Then, we claim that

**Lemma 1** Under Assumptions 1-4, we can define

\[ \tilde{\mu} = \frac{(1 + \gamma m)\chi}{1 - \gamma m} \]

such that \( \tilde{\mu} > \hat{\mu} \) and

1) for \( \mu \in [\hat{\mu}, \tilde{\mu}) \): there exists a \( \tilde{\Psi} \in (\gamma f, \gamma m) \) such that if \( \Psi_t < \tilde{\Psi} \), \( \Gamma_t \) converges towards an interior steady state denoted \( \Gamma^*(\Psi_t) \), while if \( \Psi_t \geq \tilde{\Psi} \), \( \Gamma_t \) converges towards \( \gamma m \); 

2) for \( \mu \geq \tilde{\mu} \): for all possible values of \( \Psi_t \), \( \Gamma_t \) converges towards an interior steady state denoted \( \Gamma^*(\Psi_t) \);

where \( \Gamma^*(\Psi_t) \) is the value of the \( \Gamma_t \) solution of equation \( \Gamma_t = g(\Gamma_t, \Psi_t) \) and is increasing in \( \Psi_t \).

**Proof.** See Appendix A

In Figure 2, we depict the dynamics of \( \Gamma_t \) for a given value of \( \Psi_t \) and for \( \mu \in [\hat{\mu}, \tilde{\mu}) \). Whatever its initial value, \( \Gamma_t \) converges towards a unique globally stable steady state that could belong to the interval \( (\gamma f, \gamma m) \) if \( \Psi_t \) is sufficiently low or equals \( \gamma m \) if \( \Psi_t \) is too large. In the latter case, males have full control over the household’s decisions. As mentioned in Lemma 1, the value of \( \Gamma_t \) reached in the long run is increasing in \( \Psi_t \). Indeed, all other things being equal, a higher value of \( \Psi_t \) translates to a lower level of human capital since public spending is reduced. In turn, it widens the gender wage gap and enhances the intrahousehold bargaining position of males.
3.2 The evolution of institutional empowerment

As previously explained, the lack of commitment on the part of the policy maker leads to a suboptimal equilibrium. Following Bisin and Verdier (2017), we assume that, at a given point in time, current institutions choose the power sharing for the next date to correct these imperfections in the political process. In short, institutional evolutions act as a commitment device. For instance, if the lack of commitment leads to an under-provision of human capital, current institutions are willing to transfer more political power to women to reach a situation closer to what is considered optimal by these current institutions.

Formally, at date \( t \), future institutional powers \( (\Psi_{t+1}) \) are designed to maximize the social welfare function evaluated by the current institutions (i.e., using the institutional powers as of date \( t \)). Hence, \( \Psi_{t+1} \) will be the solution of the following maximization problem:

\[
\Psi_{t+1} \in \arg \max_{\Psi} \left\{ \Psi_t \ln c^s(\Gamma_{t+1}^e, \Psi) + (1 - \Psi_t) \ln h^s(\Gamma_{t+1}^e, \Psi) \right\}
\]  

(19)

with \( \Gamma_{t+1}^e \) being the expectation, from the perspective of date \( t \), of the value of the private sphere empowerment index that would prevail in \( t + 1 \). According to (19), \( \Psi_{t+1} \) is chosen such that the static equilibrium on date \( t + 1 \) corresponds to the equilibrium that would have prevailed had the policy maker been able to commit to her choices. Thus, Bisin and Verdier (2017) show that the solution of the maximization problem (19) corresponds to the value of \( \Psi_{t+1} \) such that:

\[
\tau^s(\Gamma_{t+1}^e, \Psi_{t+1}) = \tau^c(\Gamma_{t+1}^e, \Psi_t)
\]  

(20)

where \( \tau^c(\Gamma_t, \Psi_t) \) is the level of public spending that would have been chosen at date \( t \) in the hypothetical situation in which the commitment problem is settled. Equation (20) may be interpreted as follows. Institutions today anticipate that the intrahousehold bargaining power tomorrow will be \( \Gamma_{t+1}^e \). Hence, they anticipate that the “optimal” policy, from their perspective, will be \( \tau^c(\Gamma_{t+1}^e, \Psi_t) \). Finally, they also anticipate that if institutions do not change, this level will not be reached due to the commitment problem. Hence, current institutions re-design political power (i.e., choose \( \Psi_{t+1} \)) such that the policy that will prevail tomorrow in equilibrium \( (\tau^s(\Gamma_{t+1}^e, \Psi_{t+1})) \) corresponds to the “optimal” policy \( (\tau^c(\Gamma_{t+1}^e, \Psi_t)) \).

The equilibrium with commitment may be defined as follows:

13. See also Bisin et al. (2018) or Iyigun et al. (2018).
14. According to expression (19), institutional changes are myopic in the sense that institutional powers are designed for the next date without taking into account the fact that they will be re-designed afterward (see Bisin and Verdier 2017 for a discussion).
15. See the proof of Proposition 2 in Bisin and Verdier (2017) for a proof of this statement in a general setting and Appendix B for a proof in our setting.
Definition 2 The equilibrium with commitment is defined as the pair \( \{ x^c_t, \tau^c_t \} \) such that:

\[
\tau^c_t \in \arg \max_{\tau_t} \left\{ \Psi_t \ln((1 - \tau_t)(1 - x_t)) + (1 - \Psi_t) \ln(a\tau_t + (1 - a)x_t) \right\}
\]

(21)

s.t. \( x_t \in \arg \max_{x_t} \left\{ \Gamma_t \ln(1 - \tau_t) + (1 - \Gamma_t) \ln(a\tau_t + (1 - a)x_t) \right\} \)

(22)

According to this definition, the planner chooses \( \tau^c_t \) in order to maximize the social welfare function (equation (21)), taking into account that households choose \( x_t \) in order to maximize the collective household utility function (equation (22)). Hence, to solve for the equilibrium with commitment, we plug the best response function of the household (10) into the social welfare function, and we deduce the optimal level of the tax rate. It yields:

\[
\tau^c_t = 1 - \frac{\Psi_t}{a(1 + \Psi_t)} \equiv \tau^c(\Psi_t)
\]

(23)

Combining equations (13), (20) and (23), we obtain:

\[
\Psi_{t+1} = \frac{\Psi_t}{1 + \Psi_t - \Gamma^{e}_{t+1}}
\]

(24)

Finally, assuming perfect foresight \( (\Gamma^{e}_{t+1} = g(\Gamma_t; \Psi_t)) \)\(^{16}\), we obtain the following expression for the law of motion of \( \Psi_{t+1} \):

\[
\Psi_{t+1} = \frac{\Psi_t}{1 + \Psi_t - \min\{g(\Gamma_t; \Psi_t), \gamma^m\}} \equiv h(\Psi_t; \Gamma_t)
\]

(25)

The analysis of this dynamical equation leads us to conclude that:

Lemma 2 Under Assumptions 1-4

1) for \( \mu \in [\hat{\mu}, \tilde{\mu}] \): there exists a \( \tilde{\Gamma} \in (\gamma^f, \gamma^m) \) such that if \( \Gamma_t < \tilde{\Gamma} \), \( \Psi_t \) converges towards an interior steady state denoted \( \Psi^*(\Gamma_t) \), while if \( \Gamma_t \geq \tilde{\Gamma} \), \( \Psi_t \) converges towards \( \gamma^m \);

2) for \( \mu \geq \tilde{\mu} \): for all possible values of \( \Gamma_t \), \( \Psi_t \) converges towards an interior steady state denoted \( \Psi^*(\Gamma_t) \),

where \( \Psi^*(\Gamma_t) \) is the value of the \( \Psi_t \) solution of equation \( \Psi_t = g(\Gamma_t; \Psi_t) \) and is increasing in \( \Gamma_t \).

Proof. See Appendix B

To understand the results stated in Lemma 2, let us, in a first step, analyze the process of the evolution of \( \Psi_t \) when considering the planner’s expectations \( (\Gamma^{e}_{t+1}) \) as given; then, in a second step, take into account the way expectations are formed.

\(^{16}\) The results of Lemma 2 are robust to alternative assumptions on the nature of expectations, in particular to myopic expectations.
In Figure 3, we have drawn \( \tau^c(\Psi_t) \) and \( \tau^{s,e}(\Psi_t) \equiv \tau^s(\Gamma_{t+1}^e, \Psi_t) \), where \( \tau^{s,e}(\Psi_t) \) corresponds to the public policy adopted in the static equilibrium, expressed as a function of \( \Psi_t \), when expectations \( (\Gamma_{t+1}^e) \) are fixed.\(^{17}\) This allows us to describe the institutional dynamics as stated by equation (20): The horizontal arrows correspond to the adjustment of \( \Psi_t \) between two consecutive dates, and the vertical arrows correspond to the associated change in the equilibrium public policy \( (\tau^{s,e}(\Psi_t)) \). As shown in this figure, the value of \( \Psi_t \) adjusts over time to ensure that the equilibrium value of the economic policy anticipated for tomorrow \( (\tau^{s,e}(\Psi_{t+1})) \) corresponds to the public policy that would have been chosen in equilibrium with commitment by the current institutions \( (\tau^c(\Psi_t)) \).

![Figure 3. The dynamics of \( \Psi_t \) for a given value of \( \Gamma_{t+1}^e \)](image)

We now have to take into consideration the fact that the expectations over private empowerment depend on the current values of \( \Psi_t \) and \( \Gamma_t \) (under the assumption of perfect foresight \( \Gamma_{t+1}^e = g(\Gamma_t; \Psi_t) \)). Figure 4 describes the dynamics of \( \Psi_t \) for a given value of \( \Gamma_t \) and taking into account the way expectations are formed. This figure illustrates that, whatever the initial conditions, when \( \Gamma_t \) remains fixed, women’s bargaining power in the public sphere converges towards a unique steady state.

According to Lemma 2, \( \Psi^*(\Gamma_t) \) is increasing in \( \Gamma_t \) such that private empowerment will translate into a rise in the long-term value of \( \Psi_t \). Indeed, as \( \Gamma_t \) increases, households reduce the time they invest in children \( (x_t^s) \). To compensate for this reduction, the planner increases public investment \( (\tau_t^s) \). However, due to the absence of commitment, the decrease in \( x_t^s \) and the increase in \( \tau_t^s \) are exacerbated relative to the case in which the planner is able to pre-commit to a particular public policy. Then, to make the static equilibrium closer to the equilibrium with commitment, more

\(^{17}\)To derive Figure 3, it is sufficient to note that, according to expressions (13) and (23), \( \tau^{s,e}(\Psi_t) \) is decreasing and concave, \( \tau^c(\Psi_t) \) is decreasing and convex and equation \( \tau^{s,e}(\Psi_t) = \tau^c(\Psi_t) \) admits one unique solution on \([\gamma^f, \gamma^m]\): \( \Psi_t = \Gamma_{t+1}^c \).
institutional power has to be transferred to men (i.e., Ψₜ has to increase), such that τᵣ will decrease (and, thereby, becomes closer to τᶜ), and by a substitution effect, xₛ will increase.

3.3 The joint evolution of intrahousehold and institutional empowerment

As developed in Sections 3.1 and 3.2, there exists a mutual interplay between the evolution of women’s bargaining power in the private and public spheres. The joint evolution of (Γₜ, Ψₜ) is described by the two-dimensional first-order dynamical system given by equations (17) and (25). According to Lemma 1, the stationary value of men’s bargaining power in the private sphere (Γₜ) is positively related to their institutional power in the public sphere (Ψₜ). In turn, according to Lemma 2, the stationary value of Ψₜ is increasing in Γₜ. These complementaries between the two facets of empowerment may give rise to multiple equilibria. Indeed, a low institutional power for women may impede the development of public policies allowing for their emancipation in the private sphere (see Section 3.1). In turn, limited intrahousehold bargaining power for women will prevent their enfranchisement in the public sphere (see Section 3.2).

In the following proposition, we state that several steady states can indeed co-exist, and we specify the configurations in which this is the case.

Proposition 1 Under Assumptions 1-4, there exists a μ′ ∈ (μ, ̄μ) such that:

i If μ ≥ ̄μ: (Γₜ, Ψₜ) converges towards a unique interior steady state (Γ*, Ψ*) with Ψ* = Γ*.

ii If μ ∈ [μ′, ̄μ): (Γₜ, Ψₜ) may either converge towards the interior steady state (Γ*, Ψ*) with Ψ* = Γ* or towards the patriarchal steady state (γᵐ, γᵐ).
iii If $\mu \in [\bar{\mu}, \mu')$: $(\Gamma_t, \Psi_t)$ converges towards the patriarchal steady state $(\gamma^m, \gamma^m)$, where $\Gamma^* \in \left(\frac{\gamma^f + \gamma^m}{2}, \gamma^m\right)$ is the value of $\Gamma_t$ solution of equation $\Gamma^*(\Gamma_t) = \Gamma_t$.

**Proof.** See Appendix C

The results of Proposition 1 could be intuitively represented by drawing the phase diagram associated with the joint dynamics of $\Gamma_t$ and $\Psi_t$. This diagram is given in Figure 5 for each of the configurations listed in Proposition 1. In this figure, $\Gamma$ is the stationary locus of $\Gamma_t$, defined as the set of pairs $(\Gamma_t, \Psi_t)$ such that $\Gamma_t$ is constant: $\Gamma_t = \{ (\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m] : \Gamma_{t+1} = \Gamma_t \}$, and $\Psi$ is the stationary locus of $\Psi_t$: $\Psi = \{ (\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m] : \Psi_{t+1} = \Psi_t \}$. The steady-state equilibria of the joint dynamics of $(\Gamma_t, \Psi_t)$ are given by the crossing points between the $\Gamma$ locus and the $\Psi$ locus. Finally, the motion arrows indicate how $\Psi_t$ and $\Gamma_t$ evolve off the stationary loci.

As argued in Proposition 1, the long-run evolution of women’s empowerment may dramatically differ according to the value of $\mu$. When $\mu$ is very low (lower than $\bar{\mu}$), the productivity of human capital is quite depressed with respect to $\chi$, so that, whatever the prevailing value of $\Psi_t$, the gender wage gap is large and $\Gamma_t$ converges towards $\gamma^m$. Then, $\Psi_t$ progressively adjusts and also converges towards $\gamma^m$. In the long run, all the power, in both the private and public spheres, is in the hands of males (the economy belongs to the patriarchal equilibrium $E_1$ in Figure 5(A)). In contrast, when $\mu$ is very high (higher than $\bar{\mu}$), the gender wage gap is relatively narrow even for large values of $\Psi_t$. As a consequence, in the long run, the bargaining power within the family remains relatively balanced ($\Gamma_t < \gamma^m$), even if males have all institutional power. Hence, the economy must evolve towards an interior steady state in which, in both spheres, the power is somehow shared between men and women (equilibrium $E_3$ in Figure 5(C)).

Let us now focus on the intermediate configuration: $\mu \in [\mu', \bar{\mu})$. As depicted in Figure 5(B), in that configuration, two locally stable steady states co-exist. If, initially, the bargaining power of men is high in both the private and public spheres – $(\Gamma_t, \Psi_t)$ is close to the upper-right corner – investments in human capital are markedly reduced and the gender wage gap is large. Hence, $\Gamma_t$ tends to increase, inducing further reductions in private spending devoted to human capital and widening the gender wage gap. Then, this increase is reinforced by the rise in the institutional power of males ($\Psi_t$). Finally, the economy ends up in the patriarchal equilibrium $E_1$ in which men own all the bargaining power in both the private and the public sphere. For lower initial values of $\Gamma_t$ and/or $\Psi_t$, the initial gender wage gap is sufficiently low to allow for a decrease in $\Gamma_t$ that is reinforced by a reduction in $\Psi_t$. In that case, the economy ends up in the interior equilibrium $E_3$.

Let us also emphasize that the emergence of multiple equilibria is intimately linked to this process of mutual reinforcement between the evolution of $\Gamma_t$ and $\Psi_t$. Indeed, as shown in Sections 3.1 and 3.2, when separately considering the dynamics of these two variables, each of them converges.

18.Nevertheless, due to the existence of an exogenous wage premium for men, full equality cannot be achieved in the long run: At point $E_3$, we have $\Gamma_t = \Psi_t > (\gamma^m + \gamma^f)/2$. 18
Figure 5. The phase diagram

towards one unique and globally stable steady state. Let us also remark that the two locally stable steady states (E₁ and E₃) can obviously be ranked in terms of the level of empowerment for women but also in terms of human capital (and income) and in terms of gender inequality. Indeed, \( h_{t+1}^s \) is decreasing in both \( \Psi_t \) and \( \Gamma_t \) such that the level of human capital in the economy is higher at the interior steady state \( E_3 \) than in the patriarchal equilibrium \( E_1 \). Moreover, the gender wage gap \( w_{mt}/w_{ft} \) is decreasing in the level of human capital \( h_t^s \) such that the interior steady state \( E_3 \) is less inegalitarian than the patriarchal steady state \( E_1 \). Hence, our model allows us to shed light on
a new kind of gender inequality trap (World Bank 2006), which we refer to as a *patriarchal trap*. These features, up to the fact that the most balanced bargaining power among genders may be desirable for its own sake, offer a rationale for policy interventions helping to escape the patriarchal trap and, more broadly, promoting women having a say in the long run (through a displacement of the interior steady state $E_3$ down to the left). Some of those policies are discussed in the next section.

4 Comparative statics

Two broad types of reforms have been proposed and sometimes implemented to favor women’s empowerment in developing countries. The first, related to the private sphere, consists of targeting some public transfers to women.\textsuperscript{19} The second type, related to the public sphere, consists of ensuring a minimal political representation of women. In the comparative statics exercises we propose in this section, we assess the effectiveness of each of these policies in our setup. Our comparative statics results will also serve as a basis for a discussion of path dependence in women’s empowerment.

4.1 Positive shock to women’s bargaining power in the private sphere

Within the household, the bargaining position of women improves when the gender wage gap narrows. This reduction may be captured, in the model, by a positive shock to $\mu$ (according to equation (17), all other things being equal, $\Gamma_{t+1}$ is decreasing in $\mu$).\textsuperscript{20} The effects of such a shock to the long-run situation reached by the economy, already described in Proposition 1, are summarized in the following bifurcation diagram (Figure 6).\textsuperscript{21}

When $\mu < \mu'$, $(\Gamma_t, \Psi_t)$ converges towards the patriarchal steady state $(\gamma^m, \gamma^m)$. Once $\mu$ reaches $\mu'$, two additional steady states appear. The lower one – $E_3$ in Figure 5(B) – is stable, and the second one – $E_2$ in Figure 5(B) – is unstable. When $\mu \in (\mu', \tilde{\mu})$, as $\mu$ increases, the basin of attraction of the patriarchal equilibrium shrinks (the dotted line is upward sloping), while women’s bargaining

\textsuperscript{19}Most conditional cash transfer programs (which consist of offering public benefits to poor households, conditional upon the investment, by the receivers, in the human capital of their children), such as PROGRESA/Opportunidades in Mexico, direct the transfer to women, not men. This design has been adopted for many reasons, for instance, on the grounds that, as already discussed, evidence suggests that women will be more prone to spend money on children than men. Nevertheless, the indirect benefits associated with enhanced bargaining power for women have also been proposed to justify targeting towards women (see Fiszbein and Schady 2009). Microcredit targeted towards female entrepreneurs is also considered a means to promote female empowerment (the findings by Hashemi et al 1996 or Angellucci et al 2015 support this view).

\textsuperscript{20}In fact, the negative impact of $\mu$ on $\Gamma_{t+1}$ could operate through two channels: i) a *direct channel* since $h_{t+1}$ is increasing in $\mu$ and $\Gamma_{t+1}$ is decreasing in $h_{t+1}$, and ii) an *indirect channel* since a rise in $\mu$, by enhancing the returns of human capital investments, may increase the public and private spending dedicated to human capital. However, in our setting, with a log-linear specification for the utility function, the indirect channel is canceled out ($x^*_t$ and $\tau^*_t$ are independent of $\mu$). Hence, a change in $\mu$ can be interpreted as an exogenous shock on the intrahousehold bargaining power.

\textsuperscript{21}In Figures 6 and 7, the locally stable steady states are depicted by solid lines while the unstable steady state is depicted by a dotted line.
power increases in the interior equilibrium (the solid line is downward sloping). Finally, when $\mu > \tilde{\mu}$, $(\Gamma_t, \Psi_t)$ always converges towards the interior steady state in which women’s empowerment is enhanced as $\mu$ rises.

Hence, the long-run impact of an exogenous decrease in the gender earnings gap on women’s empowerment crucially depends on the initial steady state to which the economy belongs. If the economy remains trapped in the patriarchal equilibrium, an increase in $\mu$ reduces the gender wage gap, but it does not trigger any change in $\Gamma_t$ or $\Psi_t$. In contrast, in the interior equilibrium, the initial decline in $\frac{w^m_t}{w^f_t}$ is reinforced by the downward adjustment of $\Gamma_t$ and $\Psi_t$. Finally, if an economy is initially in the patriarchal equilibrium and the rise in $\mu$ allows us to overcome the threshold $\tilde{\mu}$, this destabilizes this equilibrium and allows for a convergence towards the interior equilibrium. In that case, the process of mutual reinforcement between women’s empowerment in the public sphere and in the private sphere leads to a major increase in children’s human capital and a dramatic decline in the gender pay gap.

In summary, in countries in which women’s empowerment is very low, cash transfers targeted to women are likely to have essentially innocuous effects on female empowerment. Conversely, only a sufficiently large shock might make things change drastically. These contrasting results concerning the impact of an exogenous shock to women’s empowerment in the private sphere echo the mixed cross-country evidence for the impact of targeted transfers or credit policies on empowerment and gender equality in developing countries (see, for instance, Kabeer 2001, Mabsout and van Stereven 2010 and the discussion in Prettner and Strulik 2017).\footnote{Note that women’s empowerment in the private sphere may also come from deeply rooted socio-cultural factors. For instance, men have structurally more decision power in a patrilineal society than in a matrilineal society (see Anderen et al. 2013 or Shu et al. 2013). To that extent, the results presented in this section are also consistent with 21}
This comparative statics exercise also contributes to the debates around the long-run impacts of a temporary cash transfer policy (see Molina Millán et al. 2019). In particular, the virtuous cycle of empowerment offers a new channel through which the impact of cash transfer programs on women’s empowerment and household behavior may be sustained even after transfers end. To see this, let us consider an economy with \( \mu \in (\mu', \bar{\mu}) \) that is trapped in the patriarchal equilibrium. Let us also consider a policy allowing us to set \( \mu > \bar{\mu} \) such that the economy converges towards the interior equilibrium. Then, once the economy has reached the basin of attraction of this interior equilibrium, it will still converge towards this equilibrium even if \( \mu \) returns to its initial value. In contrast, if the scope and/or the duration of the policy is too low to allow the economy to join the basin of attraction of the interior equilibrium, gender inequalities will progressively return to their initial levels as soon as the policy is terminated.

4.2 Positive shock to women’s bargaining power in the public sphere

Many countries have adopted measures, such as quotas or reservation policies, to enforce women’s access to political positions (see Duflo 2012). In our model, these measures could be captured by an exogenous limit \( \bar{\beta} \) on the institutional bargaining power of men such that \( \beta_t \leq \bar{\beta} \) with \( \bar{\beta} \geq 1/2 \). Under this condition, the law of motion of \( \Psi_t \) may be rewritten as:

\[
\Psi_{t+1} = \min\{ h(\Psi_t; \Gamma_t), \bar{\Psi} \} \quad \text{with} \quad \bar{\Psi} = \gamma^f + \bar{\beta}\Delta\gamma,
\]

with \( h(\Psi_t; \Gamma_t) \) defined in equation (25). Under this specification, higher gender quotas are captured by a decrease in \( \bar{\Psi} \).

Regarding the long-run dynamics of the economy (as depicted by the phase diagrams in Figure 5), the only change lies in the fact that the value of \( \Psi_t \) is bounded above by \( \bar{\Psi} \). The impact of a change in \( \bar{\Psi} \) on the value of \( \Psi_t \) and \( \Gamma_t \) reached by the economy in the long run is summarized in the bifurcation diagrams depicted in Figure 7. In these diagrams, we focus on the most interesting case in which multiple equilibria arise in the absence of a public intervention: \( \mu \in [\mu', \bar{\mu}) \).

The configuration in which \( \bar{\Psi} \) is higher than \( \gamma^m \) captures the case without any affirmative action policy such that the dynamics of the economy are described by the phase diagram in Figure 5(B). Starting from \( \bar{\Psi} = \gamma^m \), a decrease in \( \bar{\Psi} \) corresponds to a more active affirmative action policy. It mechanically decreases the equilibrium value of \( \Psi_t \) (the participation of women in the political process is enforced); however, as long as \( \bar{\Psi} \) remains higher than \( \bar{\Psi} \), this does not trigger any empowerment of women within the household (\( \Gamma_t \) remains equal to \( \gamma^m \)). Then, as the participation of women in the political process increases public spending on human capital, the gender wage gap

the fact that the political participation of women is higher in matrilineal societies (see Gottlieb and Robinson 2016 or Brulé and Gaikwad 2018).

23. The derivation of the thresholds \( \bar{\Psi} \), \( \bar{\Psi} \) and \( \Psi^* \), as well as the derivation of the steady states of the economy according to the value of \( \Psi \), are presented in Appendix D.
shrinks, and as soon as $\Psi$ becomes smaller than $\hat{\Psi}$, the bargaining power of women also increases in the private sphere. Up to the point where $\Psi = \bar{\Psi}$, the reservation policy is binding, meaning that all the improvement in the political voice of women is enforced by law. Nevertheless, further reductions in $\bar{\Psi}$ will trigger endogenous declines in both $\Gamma_t$ and $\Psi_t$ along the convergence path towards the interior equilibrium. When $\Psi$ belongs to the interval $[\Psi^*, \hat{\Psi}]$, the reservation policy is no longer binding; however, since power sharing in the private sphere remains inegalitarian, men have more institutional power than women in the steady state, and there is still room for affirmative action policies. Further, when $\bar{\Psi}$ becomes lower than $\Psi^*$ (the level of $\Psi_t$ compatible with the interior equilibrium), the affirmative action policy is binding again. Hence, any decrease in $\bar{\Psi}$ translates into the empowerment of women in the public sphere, and when $\bar{\Psi}$ equals $\frac{\gamma^m + \gamma^f}{2}$, men and women end up with the same institutional power.\footnote{Note that this situation cannot be achieved in the long run without affirmative action policies (see Duflo 2012).}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Steady states of $\Psi_t$ (first panel) and $\Gamma_t$ (second panel) as a function of $\bar{\Psi}$}
\end{figure}
higher bargaining power than women in the private sphere.

Again, the effectiveness of such an affirmative action policy depends on initial conditions. For an economy subject to the inegalitarian trap, these reforms, while allowing for better representation of women’s interests in the political decision-making process, may fail to trigger female empowerment in other dimensions (see Beath et al 2013). However, such quota policies, even when temporary, can have a major impact on women’s empowerment if they are sufficiently ambitious to allow the economy to escape the inegalitarian trap. The level of $\Psi$ that permits such an escape (i.e., the threshold $\tilde{\Psi}$) obviously depends on other parameters. For instance, a given decrease in $\Psi$ is more likely to initiate a virtuous circle of empowerment when the gender wage gap is relatively low ($\mu$ is relatively high compared to $\chi$). For this reason, affirmative action policies intended to empower women in the public sphere could be more effective when combined with cash transfer programs intended to empower women within the household (and vice versa).25

4.3 Path dependency

As developed in the two previous sections, the level of women’s empowerment depends on both initial conditions (since multiple equilibria may co-exist) and parametric conditions (for instance, the value of $\mu$, $\chi$ or $\tilde{\Psi}$). Because of this, path dependence phenomena (i.e., situations in which exogenous discrepancies in the distant past may be magnified by endogenous changes) may arise. To see this, let us consider two economies that initially differ according to their value of $\chi$. This could be, for instance, attributed to technological differences, with one of the two countries being characterized by technologies relying more on physical strength such that the gender gap in productivity is higher in this country. The impact of $\chi$ is essentially symmetric to the impact of $\mu$ discussed in Section 4.1. Hence, it could be the case that for the country with the higher $\chi$ (say Country 1), $(\gamma^m, \gamma^m)$ is the unique globally stable steady state, while for the other country (say Country 2), the inegalitarian steady state co-exists with an interior steady state. If, in Country 2, the balance of power between men and women is not too unequal, it would converge towards the interior equilibrium. Then, even if, later in time, the technology relying on physical strength becomes obsolete and Country 1 adopts exactly the same technology as Country 2 (the parameter $\chi$ becomes the same in the two countries), Country 1 will remain trapped in a situation in which males hold all the power; in contrast, in Country 2, women will be, to some extent, empowered. This path dependency property may be related to the empirical findings obtained by Alesina et al. (2013), Xue (2018) or Teso (2019).26 In particular, Alesina et al. (2013) conclude that countries characterized by agricultural

25. In the opposite direction, an improvement in the bargaining position of women in the private sphere (through an increase in $\mu$, as discussed in the previous section) is more likely to take the economy out of the patriarchal trap if $\Psi$ is low. This result echoes the empirical findings of Mabsout and van Stereven (2010), according to which policies intended to improve women’s bargaining power within the family may be unproductive when women’s institutional empowerment is too low.

26. See also the enlightening survey by Giuliano (2018).
forms that rely heavily on physical strength (such as plow cultivation) at low development stages are more likely to exhibit low female participation in politics at present. Our model proposes new mechanisms related to the interplay between empowerment in the public and in the private sphere to account for the persistence of those gender inequality traps.\textsuperscript{27}

5 Conclusion

In the existing literature, the different dimensions of women’s empowerment are considered in isolation. In this paper, we argue that this should not be the case, and we propose a simple theoretical framework in which women’s decision power is endogenous, in both the private and public spheres. We conclude that the mutual interplay between these two facets of empowerment may lead to a patriarchal trap and that, for countries stuck in those traps, economic development may fail to initiate the virtuous cycle of empowerment (see Duflo 2012). However, well-targeted economic policies may help those countries escape from such a trap. In particular, we stress that public interventions intended to improve the bargaining position of women within the household and those targeting the political empowerment of women should be considered complementary.

While an emergent empirical literature emphasizes the role played by the interactions between different facets of women’s decision power as an impediment to women’s empowerment (see Mabsout and van Stereven 2010, Gottlieb and Robinson 2016 or Brulé and Gaikwad 2018), the implications in terms of public intervention have been largely ignored thus far. To that extent, some of the predictions of our model deserve to be empirically investigated. In particular, when a public policy intended to empower women in one sphere (e.g., conditional cash transfers or quotas) is evaluated, women’s bargaining position in other spheres should be more systemically taken into account. Moreover, we advocate policy experiments that combine instruments promoting women’s decision power both in the family and in politics.

Finally, the framework we propose is sufficiently tractable to be extended in several dimensions. For instance, human capital accumulation could be easily introduced to determine the interplay between the development process and the different facets of empowerment. Such an introduction is likely to reinforce our results since patriarchy tends to impede development, while low development may sustain patriarchy (see Doepke and Tertilt 2009). In the current version of the model, patriarchal traps emerge even in the absence of feedback effects between empowerment and the human capital accumulation process.

\textsuperscript{27}Hiller (2014) and Hiller and Baudin (2016) propose alternative mechanisms based on the cultural transmission of inegalitarian gender norms and preferences.
A Proof of Lemma 1

By differentiating the expression (17) with respect to $\Gamma_t$ and then $\Psi_t$ we get that:

$$\frac{\partial g(\Gamma_t; \Psi_t)}{\partial \Gamma_t} = \frac{\chi(\gamma^m - \gamma^f)}{2\mu(1-\Gamma_t^2)} > 0 \quad \text{and} \quad \frac{\partial g(\Gamma_t; \Psi_t)}{\partial \Psi_t} = \frac{\chi(\gamma^m - \gamma^f)}{2\mu(1-\Psi_t^2)} > 0$$

Hence, $g(\Gamma_t; \Psi_t)$ is increasing and convex in $\Gamma_t$ while an increase in $\Psi_t$ shifts the function $g(\Gamma_t; \Psi_t)$ upwards. Moreover, it is easy to verify that $g(\gamma^f; \gamma^f) > \gamma^f$ and that Assumption 3 ensures that the slope of $g(\Gamma_t; \Psi_t)$ with respect to $\Gamma_t$ when $\Gamma_t = \gamma^m$ is lower than one. Moreover, the threshold $\hat{\mu}$ is defined s.t. $g(\gamma^m; \gamma^m) > \gamma^m$ if and only if $\mu < \hat{\mu}$. Finally, $g(\gamma^f; \gamma^f) < \gamma^m$ if $\mu > \frac{\chi(1+\gamma^f)}{2(1-\gamma^f)}$ and this threshold is lower than $\hat{\mu}$ if $\gamma^m > \tilde{\gamma}^m$. Hence, under Assumptions 3 and 4, $g(\gamma^f; \gamma^f) < \gamma^m$.

Putting all these elements together, we deduce that:

i for $\mu \in [\hat{\mu}, \tilde{\mu})$: $g(\gamma^m; \gamma^m) > \gamma^m > g(\gamma^f; \gamma^f)$ meaning that there exists a unique value of $\Psi_t$ (denoted $\tilde{\Psi}$) such that $g(\gamma^m; \tilde{\Psi}) = \gamma^m$. Hence, if $\Psi_t < \tilde{\Psi}$, $g(\Gamma_t; \Psi_t)$ crosses the 45$\degree$ line only once between $\gamma^f$ and $\gamma^m$ and $\Gamma_t$ converges towards an interior steady state\(^{28}\); while, if $\Psi_t \geq \tilde{\Psi}$, $g(\Gamma_t; \Psi_t) > \Gamma_t$ and $\Gamma_t$ converges towards $\gamma^m$;

ii for $\mu \geq \tilde{\mu}$: $g(\gamma^f; \gamma^f) < g(\gamma^m; \gamma^m) < \gamma^m$ meaning that for all possible values of $\Psi_t$, $g(\Gamma_t; \Psi_t)$ crosses the 45$\degree$ line only once between $\gamma^f$ and $\gamma^m$ and $\Gamma_t$ converges towards an interior steady state.

Finally, $\tilde{\mu}$ is larger than $\hat{\mu}$ if and only if $\gamma^m < \tilde{\gamma}^m$ defined as:

$$\tilde{\gamma}^m \equiv \frac{\sqrt{17 + 8\gamma^f} - 1}{4}$$

Since, for all $\gamma^f \in (0, 1)$, $\tilde{\gamma}^m > \hat{\gamma}^m$, Assumption 4 ensures that $\tilde{\mu} > \hat{\mu}$.

B Proof of Lemma 2

Let us first prove that the value of $\Psi_{t+1}$ given by equation (20) corresponds to the solution of the maximization problem (19). Using, expressions (13) and (23), we deduce that equation (20) is satisfied for:

$$\Psi_{t+1} = \frac{\Psi_t}{1 + \Psi_t - \Gamma_t^e} \quad \text{(B.1)}$$

Moreover, using expressions (3), (12) and (13), the maximization problem (19) may be rewritten as:

$$\Psi_{t+1} \in \arg \max_{\Psi_t} \left\{ \ln(1 - \Psi_t) + \Psi_t \ln \Psi_t - (1 + \Psi_t) \ln(1 - \Gamma_t^e \Psi_t) \right\} \quad \text{(B.2)}$$

\(^{28}\) This interior steady state is denoted $\Gamma^*(\Psi_t)$. Since $g(\Gamma_t; \Psi_t)$ is upward shifted when $\Psi_t$ rises, $\Gamma^*(\Psi_t)$ is increasing in $\Psi_t$. 

26
The first order condition associated with the above maximization problem is:

\[
\frac{-1}{1 - \Psi_{t+1}} + \frac{\Psi_t}{\Psi_{t+1}} + \frac{\Gamma_{t+1}^e (1 + \Psi_t)}{1 - \Gamma_{t+1}^e \Psi_{t+1}} = 0
\]

after simple algebra, this equation reduced to (B.1).

Let us now prove the result stated in the proposition. Since \(g(\Gamma_t; \Psi_t)\) is symmetric, we can deduce from the proof of Lemma 1 (Appendix A), that:

i. when \(\mu \geq \tilde{\mu}\), \(g(\Gamma_t; \Psi_t) \in (\gamma^m, \gamma^f)\) for all \((\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m]^2\);

ii. when \(\mu \in [\mu, \tilde{\mu}]\), there exists a threshold value on \(\Gamma_t\) – let us denote this value \(\tilde{\Gamma} \in (\gamma^f, \gamma^m)\) – such that:
   - for \(\Gamma_t < \tilde{\Gamma}\), \(g(\Gamma_t; \Psi_t) \in (\gamma^m, \gamma^f)\) for all \(\Psi_t \in [\gamma^f, \gamma^m]\),
   - for \(\Gamma_t \geq \tilde{\Gamma}\), \(g(\Gamma_t; \Psi_t) \in (\gamma^m, \gamma^f)\) if \(\Psi_t < \tilde{\Psi}\) and \(g(\Gamma_t; \Psi_t) \geq \gamma^m\) if \(\Psi_t \geq \tilde{\Psi}\).

From this, when \(\mu \geq \tilde{\mu}\) or when \(\mu \in [\tilde{\mu}, \bar{\mu}]\) and \(\Gamma_t < \tilde{\Gamma}\), equation (20) may be rewritten as:

\[
\Psi_{t+1} = \frac{\Psi_t}{1 + \Psi_t - g(\Gamma_t; \Psi_t)} \equiv \hat{h}(\Psi_t; \Gamma_t)
\]

with

\[
\frac{\partial \hat{h}(\Psi_t; \Gamma_t)}{\partial \Psi_t} = \frac{1 - g + g' \Psi}{(1 + \Psi_t - g)^2} > 0
\]

where the inequality comes from the fact that \(g(\Gamma_t; \Psi_t) \in (\gamma^m, \gamma^f)\) and \(g'(\Gamma_t; \Psi_t) > 0\) (see Appendix A). Moreover

\[
\hat{h}(\gamma^f; \Gamma_t) = \frac{\gamma^f}{1 + \gamma^f - g(\Gamma_t; \gamma^f)} \geq \gamma^f
\]

\[
\hat{h}(\gamma^m; \Gamma_t) = \frac{\gamma^m}{1 + \gamma^m - g(\Gamma_t; \gamma^f)} \leq \gamma^m
\]

where the inequalities come from the fact that \(g(\Gamma_t; \Psi_t) \in (\gamma^f, \gamma^m)\). Hence, for all value of \(\Gamma_t\), the function \(\hat{h}(\Psi_t; \Gamma_t)\) crosses the 45° line only once, and from above, on the interval \([\gamma^f, \gamma^m]\). Hence, the value of \(\Psi_t\) corresponding to this crossing point – that we denote \(\Psi^*(\Gamma_t)\) – is the unique and globally stable steady state of the dynamics of \(\Psi_t\). Finally, since \(g(\Gamma_t; \Psi_t)\) is increasing in \(\Gamma_t\), \(\hat{h}(\Psi_t; \Gamma_t)\) is upward shifted when \(\Gamma_t\) rises such that \(\Psi^*(\Gamma_t)\) increases with \(\Gamma_t\).

Let us now consider the situation where \(\mu \in [\tilde{\mu}, \bar{\mu}]\) and \(\Gamma_t \geq \tilde{\Gamma}\). In this case, equation (20) may be rewritten as:

\[
\Gamma_{t+1} = \begin{cases} 
\hat{h}(\Psi_t; \Gamma_t) & \text{if } \Psi_t < \tilde{\Psi} \\
\hat{h}(\Psi_t) & \text{if } \Psi_t \geq \tilde{\Psi}
\end{cases}
\]

with \(\hat{h}(\Psi_t) \equiv \frac{\Psi_t}{1 + \Psi_t - \gamma^m}\)
We already know the properties of \( \tilde{h}(\Psi_t; \Gamma_t) \). Let us now observe that \( \tilde{h}(\gamma^m) = \gamma^m \) and, for all \( \Psi < \gamma^m \), \( \tilde{h}(\Psi, \Gamma) = \tilde{h}(\Psi) > \Psi \) where the equality comes from the definition of \( \Psi \) and the inequality comes from the fact that \( \Psi < \gamma^m \). Moreover

\[
\tilde{h}'(\Psi_t) = \frac{1 - \gamma^m}{(1 + \Psi_t - \gamma^m)^2} > 0 \quad \text{and} \quad \tilde{h}''(\Psi_t) = \frac{-2(1 - \gamma^m)}{(1 + \Psi_t - \gamma^m)^3} < 0
\]

so that \( \tilde{h}(\Psi_t) \) is increasing and concave. Putting all these elements together we can deduce that, in that configuration, \( h(\Psi_t; \Gamma_t) \) cross the 45\(^\circ\) line only once, in \( \Psi_t = \gamma^m \), and from above, such that this point is the unique globally stable steady state of the dynamics of \( \Psi_t \).

### C Proof of Proposition 1

The steady states of the joint dynamics of \( \Gamma_t \) and \( \Psi_t \) are defined as the crossing points of \( \Gamma \Gamma \), the stationary locus of \( \Gamma_t \) and \( \Psi \Psi \), the stationary locus of \( \Psi_t \). Hence, as a first step, we will successively analyze the shape and the properties of these \( \Gamma \Gamma \) and the \( \Psi \Psi \) loci. Then we will be in position to complete the proof of the proposition.

**The \( \Gamma \Gamma \) locus.** From equation (17) we know that the \( \Gamma \Gamma \) locus is either \( \gamma^m \) or the value of \( \Gamma_t \) solution of the equation \( g(\Gamma_t; \Psi_t) = \Gamma_t \). The differentiation of this equation yields

\[
d\Gamma_t = g'_\Psi d\Psi_t + g'_\Gamma d\Gamma_t \quad \Leftrightarrow \quad \frac{d\Psi_t}{d\Gamma_t} = \frac{1 - g'_\Gamma}{g'_\Psi} > 0 \quad \text{(C.1)}
\]

where the inequality comes from the fact that \( g'_\Psi(\Gamma_t, \Psi_t) > 0 \) and \( g'_\Gamma(\Gamma_t, \Psi_t) \leq 1 \) under Assumption 3. Moreover, the derivative of the equation (C.1) with respect to \( \Gamma_t \) yields:

\[
\frac{\partial}{\partial \Gamma_t} \left( \frac{d\Psi_t}{d\Gamma_t} \right) = \frac{-g''_{\Gamma \Gamma} g'_\Psi - g''_{\Psi \Gamma} (1 - g'_\Gamma)}{(g'_\Psi)^2} < 0 \quad \text{(C.2)}
\]

where the inequality comes from the fact that \( g''_{\Psi \Gamma} = 0 \) while \( g''_{\Gamma \Gamma} > 0 \) and \( g'_\Psi > 0 \) (see Appendix A).

Hence, in the plan \( (\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m]^2 \), the \( \Gamma \Gamma \) locus is increasing and concave. Using equation (C.1) and the expressions of \( g'_\Psi \) and \( g'_\Gamma \) provided in Appendix A, we also conclude that the slope of the \( \Gamma \Gamma \) locus at the point \( (\Gamma_t, \Psi_t) = (\gamma^m, \gamma^m) \) is lower than one if and only if \( \mu > 2\bar{\mu} \). Finally, \( \gamma^m \) is defined as the value of \( \gamma^m \) such that \( 2\bar{\mu} = \bar{\mu} \) and Assumption 4 ensures that \( \bar{\mu} > 2\bar{\mu} > \bar{\mu} > \bar{\mu} \).

Thus, the \( \Gamma \Gamma \) locus may be expressed as:

\[
\Gamma \Gamma \equiv \left\{ (\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m]^2 : \Psi_t = \min\{f_{\Gamma \Gamma}(\Gamma_t), \gamma^m\} \right\} \quad \text{(C.3)}
\]

with \( f_{\Gamma \Gamma}(\Gamma_t) \) increasing and concave in \( \Gamma_t \). Moreover, since \( g(\gamma^f, \gamma^f) > (\gamma^m + \gamma^f)/2 \) and \( g(\Gamma_t; \Psi_t) \) is increasing in both \( \Psi_t \) and \( \Gamma_t \), the value of \( \Gamma_t \) such that \( f_{\Gamma \Gamma}(\Gamma_t) = \gamma^f \) is larger than \( (\gamma^m + \gamma^f)/2 \).
In addition, \( f_{\Gamma \Gamma}(\Gamma_t) \) is upward shifted as \( \mu \) increases\(^{29}\) with

\[
\begin{align*}
  f_{\Gamma \Gamma}(\gamma^m) \begin{cases} > \\ < \end{cases} \gamma^m & \Leftrightarrow \mu \begin{cases} > \\ < \end{cases} \tilde{\mu}
\end{align*}
\]

Finally, since \( \tilde{\mu} > 2\hat{\mu} \), when \( \mu \) is close to \( \tilde{\mu} \) the slope of \( f_{\Gamma \Gamma}(\Gamma_t) \) is lower than one. All these properties of \( f_{\Gamma \Gamma}(\Gamma_t) \) are depicted on Figure 8.

![Figure 8. Properties of \( f_{\Gamma \Gamma}(\Gamma_t) \)](image)

From these properties we conclude that when \( \mu \) is low, the \( \Gamma \Gamma \) locus is always below the \( 45^\circ \) line. When \( \mu \) is lower but become sufficiently close to \( \tilde{\mu} \), the \( \Gamma \Gamma \) locus crosses the \( 45^\circ \) line twice (first from below and then from above). Finally, when \( \mu > \tilde{\mu} \), the \( \Gamma \Gamma \) locus crosses the \( 45^\circ \) line (from below) only once.

**The \( \Psi \Psi \) locus.** From equation (25) we know that the \( \Psi \Psi \) locus is either \( \gamma^m \) or the value of \( \Psi_t \) solution of the equation \( g(\Gamma_t; \Psi_t) = \Psi_t \). The differentiation of this equation yields

\[
\frac{d\Psi_t}{d\Gamma_t} = \frac{g'_\Gamma}{1 - g'_\Psi} > 0
\]

where the inequality comes from the fact that \( g'_\Gamma(\Gamma_t, \Psi_t) > 0 \) and \( g'_\Psi(\Gamma_t, \Psi_t) \leq 1 \) under Assumption 29.

\(^{29}\) To see this, let us observe that the differentiation of \( \Gamma_t = g(\Gamma_t; \Psi_t) \) with respect to \( \mu \) and \( \Psi_t \) yields \( \frac{d\Psi_t}{d\mu} = -\frac{\delta\Psi}{\delta\mu}/g'_\Psi \) which is positive since \( \frac{\delta\Psi}{\delta\mu} < 0 \) and \( g'_\Psi > 0 \).
Let us consider the parametric configuration corresponding to the point ii of the Proposition 1: $\mu \in [\mu^*, \bar{\mu}]$. In such a configuration, in the absence of any affirmative action policy (i.e., when

\[
\frac{\partial}{\partial \Gamma_t} \left( \frac{d\Psi_t}{d\Gamma_t} \right) = \frac{g_t''(1 - g_t') + g_t''g_t'}{(1 - g_t')^2} > 0
\]

where the inequality comes from the fact that $g_t'' = 0$ while $g_t'' > 0$ and $g_t' \leq 1$ (see Appendix A). Hence, in the plan $(\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m]^2$, the $\Gamma\Gamma$ locus is increasing and convex. Thus, the $\Psi\Psi$ locus may be expressed as:

\[
\Psi\Psi \equiv \{ (\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m]^2 : \Psi_t = \min\{ f_{\Psi\Psi}(\Gamma_t), \gamma^m \} \} 
\]

with $f_{\Psi\Psi}(\Gamma_t)$ increasing and convex in $\Gamma_t$. Moreover, since $g(\Gamma_t; \Psi_t) > (\gamma^m + \gamma^f)/2$ for all $(\Gamma_t, \Psi_t) \in [\gamma^f, \gamma^m]^2$, it must be the case that $f_{\Psi\Psi}(\Gamma_t) > (\gamma^m + \gamma^f)/2$ for all $\Gamma_t \in [\gamma^f, \gamma^m]$. In addition, $f_{\Psi\Psi}(\Gamma_t)$ is downward shifted as $\mu$ increases\(^{30}\) and, since the function $g(\Gamma_t; \Psi_t)$ is symmetric, the $\Psi\Psi$ locus crosses the $45^\circ$ line at exactly the same point than the $\Gamma\Gamma$ locus. It directly follows that, when $\mu$ is low, the $\Psi\Psi$ locus is always above the $45^\circ$ line. When $\mu$ is lower but become sufficiently close to $\bar{\mu}$, the $\Psi\Psi$ locus crosses the $45^\circ$ line twice (first from above and then from below). Finally, when $\mu > \bar{\mu}$, the $\Psi\Psi$ locus crosses the $45^\circ$ (from above) only once.

**Completion of the proof of Proposition 1.** It directly follows from the properties of the function $f_{\Gamma\Gamma}(\Gamma_t)$ and $f_{\Psi\Psi}(\Gamma_t)$ that:

- When $\mu > \bar{\mu}$, the $\Gamma\Gamma$ and the $\Psi\Psi$ loci cross only once. We deduce from Lemma 1 and 2 that the corresponding steady-state is globally stable. Moreover, this steady state $(\Gamma^*, \Psi^*)$ is such that $\Gamma^* = \Psi^* > (\gamma^m + \gamma^f)/2$.

- When $\mu$ is slightly lower than $\bar{\mu}$, the $\Gamma\Gamma$ and the $\Psi\Psi$ loci cross three times. Two of the corresponding steady states are interiors and the third is $(\gamma^m, \gamma^m)$. We deduce from Lemma 1 and 2 that the two extreme steady states are locally stable while the intermediate steady state is unstable. Finally, the interior and locally stable steady state $(\Gamma^*, \Psi^*)$ is such that $\Gamma^* = \Psi^* > (\gamma^m + \gamma^f)/2$.

- When $\mu$ is sufficiently low, the $\Gamma\Gamma$ and the $\Psi\Psi$ loci cross only once in $(\gamma^m, \gamma^m)$. We deduce from Lemma 1 and 2 that the corresponding steady-state is globally stable.

**D Derivation of the bifurcation diagram of the Section 4.2**

Let us consider the parametric configuration corresponding to the point ii of the Proposition 1: $\mu \in [\mu^*, \bar{\mu}]$. In such a configuration, in the absence of any affirmative action policy (i.e., when

\(^{30}\)To see this, let us observe that the differentiation of $\Psi_t = g(\Gamma_t; \Psi_t)$ with respect to $\mu$ and $\Psi_t$ yields $\frac{\partial \Psi_t}{\partial \mu} = \frac{\partial g_t}{\partial \mu} = \frac{\partial g_t}{\partial \Psi_t} \leq 0$ which is negative since $\frac{\partial g_t}{\partial \Psi_t} < 0$ and $g_t' \leq 1$. 

30.
$\bar{\Psi} \geq \gamma^m$) two locally stable steady-states ($E_1$ and $E_3$ on Figure 5(B)) co-exist with one unstable steady-state ($E_2$ on Figure 5(B)). As depicted on Figure 9, the threshold $\bar{\Psi}$ is the value of $\Psi_t$ such that $f_{TT}(\Psi_t) = \gamma^m \left( \bar{\Psi} = f_{TT}^{-1}(\gamma^m) \right)$; $\bar{\Psi}$ is the value of $\Psi_t$ corresponding to the unstable equilibrium $E_2$; and, as defined in Proposition 1, $\Psi^*$ is the value of $\Psi_t$ corresponding to the interior equilibrium $E_3$. Figure 9 also depicts the steady-states of the economy for different levels of the affirmative action policy $\bar{\Psi}$. Four configurations can be distinguished according to the value of $\bar{\Psi}$ with respect to the three thresholds $\bar{\Psi}$, $\hat{\Psi}$ and $\Psi^*$. On this figure, the dashed area corresponds to the values of $\Psi_t$ that are not reachable because of the affirmative action policy.

Figure 9. The phase diagram in the presence of an affirmative action policy
When $\bar{\Psi} \in [\bar{\Psi}, \gamma^m)$, the locally stable patriarchal equilibrium $E_1$ co-exists with the locally stable interior equilibrium $E_3$ and the unstable equilibrium $E_2$. At the patriarchal equilibrium $\Gamma_t = \gamma^m$ while $\Psi_t = \bar{\Psi}$ since the reservation policy is binding. At the interior equilibrium $\Gamma_t = \Psi_t = \Psi^*$ such that the reservation policy is not binding.

When $\bar{\Psi} \in (\bar{\Psi}, \bar{\Psi})$, the locally stable patriarchal equilibrium $E_1$ co-exists with the locally stable interior equilibrium $E_3$ and the unstable equilibrium $E_2$. At the patriarchal equilibrium $\Gamma_t < \gamma^m$ while $\Psi_t = \bar{\Psi}$ since the reservation policy is still binding. At the interior equilibrium $\Gamma_t = \Psi_t = \Psi^*$ such that the reservation policy is not binding.

When $\bar{\Psi} \in [\bar{\Psi}, \hat{\Psi})$, the interior equilibrium $E_3$ is the unique, globally stable, steady-state. At this equilibrium $\Gamma_t = \Psi_t = \Psi^*$ such that the reservation policy is not binding.

When $\bar{\Psi} \in (\hat{\Psi}, \bar{\Psi})$, the interior equilibrium $E_3$ is the unique, globally stable, steady-state. At this equilibrium $\Psi_t = \bar{\Psi}$ such that the reservation policy is binding while $\Gamma_t \in (\bar{\Psi}, \Psi^*)$.

**References**


Data Appendix (Online Appendix - Not for Publication)

In this appendix we first provide an indicative evidence of the beneficial impact of empowerment in both spheres on human capital. Then we describe the data (variables and observations) used to draw our figures.

**Empowerment and human capital achievement.** In this appendix we first provide an indicative evidence of the beneficial impact of empowerment in both spheres on human capital. To do so, we plot in Figure 10 an index of educational achievement against a measure of women’s empowerment in politics (left panel) and within the household (right panel) for a set of developing countries.

![Figure 10. Women empowerment and educational achievements](image)

(a) corr=0.28 (p-value=0.015)  
(b) corr=0.63 (p-value<0.01)

**Variables.** Here we define the three variables that we use to draw Figures 1 and 10.

*Educational achievement.* We use the subindex “Education Index” of the Human Development Index (HDI) provided by the United Nations Development Programs (UNDP). It is calculated as the average between mean years of schooling (of adults) and expected years of schooling (of children), both being rescaled to be expressed as indices between 0 and 1.\(^{31}\) The values that are reported in Figures 1 and 10 are for the year 2017. Data are freely available on the UNDP website.\(^{32}\)

*Political empowerment.* We use the subindex “Empowerment Gap” of the Gender Equity Index (GEI) developed by Social Watch. It consists in the mean of four indicators measuring the difference

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between the percentage of men and the percentage of women in technical positions, in management and governments positions, in parliaments and in ministerial posts.\textsuperscript{33} The values that are reported in Figures 1 and 10 are for the year 2012 for almost all the countries we are considering except Guinea (2008) and Nigeria (2009). Data are freely available on the Social Watch website.\textsuperscript{34}

Intrahousehold empowerment. We use the “Indicators of Women’s Empowerment” of the 7rd wave of the Demographic and Health Surveys (DHS-7). More precisely, we use the question v743a who asks to married women (age 15-49) who usually make decisions about her own health care: “respondent alone”, “respondent and husband”, “husband alone”, “someone else” and “other”.\textsuperscript{35} Then we define, for each country, our intrahousehold empowerment indice as follows: \(percentage\ of\ respondent\ alone + \frac{1}{2}(percentage\ of\ respondent\ and\ husband)\). The values that are reported in Figures 1 and 10 depend on the year at which each country has been surveyed (it varies between 2003 and 2018). Data from the DHS are available upon request.

Observations. Our data set consists of 62 developing countries for which we have observations for at least one of the two empowerment variable. These countries as well as the value of our three variables for each of them are listed in the Table 1 below.

\textsuperscript{33}See http://www.socialwatch.org/node/9280 for methodological details.
\textsuperscript{34}http://www.socialwatch.org/node/14367.
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<th>Country</th>
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<th>Political empowerment</th>
<th>Intrahousehold empowerment</th>
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