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Abstract

This paper studies the evolution of long-run growth rates in the G-7 countries. We identify a measure of long-run output growth rate with that rate of growth consistent with a constant unemployment rate. The methodology proposed also allows the derivation of the long-run growth rate associated with technical progress by separating the effects derived from movements in the rate of growth of the labour force. To measure its trajectories during the postwar period, we use time-varying parameter models that incorporate both stochastic volatility and a Heckman-type two-step estimation procedure that deals with the possible endogeneity problem in the econometric models. Our results show a significant decline in long-run growth rates that is not associated with the detrimental effects of the Great Recession, and that the rate of growth of technical progress appears to be behind the slowdown in long-run GDP growth.

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1 Introduction

The Great Recession (GR) has raised concerns about the possibility that advanced economies are entering an era of secular stagnation, that is, an era characterised by a slowdown in the rate of growth of long-run GDP. The estimation of long-run output is, however, surrounded by considerable uncertainty because its conceptualisation reflects the ongoing controversy regarding the origins of economic fluctuations.

The present paper studies the evolution of the rate of growth of output consistent with a constant unemployment rate, which can be identified with a measure of long-run GDP growth because it represents the sum of labour force and technical progress growth. We show that: (1) this measure of long-run GDP growth rate can be derived from the first difference version of Okun's law by combining a production function approach to depict the equilibrium in the goods market and a wage-setting equation to depict the equilibrium in the labour market; and (2) the methodology adopted also allows the derivation of the long-run growth rate of technical progress by separating the effects derived from movements in the rate of growth of the labour force. We then estimate both long-run growth rates for the G-7 countries during the post-war era using time-varying parameter models that incorporate stochastic volatility and a Heckman-type two step estimation procedure that deals with the possible endogeneity problem in the econometric models. In this way, the mean and the variance of the growth rates are allowed to drift gradually over time in order to capture the changes in the volatility of output that have taken place during the post-war period (*i.e.*, the "Great Moderation"), which allows to characterise the possible uncertainty around the estimates.

The main empirical results obtained can be summarised as follows. First, we document a significant decline in long-run output and technological progress growth rates. With respect to output growth rates, the fall ranges from approximately -8.6 percentage points in Japan to approximately -1.6 percentage points in the United Kingdom. Regarding the rates of growth of technological progress, the fall ranges from approximately -8.3 percentage points in Japan to -1.6 percentage points in Italy. Because our approach identifies two main sources of economic growth —labour force growth and technological progress growth, these results suggests that the slowdown in technical change has been the main driver of the decline in GDP growth. Second, besides the smoothed estimates of stochastic volatility —generated using all information available in the sample, the particle filter applied to the time-varying parameter models allows the computation of one-sided estimates —generated using real-time information, which can be considered real-time estimates of the latent processes. This allows to characterise the evolution of long-run growth rates before and after the GR. The results obtained from both estimates are similar, so that the largest decline in growth rates does not seem to be associated with the detrimental effects derived from the GR.

This article is closely related to recent studies that have documented a reduction in different measures of long-run output growth in advanced economies and that have tried to decompose long-run GDP growth into its main drivers (Antolin-Diaz et al. , 2017; Benati , 2007; Fernald , 2015; Fernald et al. , 2017; Gordon , 2010; 2012; 2014a;b; 2016; 2018). It is possible to summarise the main findings of these studies as follows:

1. There has been a gradual decline, rather than a discrete break, in different estimates of long-run output growth in developed countries.

2. The slowdown in long-run output predated the GR. With respect to the USA, Fernald et al.

(2017: 30) have argued that "the US economy suffered a deep recession superimposed on a sharply slowing trend".

3. The decline in the growth rates of technological progress and labour productivity appear to be behind the slowdown in long-run output growth in the USA. Benati (2007), Fernald (2015) and Gordon (2012; 2014a;b; 2016; 2018) describe the evolution of productivity growth in the USA as follows: high levels of productivity in the 1950s and 1960s as a consequence of the inventions derived from the second industrial revolution (airplanes, air conditioning, interstate highways); productivity growth slowed down after 1973; information technology (the third industrial revolution: computers, the web, mobile phones) created only a short-lived productivity growth revival from mid-1990s and early 2000s; and productivity growth slowed again before the GR and it has practically vanished during the past decade.[1]

4. More recently, Cette et al. (2016), Antolin-Diaz et al. (2017) and Gordon (2018) have shown that the weakening in technical progress and labour productivity prior to the GR also appear to be a global phenomenon.

Our main focus in this paper is to study the possible changes in growth rates that have been permanent in nature (*i.e.*, non-mean-reverting changes), as in Beveridge and Nelson (1981) and Antolin-Diaz et al. (2017). Therefore, we interpret the long-run as frequencies lower than the business cycle and we see our paper as extending the aforementioned literature.

Specifically, our approach is similar to that of Gordon (2010; 2014b), who pointed out that Okun's law can be used to identify the breakdown of trend growth and changes in cyclical fluctuations despite the simplicity of the approach and the restrictive assumptions. Gordon (2018) calculates the long-run trends of the slowdown in GDP growth and, in order to avoid any influence of the ups and downs of the business cycle, these are calculated between years with roughly the same unemployment rate. Our article formally shows how the rate of growth of output consistent with a constant unemployment rate can be estimated via the first difference version of Okun's law by deriving the latter from a growth model that combines the equilibrium in the goods and labour markets, thus demonstrating that the underlying parameters of this specification contain important information associated with an economy's long-run growth performance.

On the other hand, our econometric procedures emphasise the importance of stochastic volatility and the possible endogeneity problems that stem from the estimation of the reduced-form models. Antolin-Diaz et al. (2017) have also stressed the importance of changing volatility for describing long-run growth in the context of a dynamic factor model that incorporated four business cycle variables measured at quarterly frequency (output, consumption, investment and aggregate hours worked) and a set of 24 monthly indicators. Relative to this study, our methodology is simple in terms of the number of variables employed, which allows, first, to derive a clear interpretation of long-run output and technical progress growth rates; and, second, to conduct frequentist inference, which is important since the treatment of stochastic volatility in Bayesian models can be subject to strong prior beliefs.

The rest of the paper comprises four sections. Section 2 describes the methodology employed by presenting a theoretical derivation of Okun's law using a simple growth model. Section 3

[1] Byrne et al. (2016) have shown that there is little evidence that the slowdown in the growth rates of labour productivity and total factor productivity arises from growing mismeasurement of the gains from innovation and information technology-related goods and services.

provides a description of the econometric techniques used. The main empirical findings are presented and discussed in Section 4. Finally, Section 5 summarises the main conclusions and mentions some potentially relevant areas for future research.

2 Okun's law and long-run output growth rates

Our main purpose in this section is to show that the underlying parameters in the first difference version of Okun's law contain relevant information associated with the long-run properties of GDP growth rates. Prachowny (1993) provided some theoretical foundations of Okun's law by deriving the relationship between changes in output and unemployment from a production function that included changes in weekly hours and capacity utilization. In this paper, however, we incorporate both the direct effect of changes in the unemployment rate on output and the indirect effects through changes in weekly hours as well as capacity utilization that accompany the change in the unemployment rate.

Our derivation of Okun's law is similar to that of Adachi et al. (2015), and is composed of two main features: a production function that explicitly incorporates capacity utilization and working hours in order to derive the equilibrium in the goods market; and a wage-setting equation that depicts a negative relationship between real wages and the unemployment rate in order to derive the equilibrium in the labour market.

First, we analyse the goods market. Consider that the production function is given by:

$$Y = F\left(AhN, vK\right), \tag{1}$$

where Y, A, h, N, v and K denote output, the level of technological progress, working hours per worker, labour employment, the utilization rate of capital, and the capital stock, respectively.

Let us assume that an increase in v is positively associated with an increase in h, so that h is an increasing function of v: $h = \overline{h}v^{\gamma}$, where $\overline{h}$ denotes "normal" working hours (that is, when $v = 1$) and γ is the elasticity of working hours with respect to the rate of capacity utilization. For convenience, we set $\overline{h} = 1$ and, therefore, $h = v^{\gamma}$.

Since $F(.)$ is assumed to be a standard constant returns to scale production function, we can express equation (1) as output per unit of capital $y = Y/K$:

$$y = vf\left(v^{\gamma-1}n\right), \tag{2}$$

where $n = AN/K$ represents effective labour per unit of capital; $f(.)$ is assumed to be an ordinary well-behaved concave function: $f(0) = 0$, $f' > 0$, $f'' < 0$, and $f(\infty) = \infty$; and the production function in (2) exhibits diminishing returns with respect to v.[2]

We consider a situation in which each firm uses one unit of capital and the economy has K capital stocks. Therefore, K is equal to the number of firms in the economy, changes in the latter correspond to the entry and exit decisions of firms, and each firm's output is given by the production function shown in (2). We also assume that each firm determines v to achieve y sufficient for its demand level y^d. Hence: $y = y^d = vf\left(v^{\gamma-1}n\right)$; and the equilibrium in the goods market can be represented as follows:

$$Y = Y^d = Ky = Ky^d = Kvf\left(v^{\gamma-1}n\right), \tag{3}$$

[2] Otherwise, output can be increased as much as possible by increasing the utilization rate with constant capital and labour. As shown in Appendix A.1, this implies that $0 < \gamma < 1$.

where $Y = Ky$ represents aggregate output and $Y^d = Ky^d$ represents aggregate demand.

Second, we analyse the equilibrium in the labour market. We assume that the economy is composed of monopolistically competitive price-setting firms and that each firm faces a downward sloping demand curve. The demand for its good is given, in inverse form, by:

$$\frac{p}{P} = \left(\frac{y}{\tilde{y}}\right)^{-\frac{1}{\eta}}, \tag{4}$$

where p is the price charged by the firm; P is the aggregate price level; $\tilde{y}$ is the average output of all firms; and $\eta > 1$ is the price elasticity of demand.

Each firm determines the price of goods p and the amount of labour employment n in order to maximize profits, π, subject to the demand function in (4) and the production function in (2):

$$\pi = \frac{p}{P}y - \frac{W}{PA}hn,$$

where W is the nominal wage rate; and W/PA is the real wage rate per efficiency unit of labor.

It follows that the first order condition for profit maximization is[3]:

$$\frac{p}{P}\left(1 - \frac{1}{\eta}\right) f'(v^{\gamma-1}n) = \frac{W}{PA}.$$

The symmetry condition that all firms must charge the same price, so that $p = P$, implies that:

$$\frac{1}{1+\mu} f'(v^{\gamma-1}n) = \frac{W}{PA}, \tag{5}$$

where $\mu = 1/(\eta - 1)$ represents the mark-up of price over costs.[4] Therefore, given W/PA, equation (5) determines each firm's demand for labour n; and aggregate labour demand is given by $N = nK/A$.

On the other hand, we assume that the supply side of the labour market is given by a wage-setting function derived from the efficiency wage and wage bargaining theories, thus considering that the real wage rate is a positive function of the employment rate —or a negative function of the unemployment rate. We adopt the simple wage-setting formulation presented by Blanchard (1997):

$$\frac{W}{PA} = \alpha\left(\frac{N}{L}\right)^{\omega}, \tag{6}$$

where L is the labour force; α is a parameter that reflects the bargaining power of labour or the reservation wage; and ω is the elasticity of real wages W/PA with respect to the employment rate N/L.

If we denote the ratio of labour force in efficiency units LA to capital K by $l = LA/K$ then it is possible to express (6) as:

$$\frac{W}{PA} = \alpha\left(\frac{n}{l}\right)^{\omega}, \tag{7}$$

[3] See Appendix A.2.

[4] As Adachi et al. (2015) emphasise, the condition in (5) is different from the traditional equality between marginal productivity of labour and real wage rate because of the inclusion of the mark-up term μ, and because the marginal product of labor W/PA is determined by both the amount of labour employed n and the utilization rate of capital v.

where n/l is the employment rate. Since the level of employment cannot exceed the labor force, then $n \leq l$.

The equilibrium in the labour market can be represented as follows:

$$\frac{1}{1+\mu} f'(v^{\gamma-1}n) = \alpha \left(\frac{n}{l}\right)^{\omega}. \tag{8}$$

It is possible to introduce the unemployment rate u into the analysis of the goods market and the labour market because, by definition, $n/l \equiv 1-u$. Therefore, equations (3) and (8) can be expressed as follows:

$$Y = Y^d = Kvf(v^{\gamma-1}(1-u)l). \tag{9}$$

$$\frac{1}{1+\mu} f'(v^{\gamma-1}(1-u)l) = \alpha(1-u)^{\omega}. \tag{10}$$

Let us now consider the case of a growing economy in which A, K and L are growing at constant rates: $\dot{A}/A = \tau$; $\dot{K}/K = \kappa$; $\dot{L}/L = \lambda$, where τ is the rate of growth of technical progress; κ is the rate of growth of capital; and λ is the rate of growth of labour force.

Since $l = LA/K$, the growth rate of l can be expressed as:

$$\frac{\dot{l}}{l} = \lambda + \tau - \kappa. \tag{11}$$

Taking the logarithm of (9) and (10), differentiating with respect to time, and using equation (11) to denote $\dot{l}/l$ we obtain:

$$\frac{\dot{Y}}{Y} = \kappa + [1-\theta(1-\gamma)]\frac{\dot{v}}{v} - \theta\frac{\dot{u}}{1-u} + \theta(\lambda+\tau-k), \tag{12}$$

$$\frac{1-\theta}{\sigma}(1-\gamma)\frac{\dot{v}}{v} = -\left(\frac{1-\theta}{\sigma}+\omega\right)\frac{\dot{u}}{1-u} + \frac{1-\theta}{\sigma}(\lambda+\tau-k), \tag{13}$$

where $0 < \theta < 1$ represents the elasticity of output with respect to employment and $\sigma > 0$ represents the elasticity of substitution between labour and capital.[5]

Equations (12) and (13) contain three important variables for our analysis: the rate of growth of output, $\dot{Y}/Y$; the rate of growth of capacity utilization, $\dot{v}/v$; and changes in —first differences of— the unemployment rate, $\dot{u}$. Therefore, it is possible to eliminate $\dot{v}/v$ from these equations by substituting equation (13) into equation (12) and solving it with respect to $\dot{Y}/Y$:

$$\frac{\dot{Y}}{Y} = \kappa + \frac{1}{1-\gamma}(\lambda+\tau-\kappa) - \frac{1}{(1-u)(1-\gamma)}\left[1+\frac{1-\theta(1-\gamma)}{1-\theta}\omega\sigma\right]\dot{u},$$

which represents an equation that expresses the relationship between $\dot{Y}/Y$ and $\dot{u}$.

Further, if we assume that in the long-run the economy is on a balanced-growth path, then $\kappa = \lambda + \tau$. This allows us to re-write the equation above as follows:

$$\frac{\dot{Y}}{Y} = \lambda + \tau - \frac{1}{(1-u)(1-\gamma)}\left[1+\frac{1-\theta(1-\gamma)}{1-\theta}\omega\sigma\right]\dot{u}. \tag{14}$$

[5] Appendix A.3 presents the most important steps to derive equations (12) and (13).

Equation (14) shows the relationship between $\dot{Y}/Y$ and $\dot{u}$ by emphasising two important results. First, the coefficient on $\dot{u}$ is the Okun coefficient on unemployment, which can be represented as follows:

$$\beta_1 = \frac{1}{(1-u)(1-\gamma)}\left[1+\frac{1-\theta(1-\gamma)}{1-\theta}\omega\sigma\right].$$

It is possible to observe that the β_1 coefficient represents a mixture of different effects: the actual employment rate $1-u$; and four different parameters: (1) the elasticity of hours worked with respect to the capacity utilization rate γ; (2) the elasticity of output with respect to labour employment θ; (3) the elasticity of real wages with respect to the employment rate ω; and (4) the elasticity of substitution between labour and capital σ.[6]

Second, the constant term in equation (14) is given by:

$$\beta_0 = \lambda + \tau.$$

The parameter β_0 can be identified with a measure of long-run output growth rate because it represents the sum of the rates of growth of labour force λ and of technical progress τ. This means that, assuming that there are no changes in the unemployment rate —in other words, if $\dot{u}=0$— the rate of growth of output $\dot{Y}/Y$ is equal to the sum of λ and τ.

Nevertheless, there is substantial empirical evidence that shows the presence of asymmetric behaviour between output and unemployment. Following the derivation of equation (14) it is possible to observe that the parameter β_1, the Okun coefficient on unemployment, can be affected by changes in the employment rate, the elasticity of hours worked with respect to the capacity utilization rate, the elasticity of output with respect to labour employment, the elasticity of real wages with respect to the employment rate, and the elasticity of substitution between labour and capital. It is conceivable that the introduction of new technologies, the globalization process and the changing dynamics of the labour market have affected these parameters over time.

Likewise, as discussed in the previous section, it is likely that the long-run output growth rate has experienced fluctuations over time. The parameter β_0 in equation (14) could also be a time-varying parameter because λ has changed over time, τ has changed over time, or both components have experienced fluctuations during the post-war period.

Thus, we explicitly consider the possibility that both β_0 and β_1 are time-varying parameters —that is, not fixed over time:

$$g_t = \beta_{0,t} - \beta_{1,t}\Delta u_t + \varepsilon_t, \tag{15}$$

where $g_t \equiv \dot{Y}/Y$; $\Delta u_t \equiv \dot{u}$; and ε_t represents an idiosyncratic shock hitting the underlying economic system.

In brief, equation (15) depicts the first difference version of Okun's law using a time-varying parameter model (TVPM). The latter equation can be used as an econometric device for estimating the long-run output growth rate if ε_t satisfies certain statistical properties. The $\beta_{1,t}$ coefficient represents the time-varying Okun coefficient, which measures the inverse relationship between the change in the unemployment rate Δu_t and output growth g_t. The variation in the long-run output growth rate is captured by the parameter $\beta_{0,t}$, composed of the sum of the (potentially) time-varying rates of growth of technical progress τ_t and of the labour force λ_t.[7]

[6]Note also that the Okun coefficient β_1 is an increasing function of these four parameters.

[7]Other studies (IMF , 2010; Klump et al. , 2008; León-Ledesma and Thirlwall , 2002; Mendieta-Muñoz , 2017;

Finally, the derivation of the long-run growth rate in equation (15) also allows the computation of the rate of growth of technical progress component by subtracting λ_t from both sides of equation (15):

$$g_t - \lambda_t = \beta^*_{0,t} - \beta^*_{1,t}\Delta u_t + \varepsilon^*_t, \tag{16}$$

where the parameter $\beta^*_{0,t} = \tau_t$ measures now the rate of growth of technical progress; $\beta^*_{1,t}$ represents the time-varying Okun coefficient that measures the inverse relationship between Δu_t and $g_t - \lambda_t$; and ε^*_t represents the stochastic disturbance term. As in Antolin-Diaz et al. (2017), the estimate of $\beta^*_{0,t}$ in our framework captures both technological factors and other factors, such as capital deepening and labour quality.

3 Econometric techniques

Our main interest consists in estimating the parameters $\beta_{0,t}$ and $\beta^*_{0,t}$ for the G-7 countries from the TVPMs shown in equations (15) and (16), respectively. It is noteworthy that standard unit root tests suggests that the unemployment rates of the G-7 countries are non-stationary series. The previous section showed that the first-difference version of Okun's law allows the computation of the long-run output growth rates, thus avoiding the spurious regression problem. Nevertheless, there are two possible problems associated with the estimation of these models that are necessary to consider.

First, both TVPMs relate output growth rates to changes in the unemployment rate using a partial equilibrium framework. In an econometric model, however, it is also necessary to include other dynamics of g_t and $g_t - \lambda_t$ that are not explained by Δu_t. To this end, we make the error dynamics agnostic to possible model misspecification by incorporating moving average error terms of order 1 (MA(1)) with stochastic volatility (SV).[8] Thus, we estimated time-varying parameter models with stochastic volatility (TVPMs-SV) instead of the traditional TVPMs. The TVPMs-SV satisfied the standard correct specification tests (see below).

Second, it is likely that the TVPMs and the TVPMs-SV present endogeneity problems because of the possible correlation between Δu_t and the measurement disturbance terms. Kim (2006) shows that the Kalman filter applied to a TVPM leads to invalid inferences of the model (*i.e.*, inferences on the hyperparameters and time-varying coefficients or stochastic state variables) using maximum likelihood (ML) estimation if the regressors are endogenous. This may arise if Δu_t is correlated with other omitted variables that also affect g_t and $g_t - \lambda_t$.[9] In order to correct the

Schnabel , 2002; Thirlwall , 1969) have also identified the rate of growth consistent with a constant unemployment rate derived from the first difference version of Okun's law as a measure of a "potential" or "natural" output growth rate, without focusing on the evolution of the latter over time. Note also that equation (15) reverses the dependent and independent variables in the traditional Okun's law specification. Thirlwall (1969) and Barreto and Howland (1993) also justified this by emphasising that reversing the order of the variables can be used to avoid estimation biases caused by labour hoarding and that the best predictor of the output growth rate can be found by regressing g_t on Δu_t, respectively.

[8]It may also be possible to incorporate MA(p) error dynamics, but in our empirical results we find MA(1) error terms to be sufficient.

[9]For example, according to Holston et al. (2017), changes in g_t are expected to be Granger-caused by the unobserved inflation gap and the real interest rate gap, via the Phillips curve and the IS curve, respectively. Both gaps are likely to be correlated with Δu_t since they also measure the phase of the business cycle.

possible endogeneity problem and to obtain consistent estimates of the TVPMs-SV, we employ the Heckman-type two-step bias correction developed by Kim (2006).

The rest of this section describes the implementation of the TVPMs-SV and the Heckman-type two-step bias correction method. For simplicity, we will consider only the estimation of model (15); and the notation that we employ in this section is independent of the notation employed in Section 2.

3.1 Time-varying parameter models with stochastic volatility

We propose the following homogeneous TVPM-SV[10] composed of the observed variables Δu_t and g_t, and of the unobserved parameters or state vector $\beta_{0,t}$ and $\beta_{1,t}$. The measurement equation of the model is

$$g_t = \beta_{0,t} - \beta_{1,t}\Delta u_t + \phi\varepsilon_{t-1} + \varepsilon_t, \quad t = 2, ..., T, \tag{17}$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$ for all t with the log-variance evolving as a random walk, *i.e.*

$$\log\sigma_{t+1} = \log\sigma_t + \sigma_\varepsilon\zeta_t, \quad t = 2, ..., T-1,$$

where σ_ε is called the volatility of volatility. The state variable $\beta_{0,t}$ measures the long-run output growth rate, which follows an integrated random walk or a smooth trend dynamics; whereas the time-varying Okun coefficient on unemployment $\beta_{1,t}$ follows a random walk. Hence,

$$\begin{aligned} \beta_{0,t+1} &= \beta_{0,t} + \beta_t \\ \beta_{t+1} &= \beta_t + \lambda_0\sigma_t\eta_{0,t} \\ \beta_{1,t+1} &= \beta_{1,t} + \lambda_1\sigma_t\eta_{1,t}, \quad t = 2, ..., T-1. \end{aligned} \tag{18}$$

The parameters λ_0^2 and λ_1^2 in equation (18) are the signal-to-noise ratios (SNR) for $\beta_{0,t}$ and $\beta_{1,t}$. The innovation terms ζ_t, $\eta_{0,t}$, and $\eta_{1,t}$ are *i.i.d.* standard normal random variables for all t.

Because of nonstationarity in the state dynamics, we use diffuse initialisation for the log-variance and state variables (Koopman , 1997). Note that $\eta_{0,t}$ and $\eta_{1,t}$ are permanent shocks to the system that try to capture possible systematic changes in the time-preferences of consumers, composition of production factors, and technological development; and that ε_t incorporates transitory shocks such as financial crisis and central bank interventions (Laubach and Williams , 2003).

The choice of the dynamics of $\beta_{0,t}$ is derived from the study by Harvey (2011) and Holston et al. (2017), who discovered that the logarithm of the US GDP follows an integrated random walk of order 2. Also, it is agnostic to model specification of g_t as we do not need to model a time-varying mean.

The modelling of SV plays an important role in model specification. For simplicity, let us illustrate this point by considering only the results for the USA. Figure 1 below shows the cumsum statistic of squared standardised residuals from the estimated TVPMs-SV and TVPMs. If a model

[10] A state-space model is said to be homogeneous if the variance of measurement disturbances is proportional to that of the state innovations. See Harvey (1989) for some examples. Homogeneous stochastic volatility models are parsimonious because they impose a constant signal-to-noise ratio. We also considered heterogeneous stochastic volatility for our models, but no significant differences in the final estimates were found. The results are available upon request.

is correctly specified with respect to the second moment of error terms, the cumulative sum of squared standardised residuals should be proportional to their total sum, thus laying on a 45 degree line. This is the case of the TVPMs-SV presented in the left graph of Figure 1 below. On the contrary, without SV we have a non-constant increase in the cumulated sum, as shown by the estimation of the TVPMs presented in the right graph below.

[INSERT FIGURE 1 ABOUT HERE]

We also find that, if estimated unrestrictedly, the ML estimate of λ_0 tends to zero. This is the "pile-up" or "limited variation" problem documented by Stock and Watson (1998) and found in other empirical macroeconomic studies. To overcome this, we use the unbiased median estimator developed in that paper. This requires an auxiliary first-stage model to determine λ_0, after which the other parameters are estimated based on the full model with λ_0 fixed. Therefore, our first-stage model is the following TVPM:

$$\begin{aligned} g_t &= \beta_0 + \beta_{1,t}\Delta u_t + \sigma_\xi(\phi\xi_{t-1} + \xi_t), \\ \beta_{1,t+1} &= \beta_{1,t} + \sigma_1\eta_{1,t}, \end{aligned} \tag{19}$$

where ξ_t and $\eta_{1,t}$ are *i.i.d.* standard normal random variables for all t. Based on the exponential Wald statistic for testing structural breaks with unknown break dates of the constant intercept β_0, we can determine

$$\hat{\lambda}_0 = \frac{\hat{\sigma}_0}{\hat{\sigma}_\xi},$$

where $\hat{\sigma}_0^2$ is the estimated standard deviation of innovation for $\beta_{0,t}$ if one rejects the null of a constant intercept.

In the second-stage, the full model shown in equations (17) and (18) is estimated via simulated ML estimation with the vector of free parameters $\theta = (\sigma_\varepsilon, \lambda_1, \phi)'$, keeping $\lambda_0^2 = \hat{\lambda}_0^2(1+\phi^2)$. The model becomes nonlinear because of the presence of SV, rendering the Kalman filter infeasible.[11] Nevertheless, the model is conditionally linear, meaning that given h_t, $t = 1,...,T$, the Kalman filter can integrate out both $\beta_{0,t}$ and $\beta_{1,t}$ to calculate the conditional likelihood.

To conduct inference, the simulated ML estimation is based on the numerically accelerated importance sampling (NAIS) developed by Koopman et al. (2015).[12] Denoting $Y = \{y_1,...,y_T\}$, $X = \{\Delta u_1,...,\Delta u_T\}$ and $H = \{\log\sigma_1,...,\log\sigma_t\}$, we can write the likelihood as

$$\begin{aligned} L(Y|X;\theta) &= g(Y|X;\theta)\int_H \frac{p(Y,H|X;\theta)}{g(Y,H|X;\theta)} g(H|Y,X;\theta)dH \\ &= g(Y|X;\theta)\int_H \omega_\theta(H)g(H|Y,X;\theta)dH \end{aligned}, \tag{20}$$

where $p(.)$ denotes densities related to the true TVPM-SV model (equations (17) and (18)); and $g(.)$ is an efficient linear and Gaussian importance density constructed using the NAIS.[13] The

[11] Note that without SV the model is a linear Gaussian state space model that can be efficiently estimated using the Kalman filter.

[12] See Appendix B.1 for a description.

[13] Thereby, it is possible to evaluate $g(Y|X;\theta)$ using the Kalman filter and sample H from $g(H|Y,X;\theta)$ using the simulation smoother of De Jong and Shephard (1995).

importance weight is given by

$$\omega_\theta(H) = \frac{p(Y,H|X;\theta)}{g(Y,H|X;\theta)} = \frac{p(Y|X,H;\theta)}{g(Y|X,H;\theta)},$$

which is a function of the data contained in X, Y and of the parameter vector θ.[14] Under regularity conditions specified in Geweke (1989), we have the following unbiased and consistent Monte Carlo estimate for the likelihood

$$\hat{L}(Y|X;\theta) = g(Y|X;\theta)\bar{\omega}_\theta, \quad \bar{\omega}_\theta = \frac{1}{M}\sum_{j=1}^{M}\omega_\theta(H^{(j)}), \tag{21}$$

where $H^{(j)}$ is drawn from $g(H|Y,X;\theta)$. In practice, we maximise the bias-corrected simulated log-likelihood $\hat{l}(Y|X;\theta)$ with respect to θ, where

$$\hat{l}(Y|X;\theta) = \log g(Y|X;\theta) + \log\bar{\omega}_\theta + \frac{M-1}{2M\bar{\omega}_\theta^2}\sum_{j=1}^{M}\left(\omega_\theta(H^{(j)}) - \bar{\omega}_\theta\right)^2.$$

For inference, Geweke (1989) argues that if the variance of importance weights exists, we have a central limit theorem for smoothed estimate of time-varying parameters. Let $E_{Y,X}(.)$ and $V_{Y,X}(.)$ denote the smoothed estimate of the mean and variance. For the SV, we have

$$\begin{aligned} E_{Y,X}(\sigma_t) &= \sum_{j=1}^{M}\omega_{\hat{\theta}}^*(H^{(j)})\sigma_t^{(j)}, \\ V_{Y,X}(\sigma_t) &= \sum_{j=1}^{M}\omega_{\hat{\theta}}^*(H^{(j)}){\sigma_t^{(j)}}^2 - \left(E_{Y,X}(\sigma_t^{(j)})\right)^2, \end{aligned} \tag{22}$$

where $\omega_{\hat{\theta}}^*(H^{(j)})$ is the normalised importance weight evaluated at the simulated ML estimate $\hat{\theta}$. Similarly, for the estimates of $\beta_{0,t}$ and $\beta_{1,t}$ we have that

$$\begin{aligned} E_{Y,X}(\beta_{i,t}) &= \sum_{j=1}^{M}\omega_{\hat{\theta}}^*(H^{(j)})E_p(\beta_{i,t}|H^{(j)}), \\ V_{Y,X}(\beta_{i,t}) &= \sum_{j=1}^{M}\omega_{\hat{\theta}}^*(H^{(j)})V_p(\beta_{i,t}|H^{(j)}) + \sum_{j=1}^{M}\omega_{\hat{\theta}}^*(H^{(j)})\left(E_p(\beta_{i,t}|H^{(j)})\right)^2 - \left(E_{Y,X}(\beta_{i,t})\right)^2, \end{aligned} \tag{23}$$

where $i = 0,1$. $E_p(.|H^{(j)})$ and $V_p(.|H^{(j)})$ are the smoothed mean and variance of $\beta_{i,t}$ derived from the Kalman smoother based on the TVPM-SV with $H^{(j)}$ given; and $V_{Y,X}(\beta_{i,t})$ comes from the law of total variance.

[14]Note that the last equation holds because $p(H|X;\theta) = g(H|X;\theta)$. The former is Gaussian as $\log\sigma_t$ is a random walk with Gaussian innovations.

3.2 Heckman-type two-step bias correction

We employ the Heckman-type two-step bias correction developed by Kim (2006) in order to deal with the possible endogeneity problem in the TVPMs-SV.[15] Suppose that a set of instrumental variables (IVs) $z_t \in \mathbb{R}^p$, $t = 1, \dots, T$, is available; and that there is a linear relationship between the endogenous regressor Δu_t and z_t via a standard TVPM. Hence, for $t = 1, \dots, T$,

$$\begin{aligned} \Delta u_t &= z_t' \gamma_t + e_t, \quad e_t \sim N(0, \sigma_e^2), \\ \gamma_{t+1} &= \gamma_t + \xi_t, \quad \xi_t \sim N(0, \Sigma_\xi), \end{aligned} \tag{24}$$

where γ_t is a $p \times 1$ vector of time-varying parameters with diagonal innovation covariance matrix Σ_ξ. The orthogonal projection lemma and the Kalman filter allows us to decompose Δu_t into a predicted value $E(\Delta u_t | \mathscr{F}_{t-1})$ and an orthogonal prediction error $\hat{e}_t$:

$$\Delta u_t = E(\Delta u_t | \mathscr{F}_{t-1}) + \hat{e}_t, \quad \hat{e}_t = \sigma_e \hat{e}_t^*, \quad \hat{e}_t \sim N(0,1),$$

where $\hat{e}_t^*$ is the standardised prediction error and $\mathscr{F}_{t-1}$ denotes the information set at $t-1$. The standard deviation σ_e of the TVPM (24) is derived from the Kalman recursions.

If we assume that $E(\hat{e}_t^* \varepsilon_t) = \rho \sigma_t$, the regression lemma yields

$$\varepsilon_t = \rho \sigma_t \hat{e}_t^* + \varepsilon_t^*, \quad \varepsilon_t^* \sim N\big(0, (1-\rho^2)\sigma_t^2\big). \tag{25}$$

Equation (25) shows the two components of ε_t. The endogenous regressor Δu_t is correlated with $\hat{e}_t^*$, but not correlated with the orthogonal component ε_t^*. Substituting (25) into (17) results in

$$g_t = \beta_{0,t} + \beta_{1,t} \Delta u_t + \rho(\phi \sigma_{t-1} \hat{e}_{t-1}^* + \sigma_t \hat{e}_t^*) + \phi \varepsilon_{t-1}^* + \varepsilon_t^*. \tag{26}$$

Thus, the standardised prediction errors $\hat{e}_t^*$ and $\hat{e}_{t-1}^*$ in the equation above augment the original measurement equation (17) as bias correction terms in the spirit of Heckman (1976)'s two-step procedure for a sample selection model.[16]

To summarise, the TVPM-SV with bias correction terms are estimated via the following three steps:

1. *Error decomposition:* the IV equation (24) is estimated using the Kalman filter and the standardised one step-ahead prediction errors $\hat{e}_t^*$, $t = 1, \dots, T$, are obtained.

2. *Median unbiased estimator:* the first-stage equation (19) is augmented with $\hat{e}_t^*$. We then estimate the following model using the Kalman filter:

$$\begin{aligned} g_t &= \beta_0 + \beta_{1,t} \Delta u_t + \rho \sigma_\xi (\phi \hat{e}_{t-1}^* + \hat{e}_t^*) + \sqrt{1-\rho^2} \sigma_\xi (\phi \varepsilon_{t-1}^* + \varepsilon_t^*) \\ \beta_{1,t+1} &= \beta_{1,t} + \sigma_1 \eta_{1,t}, \end{aligned}$$

and determine $\hat{\lambda}_0 = \hat{\sigma}_0 / \hat{\sigma}_\xi$ based on the exponential Wald test statistic.

[15] As shown in Appendix B.2, the TVPM-SV is conditionally linear, meaning that the Kalman filter can integrate out $\beta_{0,t}$ and $\beta_{1,t}$ for a given trajectory of $\{\log \sigma_t\}_{t=1}^T$. Therefore, if the exogeneity assumption were violated, the Monte Carlo estimate (21) would not be consistent.

[16] Note that the SV, σ_t, is part of the measurement equation (26), which resembles the *SV in mean* model of Koopman and Hol Uspensky (2002), with the state transition shown in equation (18). Nevertheless, the TVPM-SV model with bias correction is still conditionally linear, so that we use the simulated ML method introduced in the previous section for estimation. Note also that the bias correction term $\rho(\phi \sigma_{t-1} \hat{e}_{t-1}^* + \sigma_t \hat{e}_t^*)$ serves as a time-varying intercept.

3. *Simulated MLE:* with $\lambda_0^2 = \hat{\lambda}_0^2 \frac{1+\phi^2}{1-\rho^2}$, the full model with measurement equation (26) and state transition equation (18) is estimated using simulated ML with parameter vector $\theta = (\sigma_\varepsilon, \lambda_1, \phi, \rho)$.

4 Estimation results

We estimated the TVPMs-SV for the G-7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the USA) for the longest possible periods, selected according to the availability of data. We used quarterly data constructed as follows: g_t denotes the quarterly growth rate of GDP; Δu_t is the quarter-to-quarter first difference of the u_t; and λ_t denotes the quarterly growth rate of the labour force.[17] We included pulse dummy variables (D) for the 4 quarters of 1991 ($D = 1$ in 1991q1, 1991q2, 1991q3, and 1991q4; and $D = 0$ otherwise) in the estimation for Germany in order to control for the re-unification.

Regarding the IVs employed for Δu_t, we used different combinations of the lags of Δu_t; $g_t - \lambda_t$; λ_t; and of the rate of growth of hours worked per worker (percent change from same quarter a year ago) $\hat{h}_t$, which was only available for the US business sector. The lags of these variables reflect relevant characteristics of the labour market that can be regarded as exogenous with respect to the current existent relationships presented in the TVPMs-SV (that is, between g_t and Δu_t and between $g_t - \lambda_t$ and Δu_t). The final combination of instruments for each country was selected according to two criteria based on the standard two-stage least square estimation: (1) the instruments employed needed to be valid —that is, uncorrelated with the error term— according to Hansen's J-statistic[18]; and (2) the instruments employed needed to be jointly significant according to the first-stage F-statistic.[19] Moreover, since we incorporated MA(1) dynamics in the error terms of the TVPMs-SV, we employed lagged values prior to $t-1$ to consider only predetermined lagged variables.[20]

Tables 1 and 2 below present the estimates of the innovation variances for the TVPMs-SV for the long-run output growth rates and the long-run technical progress growth rates, respectively. The estimates satisfied the correct specification tests (no serial correlation, no heteroskedasticity and normality) in the standardised one-period-ahead-forecast errors at the 95% confidence level.[21]

[INSERT TABLE 1 ABOUT HERE]
[INSERT TABLE 2 ABOUT HERE]

[17] Table B.1 in Appendix C shows a full description of the series for each country.

[18] Hansen's J-statistic is a test for over-identifying restrictions that is consistent in the presence of heteroskedasticity and autocorrelation. Under the assumption of conditional homoskedasticity, Hansen's J-statistic becomes the well-known Sargan statistic of overidentifying restrictions (Hayashi , 2000).

[19] For the case of a single endogenous regressor, the first-stage F-statistic corresponds to the Cragg–Donald F-statistic, which tests for weak identification (that is, it tests if instruments are only marginally relevant) (Stock and Yogo , 2005).

[20] The sets of instruments selected in this way for each country are presented in Tables 1 and 2. A full description of the two-stage least square estimation results obtained for each country is available on request.

[21] The innovation variances obtained from the TVPMs without SV are presented in Tables C.2 and C.3 in Appendix C. The majority of these models presented both heteroskedasticity and normality problems, which corroborates the importance of introducing the SV component. A full report showing all the correct specification tests for both the TVPMs-SV and the TVPMs is also available on request.

Regarding the endogeneity problem of the regressor Δu_t, from Tables 1 and 2 it possible to observe that the estimated coefficient of the correction term bias, ρ, is statistically significant in the majority of countries when models (15) and (16) were estimated, the only exceptions being Canada and Japan. Hence, the endogeneity problem seems to be important in France, Germany, Italy, the United Kingdom and the USA; and, for these countries, the final estimates need to be retrieved from the estimations that include the bias correction terms.

The time-varying long-run growth rates are presented in Figures 2 to 8. As mentioned before, we computed both the smoothed estimates and the one-sided estimates. The latter can be regarded as the real-time estimates of the latent processes and, thus, are important in order to consider the periods for each country that do not incorporate the effects of the GR. We also plot the rate of growth of potential output estimated by the Congressional Budget Office (CBO) for the USA in Figure 8 in order to compare our estimation results. It is worth noting that the CBO's estimates lie within our estimated 95% confidence intervals during the period of study.[22]

[INSERT FIGURE 2 ABOUT HERE]
[INSERT FIGURE 3 ABOUT HERE]
[INSERT FIGURE 4 ABOUT HERE]
[INSERT FIGURE 5 ABOUT HERE]
[INSERT FIGURE 6 ABOUT HERE]
[INSERT FIGURE 7 ABOUT HERE]
[INSERT FIGURE 8 ABOUT HERE]

Figures 2 to 8 show that both long-run growth rates have been declining in the G-7 countries during the periods of study and that this result is robust to both the smoothed estimates and the one-sided estimates, so that the long-run growth rates were already declining before the GR. In order to calculate the relative magnitudes, Table 3 below calculates the percentage point (pp) changes in the estimated long-run growth rates for the complete periods of study and for the period up until 2006Q4:

[INSERT TABLE 3 ABOUT HERE]

Upon inspection of Figures 2 to 8 and Table 3, it is possible to summarise the main two findings as follows. First, long-run output growth rates have fallen in the G-7 countries during the post-war era because of reasons unrelated to the effects of the GR. The estimates show that the long-run output growth rates have been falling since the late 1960s in the USA; since the early 1970s in Canada, Germany and Japan; and since the mid-1980s in France, Italy and the United Kingdom.

[22]Both the time-varying Okun coefficients on unemployment and the respective SV coefficients obtained from model (15) are presented in Figures D.1 to D.7 in Appendix D. We find a reduction of the SV coefficients in the majority of countries, which corroborates the findings of the literature on the Great Moderation (that is, a reduction in the volatility of business cycle fluctuations) (Stock and Watson , 2002). On the other hand, the results obtained for the Okun coefficients need to be interpreted in the light of a mix of components such as the demographic structure of each country, its labour market flexibility, its labour market policies, and its policy implementation timing because, as shown in Section 2, the Okun coefficient on unemployment is a mixture of different effects. This exceeds the purpose of the current paper. Nevertheless, it is possible to say that, with the exception of Germany, the results obtained corroborate previous findings by Mendieta-Muñoz (2017), who documented a reduction (increase) in the Okun coefficient on unemployment in the USA (Canada, France, Italy, Japan and the United Kingdom) for the period 1981-2011 using a penalised regression spline estimator.

If we consider the average of the respective estimation periods, Japan is the country with the most important fall in long-run output growth (approximately -8.6 pp) during the post-war era, followed by Canada (-3.5 pp), Germany (-3.3 pp), the USA (-3.0 pp), France (-2.5 pp), Italy (-2.2 pp) and the United Kingdom (-1.6 pp).

Second, long-run technical progress growth rates have also fallen in the G-7 countries during the post-war era because of reasons unrelated to the effects of the GR. The estimation results show that technical progress growth rates have been falling since the early 1960s in Canada; since the late 1960s in the USA; since the early 1970s in Germany and Japan; since the late 1980s in Italy; and over the last decade in France. It is worth mentioning that the results for the USA are broadly consistent with the other studies (mentioned in Section 1) that have discussed the evolution of productivity growth in the USA: high growth rates in the 1950s and 1960s, lower growth rates in the 1970s and 1980s, relatively higher growth rates in the 1990s and 2000s, and a further reduction in productivity growth rates since then. Again, if the averages of the respective periods of study are considered, Japan is the country with the most important fall in technical progress growth (approximately -8.3 pp), followed by Germany (-4.9 pp), Canada (-3.1 pp), the USA (-3.1 pp), France (-2.7 pp), the United Kingdom (-2.1 pp) and Italy (-1.6 pp). Because our approach identifies two main sources of economic growth —labour force growth and technical progress growth, these results show that the main reason behind the fall in long-run output growth rates is associated with the permanent fall in long-run technical progress growth rates.

5 Concluding remarks

The present article is related to the recent literature that has studied the possibility of permanent losses of long-run GDP growth in developed countries. We studied the evolution of the rate of growth of output consistent with a constant unemployment rate, which can be identified with a measure of long-run GDP growth because it represents the sum of labour force and technical progress growth. We showed that the latter can be estimated using the first-difference version of Okun's law by combining a production function that included capacity utilization and working hours with a wage-setting equation in order to depict the equilibrium in the goods and labour markets. This analysis illustrates that, despite the simplicity of the approach and the restrictive assumptions, the Okun coefficient on unemployment represents a mixture of different effects and, more importantly, that an estimate of the intercept derived from this specification represents the sum of the rates of growth of labour force and of technical progress. The methodology proposed also allowed the computation of the long-run growth rate associated with technical progress by separating the effects derived from movements in the rate of growth of the labour force.

The long-run growth rates were estimated for the G-7 countries during the post-war era using time-varying parameter models that incorporate both stochastic volatility and a Heckman-type two-step estimation procedure that deals with the issue of endogenous regressors in the econometric models. The results show a permanent reduction in long-run output and technical progress growth rates during the post-war era in the G-7 countries that is not associated with the detrimental effects of the Great Recession. Although each country has experienced its own growth dynamics, we document that long-run output growth rates began to fall since the late 1960s and that long-run technical progress growth began to fall since the early 1960s. With respect to the former, we quantify that Japan is the country that has experienced the largest decline (-8.6 percentage points), followed by Canada (-3.5 percentage points), Germany (-3.3 percentage points), the USA (-3.0 percentage points), France (-2.5 percentage points), Italy (-2.2 percentage points) and the United Kingdom (-1.6 percentage points). Likewise, we quantify that the fall in long-run technical progress growth rates has been approximately -8.3 percentage points in Japan, -4.9 percentage points in Germany, -3.1 percentage points in Canada and the USA, -2.7 percentage points in France, -2.1 percentage points in the United Kingdom, and -1.6 percentage points in Italy. These findings suggest that that the slowdown in productivity —and not demographic factors— has been the main driver of the decline in long-run GDP growth.

Our results also raise questions about the underlying properties of output and technical progress. Future theoretical and empirical research should try to study the deep causes of the secular decline in economic growth. One potentially fruitful line for future research is to try to identify the main sectors in which the largest declines in technical progress have taken place in each country. Another alternative is to consider more seriously the possibility of strong hysteresis effects —given that some recessions can have systematic permanent effects on economic growth, as shown by Cerra and Saxena (2008); DeLong and Summers (2012); Reifschneider et al. (2015)— and to try to identify the relevant short-run fluctuations that might have affected the evolution of technical progress in the long-run.

References

Adachi, Hideyuki, Shin Imoto, and Kazuyuki Inagaki. (2015). “Economic growth and unemployment: Theoretical foundations of Okun’s law.” In Adachi, Hideyuki, Tamotsu Nakamura, and Yasuyuki Osumi (eds.). *Studies in Medium-Run Macroeconomics. Growth, Fluctuations, Unemployment, Inequality and Policies*, pp. 69-85. World Scientific Publishing: Singapore.

Antolin-Diaz, Juan, Thomas Drechsel, and Ivan Petrella. (2017). “Tracking the slowdown in long-run GDP growth.” *Review of Economics and Statistics* 99, 343-356.

Barreto, Humberto, and Frank Howland. (1993). “There are two Okun’s law relationship between output and unemployment.” Wabash College Working Paper.

Benati, Luca. (2007). “Drift and breaks in labor productivity.” *Journal of Economic Dynamics and Control* 31, 2847-2877.

Beveridge, Stephen, and Charles Nelson. (1981). “A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle.” *Journal of Monetary Economics* 7, 151-174.

Blanchard, Olivier. (1997). “The Medium Run.” *Brooking Papers on Economic Activity*, 28(2): 89–158.

Byrne, David, John Fernald, and Marshall Reinsdorf. (2016). “Does the United States have a productivity slowdown or a measurement problem?” *Brookings Papers on Economic Activity* 47, 109-157.

Cerra, Valerie, and Sweta Saxena. (2008). “Growth dynamics: The myth of economic recovery.” *American Economic Review* 98, 439-457.

Cette, Gilbert, John Fernald, and Benoît Mojon (2016). “The pre-Great Recession slowdown in productivity.” *European Economic Review* 88, 3-20.

De Jong, Piet, and Neil Shephard. (1995). “The simulation smoother for time series models.” *Biometrika* 82, 339-350.

DeLong, J. Bradford, and Lawrence Summers. (2012). “Fiscal policy in a depressed economy.” *Brooking Papers on Economic Activity* 43, 233-274.

Doucet, Arnaud, Nando de Freitas, and Neil Gordon. (Eds.) (2001). *Sequential Monte Carlo Methods in Practice*. New York, NY.: Springer (Series Statistics For Engineering and Information Science).

Fernald, John. (2015). “Productivity and potential output before, during, and after the Great Recession.” *NBER Macroeconomics Annual* 29, 1-51.

Fernald, John, Robert Hall, James Stock, and Mark Watson. (2017). “The disappointing recovery of output after 2009.” *Brooking Papers on Economic Activity* 48, 1-58.

Geweke, John. (1989). "Bayesian inference in econometric models using Monte Carlo integration." *Econometrica* 57, 1317-1339.

Gordon, Robert. (2010). "Okun's law and productivity innovations." *American Economic Review. Papers and Proceedings* 100, 11-15.

Gordon, Robert. (2012). "Is US economic growth over? Faltering innovation confronts the six headwinds." NBER Working Papers No. 18315.

Gordon, Robert. (2014a). "The demise of U.S. economic growth: Restatement, rebuttal and reflections." NBER Working Papers No. 19895.

Gordon, Robert. (2014b). "A new method of estimating potential real GDP growth: Implications for the labor market and the debt/GDP ratio." NBER Working Papers No. 20423.

Gordon, Robert. (2016). *The Rise and Fall of American Growth*. Princeton, NJ.: Princeton University Press.

Gordon, Robert. (2018). "Declining American economic growth despite ongoing innovation." *Explorations in Economic History* 69 (C): 1-12.

Harrod, Roy. (1939). "An essay in dynamic theory." *Economic Journal* 49, 14-33.

Harvey, Andrew. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge, UK.: Cambridge University Press.

Harvey, Andrew. (2011). "Modelling the Phillips curve with unobserved components." *Applied Financial Economics* 21, 7-17.

Hayashi, Fumio. (2000). *Econometrics*. Princeton, NJ.: Princeton University Press.

Heckman, James. (1976). "The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models." *Annals of Economic and Social Measurement* 5, 475-492.

Holston, Kathryn, Thomas Laubach, and John C. Williams. (2017). "Measuring the natural rate of interest: International trends and determinants." *Journal of International Economics* 108, S59-S75.

International Monetary Fund. (2010). "Chapter 3. Unemployment dynamics during recessions and recoveries: Okun's law and beyond." In *World Economic Outlook. Rebalancing Growth*, pp. 69-107. Washington, D.C.: World Economic and Financial Surveys.

Kim, Chang-Jin. (2006). "Time-varying parameter models with endogenous regressors." *Economics Letters* 91, 21-26.

Koopman, Siem Jan. (1997). "Exact initial Kalman filtering and smoothing for non-stationary time series models." *Journal of American Statistical Association* 92, 1630-1638.

Koopman, Siem Jan, and Eugenie Hol Uspensky. (2002). "The stochastic volatility in mean model: Empirical evidence from international stock markets." *Journal of Applied Econometrics* 17, 667-689.

Koopman, Siem Jan, André Lucas, and Marcel Scharth. (2015). "Numerically accelerated importance sampling for nonlinear non-Gaussian state-space models." *Journal of Business and Economic Statistics* 33, 114-127.

Klump, Rainer, Peter McAdam, and Alpo Willman. (2008). "Unwrapping some euro area growth puzzles: Factor substitution, productivity and unemployment." *Journal of Macroeconomics* 30, 645-666.

Laubach, Thomas, and John C. Williams. (2003). "Measuring the natural rate of interest." *Review of Economics and Statistics* 85, 1063-1070

León-Ledesma, Miguel, and Anthony Thirlwall (2002). "The endogeneity of the natural rate of growth." *Cambridge Journal of Economics* 26, 441-459.

Mendieta-Muñoz, Ivan. (2017). "On the interaction between economic growth and business cycles." *Macroeconomic Dynamics* 21, 982-1022.

Prachowny, Martin F. J. (1993). "Okun's law: Theoretical foundations and revised estimates." *Review of Economics and Statistics* 75, 331-336.

Reifschneider, David, William Wascher, and David Wilcox. (2015). "Aggregate supply in the United States: Recent developments and implications for the conduct of monetary policy." *IMF Economic Review* 63, 71-109.

Scharth, Marcel, and Robert Kohn. (2016). "Particle efficient importance sampling." *Journal of Econometrics* 190, 133-147.

Schnabel, G. (2002). "Output trends and Okun's law." BIS Working Papers No. 111.

Stock, James, and Motohiro Yogo. (2005). "Testing for weak instruments in linear IV regression." In *Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg*, edited by Donald Andrews and James Stock, pp. 80-108. New York, NY.: Cambridge University Press.

Stock, James, and Mark Watson. (1998). "Median unbiased estimation of coefficient variance in a time-varying parameter model." *Journal of the American Statistical Association* 93, 349-358.

Stock, James, and Mark Watson. (2002). "Has the business cycle changed and why?" *NBER Macroeconomics Annual* 17, 159-218.

Thirlwall, Anthony. (1969). "Okun's law and the natural rate of growth." *Southern Economic Journal* 36, 87-89.

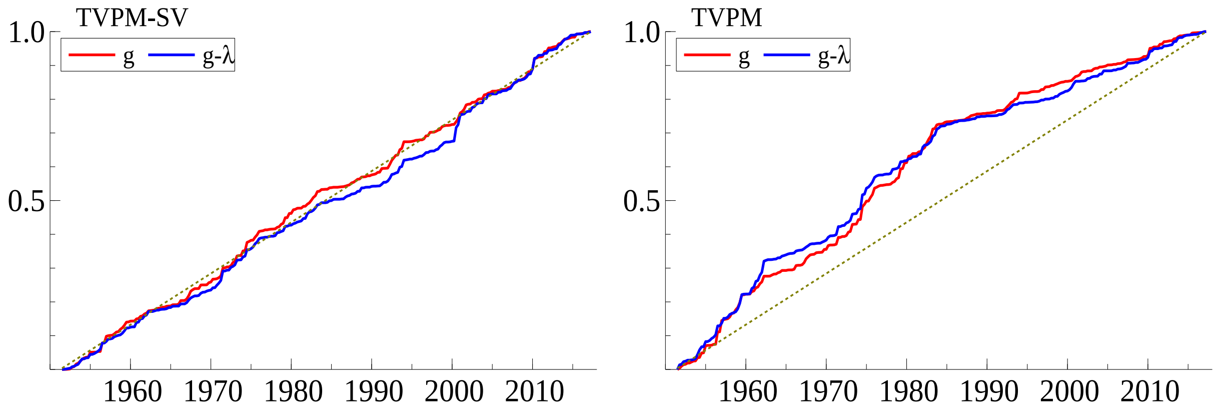


Figure 1. Cumulative sum of squared standardised residuals of model (15), denoted by g, and model (16), denoted by $g-\lambda$, for the USA. Left: residuals from the Time-Varying Parameter Models with Stochastic Volatility (TVPMs-SV). Right: residuals from the Time-Varying Parameter Models (TVPMs). The cumsum statistic of squared standardised residuals detects heteroskedasticity left in the residuals of the latter.

Table 1. Long-run output growth rates (model (15)): estimation of the hyper-parameters for the time-varying parameter models with stochastic volatility

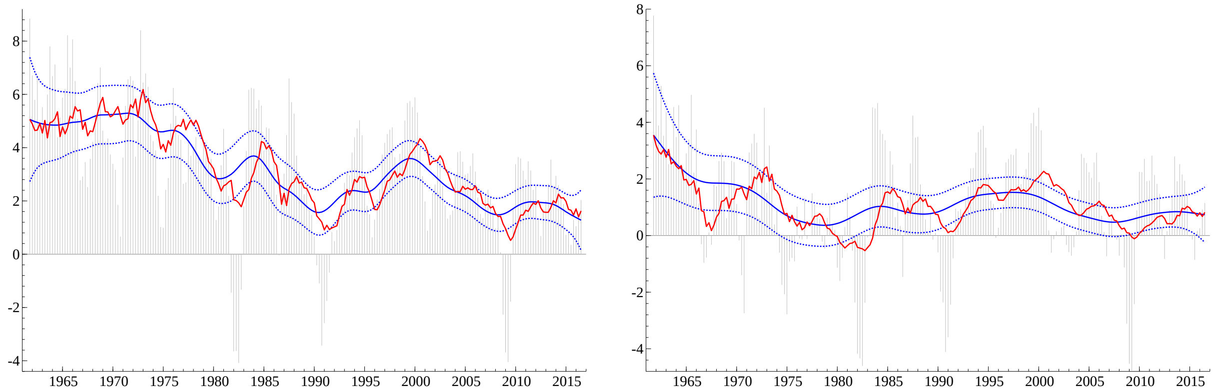
Hyper-parameters	Canada, 1961Q1-2016Q4	France, 1984Q1-2016Q4	Germany, 1963Q1-2016Q4	Italy, 1984Q1-2016Q4	Japan, 1961Q1-2016Q4	United Kingdom, 1972Q1-2016Q4	USA, 1951Q1-2016Q4
Models without bias correction terms[a]							
σ_ε	0.06*	0.18**	0.05**	0.22**	0.18*	0.05*	0.02*
	(0.03)	(0.06)	(0.02)	(0.05)	(0.09)	(0.02)	(0.01)
λ_1	0.0	0.11	0.25**	0.25	0.30*	0.29*	0.04
	(0.0)	(0.16)	(0.09)	(0.16)	(0.15)	(0.13)	(0.02)
ϕ	0.82*	0.84**	0.60**	0.84**	0.77**	0.64^	0.76**
	(0.35)	(0.13)	(0.18)	(0.08)	(0.11)	(0.33)	(0.16)
L^b	-232.40	-98.74	-227.95	-131.12	-225.65	-179.63	-295.27
Models with bias correction terms[a,c]							
σ_ε	0.07**	0.20*	0.23**	0.38**	0.13*	0.14**	0.07**
	(0.03)	(0.09)	(0.06)	(0.73)	(0.05)	(0.04)	(0.02)
λ_1	0.0	0.13	0.22*	0.12^	0.22**	0.41*	0.05
	(0.01)	(0.19)	(0.10)	(0.07)	(0.07)	(0.19)	(0.03)
ϕ	0.82**	0.82**	0.64**	0.91**	0.73**	0.79**	0.68**
	(0.10)	(0.18)	(0.07)	(0.13)	(0.09)	(0.12)	(0.06)
ρ	0.02	-0.35**	-0.24**	-0.20*	-0.07	-0.17*	-0.25**
	(0.07)	(0.11)	(0.07)	(0.10)	(0.06)	(0.07)	(0.06)
L^b	-222.48	-95.25	-214.39	-122.65	-223.14	-168.53	-275.11

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood; [c]The combination of instruments employed for Δu_t in each country was the following:
1) Canada: $(g_{t-2}-\lambda_{t-2})$, $(g_{t-3}-\lambda_{t-3})$, and $(g_{t-4}-\lambda_{t-4})$.
2) France: Δu_{t-2}, λ_{t-3}, λ_{t-4}, λ_{t-5}, and λ_{t-6}.
3) Germany: Δu_{t-2}, λ_{t-4}, λ_{t-5} , λ_{t-6}, and λ_{t-7}.
4) Italy: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, $(g_{t-7}-\lambda_{t-7})$, and $(g_{t-8}-\lambda_{t-8})$.
5) Japan: Δu_{t-2}, Δu_{t-3}, and Δu_{t-4}.
6) United Kingdom: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, and $(g_{t-7}-\lambda_{t-7})$.
7) USA: $\hat{h}_{t-3}$, $\hat{h}_{t-4}$, $\hat{h}_{t-5}$, $\hat{h}_{t-6}$, and $\hat{h}_{t-7}$.
^,*, and ** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 2. Long-run technical progress growth rates (model (16)): estimation of the hyper-parameters for the time-varying parameter models with stochastic volatility

Hyper-parameters	Canada, 1961Q1-2016Q4	France, 1996Q1-2016Q4	Germany, 1963Q1-2016Q4	Italy, 1984Q1-2016Q4	Japan, 1961Q1-2016Q4	United Kingdom, 1972Q1-2016Q4	USA 1951Q1-2016Q4
Models without bias correction terms[a]							
σ_ε	0.07*	0.31*	0.05	0.25**	0.10**	0.07**	0.03*
	(0.03)	(0.14)	(0.03)	(0.07)	(0.03)	(0.02)	(0.00)
λ_1	0.03	0.11	0.24*	0.47**	0.14*	0.47**	0.05
	(0.04)	(0.14)	(0.12)	(0.16)	(0.06)	(0.12)	(0.04)
ϕ	0.92**	0.79**	0.61**	0.92**	0.96**	0.68^	0.83**
	(0.11)	(0.08)	(0.19)	(0.18)	(0.24)	(0.36)	(0.16)
L^b	-228.33	-100.02	-228.25	-129.85	-223.24	-175.59	-290.31
Models with bias correction terms[a,c]							
σ_ε	0.09**	0.34**	0.15**	0.31**	0.11**	0.15**	0.07*
	(0.03)	(0.13)	(0.04)	(0.08)	(0.03)	(0.04)	(0.03)
λ_1	0.03	0.21	0.18*	0.22^	0.15*	0.58*	0.04
	(0.03)	(0.18)	(0.09)	(0.13)	(0.07)	(0.25)	(0.03)
ϕ	0.96**	0.86**	0.57**	0.86**	0.96**	0.77**	0.71**
	(0.04)	(0.30)	(0.07)	(0.12)	(0.14)	(0.10)	(0.07)
ρ	-0.02	-0.37**	-0.28**	-0.17^	-0.10	-0.17*	-0.32**
	(0.06)	(0.10)	(0.06)	(0.10)	(0.06)	(0.07)	(0.06)
L^b	-217.04	-94.98	-218.55	-124.68	-226.60	-169.03	-283.29

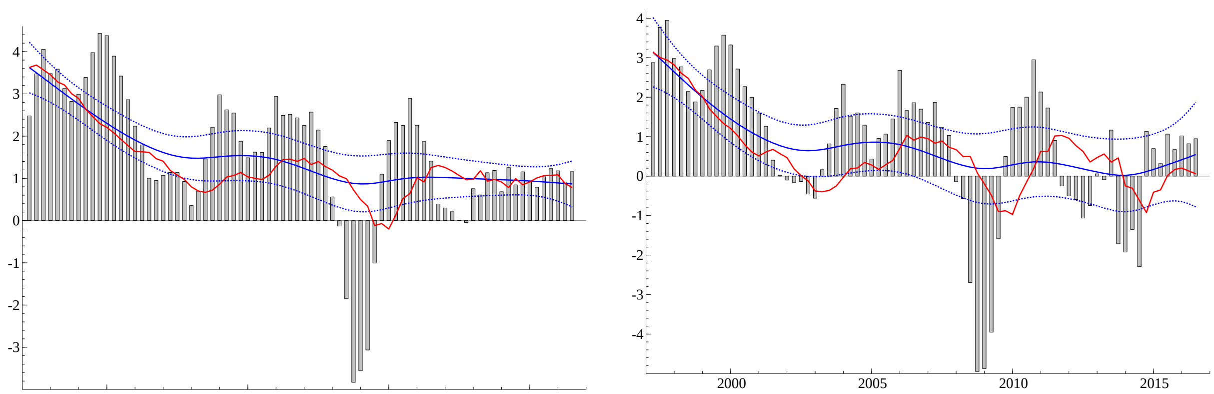
Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood; [c]The combination of instruments employed for Δu_t in each country was the following:
1) Canada: $(g_{t-2}-\lambda_{t-2})$, $(g_{t-3}-\lambda_{t-3})$, and $(g_{t-4}-\lambda_{t-4})$.
2) France: Δu_{t-2}, λ_{t-3}, λ_{t-4}, λ_{t-5}, and λ_{t-6}.
3) Germany: Δu_{t-2}, λ_{t-4}, λ_{t-5} , λ_{t-6}, and λ_{t-7}.
4) Italy: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, $(g_{t-7}-\lambda_{t-7})$, and $(g_{t-8}-\lambda_{t-8})$.
5) Japan: Δu_{t-2}, Δu_{t-3}, and Δu_{t-4}.
6) United Kingdom: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, and $(g_{t-7}-\lambda_{t-7})$.
7) USA: $\hat{h}_{t-3}$, $\hat{h}_{t-4}$, $\hat{h}_{t-5}$, $\hat{h}_{t-6}$, and $\hat{h}_{t-7}$.
^,*, and ** denote statistical significance at the 10%, 5%, and 1% levels, respectively.



(a) Long-run growth rate, model (15)

(b) Long-run technical progress growth rate, model (16)

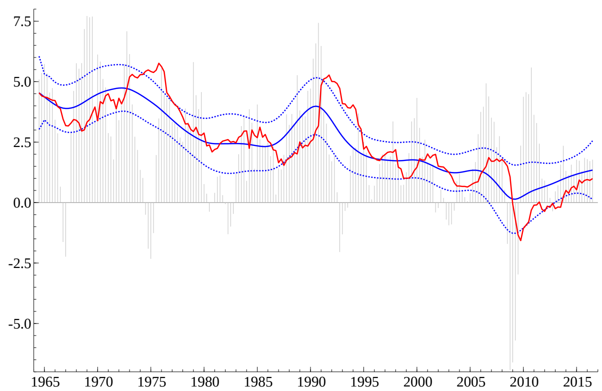
Figure 2. Canada, 1961Q1-2016Q4. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series.



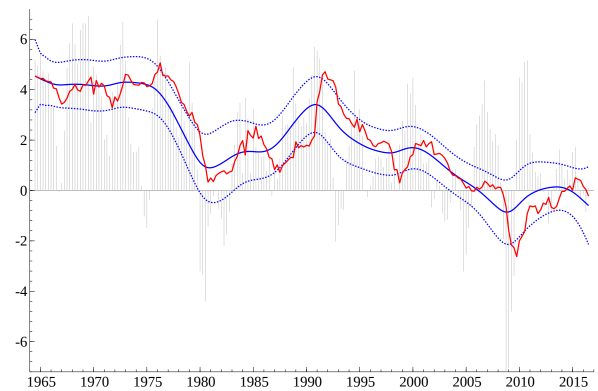
(a) Long-run growth rate, model (15), 1984Q1-2016Q4

(b) Long-run technical progress growth rate, model (16), 1996Q1-2016Q4

Figure 3. France. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series.

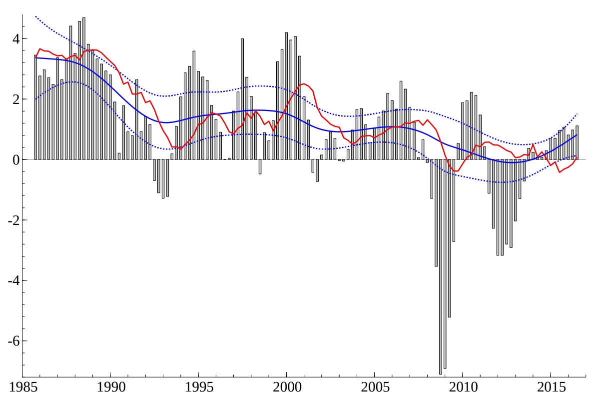


(a) Long-run growth rate, model (15)

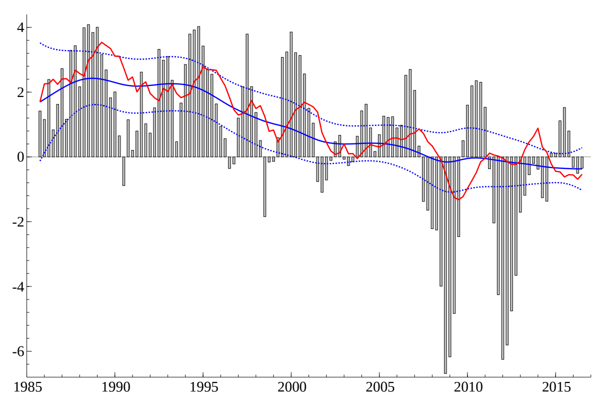


(b) Long-run technical progress growth rate, model (16)

Figure 4. Germany, 1963Q1-2016Q4. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series.

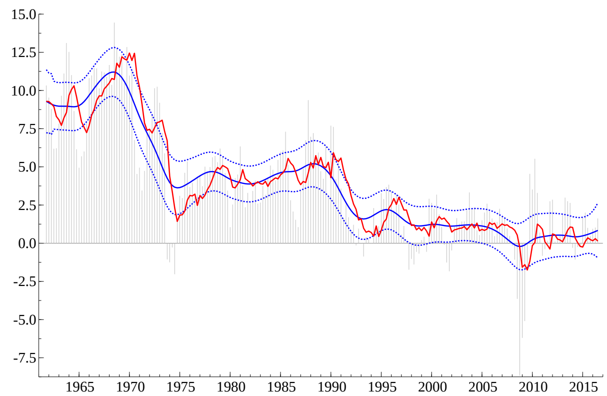


(a) Long-run growth rate, model (15)



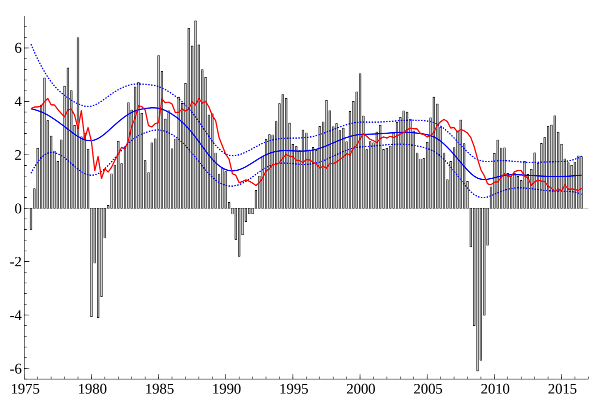
(b) Long-run technical progress growth rate, model (16)

Figure 5. Italy, 1984Q1-2016Q4. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series.



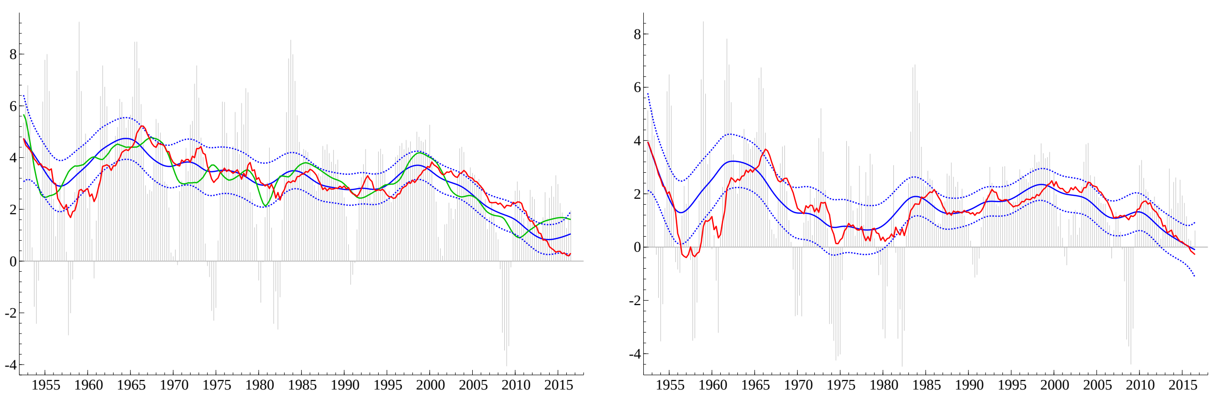
(a) Long-run growth rate, model (15) **(b)** Long-run technical progress growth rate, model (16)

Figure 6. Japan, 1961Q1-2016Q4. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series.



(a) Long-run growth rate, model (15) **(b)** Long-run technical progress growth rate, model (16)

Figure 7. United Kingdom, 1972Q1-2016Q4. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series.



(a) Long-run growth rate, model (15) **(b)** Long-run technical progress growth rate, model (16)

Figure 8. USA, 1951Q1-2016Q4. Blue straight lines are the smoothed estimates. Blue dotted lines are the 95% confidence intervals around the latter. Red straight lines are the one-sided estimates. Gray rectangular bars show the actual series. The green dotted line in Figure 8a shows the potential output growth rate estimated by the Congresional Budget Office (CBO).

Table 3. Percentage point changes in the estimated long-run growth rates

	Long-run output growth rates		Long-run technical progress growth rates[a]	
	Smoothed estimates	One-sided estimates	Smoothed estimates	One-sided estimates
For the whole period				
Canada, 1961Q1-2016Q4	-4.19	-3.47	-3.54	-2.66
France, 1984Q1-2016Q4	-2.97	-2.87	-2.82	-3.08
Germany, 1963Q1-2016Q4	-3.72	-3.45	-5.76	-4.79
Italy, 1984Q1-2016Q4	-1.92	-3.16	-1.89	-2.19
Japan, 1961Q1-2016Q4	-8.45	-9.19	-8.60	-9.01
United Kingdom, 1972Q1-2016Q4	-1.93	-2.85	-2.16	-2.94
USA, 1951Q1-2016Q4	-3.14	-4.12	-3.30	-4.23
Before the Great Recession				
Canada, 1961Q1-2006Q4	-3.68	-2.79	-3.74	-2.55
France, 1984Q1-2006Q4	-2.21	-2.03	-2.63	-2.22
Germany, 1963Q1-2006Q4	-3.18	-2.89	-4.71	-4.40
Italy, 1984Q1-2006Q4	-1.65	-2.22	-1.19	-1.06
Japan, 1961Q1-2006Q4	-8.54	-8.13	-8.04	-7.53
United Kingdom, 1972Q1-2006Q4	-0.99	-0.53	-1.59	-1.52
USA, 1951Q1-2006Q4	-2.35	-2.58	-2.47	-2.43

Notes: [a]The percentage point change in France corresponds to the period 1996Q1-2006Q4.

A Theoretical details

A.1 Diminishing returns with respect to the rate of capacity utilization

The production function shown in equation (2) is $y = vf\left(v^{\gamma-1}n\right) = vf$. The second-order partial derivative with respect to v in the latter is given by:

$$\frac{\delta^2 y}{\delta v^2} = (\gamma - 1)\, nv^{\gamma-2}\left[\gamma f' + (\gamma - 1)\, nv^{\gamma-1} f''\right]$$
$$\frac{\delta^2 y}{\delta v^2} = -(1-\gamma)\, nv^{\gamma-2}\left[\gamma f' - (1-\gamma)\, nv^{\gamma-1} f''\right].$$

If $f(.)$ is assumed to be an ordinary well-behaved concave function then the right hand-side of this equation will be negative if and only if $\gamma < 1$.

A.2 First order condition for profit maximization

The profit maximization problem is:

$$\pi = \frac{p}{P}y - \frac{W}{PA}hn.$$

Since equation (2) shows that $y = vf\left(v^{\gamma-1}n\right) = vf$; we know that $h = v^{\gamma}$; and equation (4) shows that $p/P = (y/\tilde{y})^{-1/\eta}$, then it is possible to express the profit maximization problem as follows:

$$\pi = \left(\frac{vf}{\tilde{y}}\right)^{-\frac{1}{\eta}} vf - \frac{W}{PA}v^{\gamma}n.$$

Therefore:

$$\frac{\delta \pi}{\delta n} = \left(1 - \frac{1}{\eta}\right)\frac{v^{-\frac{1}{\eta}} f^{-\frac{1}{\eta}} v^{\gamma} f'}{\tilde{y}^{-\frac{1}{\eta}}} - \frac{W}{PA}v^{\gamma}$$
$$\frac{\delta \pi}{\delta n} = \left(1 - \frac{1}{\eta}\right)\left(\frac{vf}{\tilde{y}}\right)^{-\frac{1}{\eta}} v^{\gamma} f' - \frac{W}{PA}v^{\gamma}.$$

If $\delta\pi/\delta n = 0$ then:

$$\left(1 - \frac{1}{\eta}\right)\left(\frac{vf}{\tilde{y}}\right)^{-\frac{1}{\eta}} f' = \frac{W}{PA}.$$

Finally, we use again the fact that $y = vf = vf\left(v^{\gamma-1}n\right)$ and $p/P = (y/\tilde{y})^{-1/\eta}$ to obtain:

$$\frac{p}{P}\left(1 - \frac{1}{\eta}\right) f'(v^{\gamma-1}n) = \frac{W}{PA}.$$

A.3 Derivation of equations (12) and (13)

For simplicity, let us define $x \equiv v^{\gamma-1}(1-u)l$. The logarithmic differentiation of x with respect to time is:

$$\frac{\dot{x}}{x} = (\gamma-1)\frac{\dot{v}}{v} - \frac{\dot{u}}{1-u} + \frac{\dot{l}}{l}. \tag{A.1}$$

Note that $x \equiv v^{\gamma-1}(1-u)l \equiv v^{\gamma-1}n \equiv v^{\gamma}AN/vK \equiv AhN/vK$ represents total working hours of effective labour per utilized capital stock. Considering the definition of x, the logarithmic differentiation of equation (9) with respect to time yields:

$$\begin{aligned} \frac{\dot{Y}}{Y} &= \frac{\dot{K}}{K} + \frac{\dot{v}}{v} + \frac{xf'(x)}{f(x)}\frac{\dot{x}}{x} \\ \frac{\dot{Y}}{Y} &= \frac{\dot{K}}{K} + \frac{\dot{v}}{v} + \theta\frac{\dot{x}}{x}, \end{aligned} \tag{A.2}$$

where $\theta \equiv xf'(x)/f(x)$ represents the elasticity of output with respect to labour (measured in working hours).

Substituting (A.1) into (A.2) yields:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \frac{\dot{v}}{v} + \theta\left[(\gamma-1)\frac{\dot{v}}{v} - \frac{\dot{u}}{1-u} + \frac{\dot{l}}{l}\right]. \tag{A.3}$$

Considering that $\dot{K}/K = \kappa$ and that $\dot{l}/l = \lambda + \tau - \kappa$, we derive equation (12) from equation (A.3) after some rearrangements.

On the other hand, taking the logarithm of equation (10) and differentiating with respect to time we obtain:

$$\frac{xf''(x)}{f'(x)}\frac{\dot{x}}{x} = -\omega\frac{\dot{u}}{1-u}, \tag{A.4}$$

where x is defined as above; and we assume that both the mark-up μ and the bargaining power of labour (or the reservation wage) α are constant.

It is possible to express $xf''(x)/f'(x)$ in equation (A.4) as follows:

$$\begin{aligned} \frac{xf''(x)}{f'(x)} &= \frac{xf''(x)f(x)}{f'(x)(f(x)-xf'(x))}\frac{f(x)-xf'(x)}{f(x)} \\ \frac{xf''(x)}{f'(x)} &= \frac{xf''(x)f(x)}{f'(x)(f(x)-xf'(x))}\left(1-\frac{xf'(x)}{f(x)}\right) \\ \frac{xf''(x)}{f'(x)} &= -\frac{1}{\sigma}\left(1-\frac{xf'(x)}{f(x)}\right) = -\frac{1}{\sigma}(1-\theta), \end{aligned} \tag{A.5}$$

where θ is defined as above; and $\sigma \equiv -f'(x)(f(x)-xf'(x))/xf''(x)f(x)$ represents the elasticity of substitution between labour and capital.

Substituting (A.1) and (A.5) into (A.4) we have:

$$-\frac{1}{\sigma}(1-\theta)\left[(\gamma-1)\frac{\dot{v}}{v} - \frac{\dot{u}}{1-u} + \frac{\dot{l}}{l}\right] = -\omega\frac{\dot{u}}{1-u}. \tag{A.6}$$

Considering that $\dot{l}/l = \lambda + \tau - \kappa$ we derive equation (13) from equation (A.6) after some rearrangements.

B Econometric details

In this section, we outline the numerically accelerated importance sampling (NAIS) of Koopman et al. (2015) and the particle filter applied to the proposed TVPMs-SV. As before, for simplicity we only consider the TVPM-SV applied to model (15).

B.1 Numerically accelerated importance sampling

We define $h_t = \log \sigma_t$, and the rest of the notations are kept consistent with the main text. Starting from (20), we write the likelihood as

$$L(Y|X;\theta) = g(Y|X;\theta) \int_H \omega_\theta(H) g(H|Y,X;\theta) dH. \tag{B.1}$$

We propose a linear Gaussian state space model with state h_t as follows,

$$\begin{aligned} y_t^* &= h_t + \varepsilon_t^*, \quad y_t^* = \frac{c_t}{b_t}, \quad \varepsilon_t^* \sim N(0, \frac{1}{b_t}), \\ h_{t+1} &= h_t + \sigma_\varepsilon \eta_t, \quad t = 1, ..., T, \end{aligned} \tag{B.2}$$

where c_t and b_t are the importance parameters to be determined, which are implicit functions of Y, X and θ. Model (B.2) implies a (conditional) Gaussian importance density

$$g(y_t^*|h_t) = \exp(a_t + b_t h_t - \frac{1}{2} c_t h_t^2),$$

where $a_t = -\frac{1}{2}(\log(2\pi) - \log c_t + c_t b_t^2)$ is the integrating constant that is not associated with h_t. We then decompose the conditional measurement likelihood as follows:

$$p(Y|X,H;\theta) = \prod_{t=1}^{T} f_{H,t}(v_t(H)), \quad f_{H,t}(.) \overset{\text{d.}}{=} N(0, u_t(H)),$$

where $\overset{\text{d.}}{=}$ denotes equivalence in distribution, and $v_t(H)$ and $u_t(H)$ are the prediction error and its variance at time time t, delivered by the Kalman filter based on the TVPM-SV shown in equations (17) and (18) with H given. Therefore, the importance weight in (B.1) can be factorised by

$$\omega_\theta(H) = \frac{p(Y|X,H;\theta)}{g(Y|X,H;\theta)} = \prod_{t=1}^{T} \frac{f_{H,t}(v_t(H))}{g(y_t^*|h_t)} = \prod_{t=1}^{T} \omega_{\theta,t}.$$

A convenient way of constructing a globally efficient importance density is by minimising the variance of the importance weights $\omega_\theta(H^{(j)})$, $j = 1, ..., M$, in line with the assumption of finite variance suggested by Geweke (1989).[23] This minimisation can be closely approximated because of the decomposition described above. That is, for $t = 1, ..., T$ we solve the minimisation

$$\min_{c_t, b_t} \int \lambda_t^2(c_t, b_t) \omega_{\theta,t} g(h_t|Y,X;\theta) dh_t,$$

[23] Which is globally efficient in a χ^2-divergence sense between $p(.)$ and $g(.)$.

where

$$\lambda_t(c_t, b_t) = \log f_{H,t}(v_t(H)) - \log g(y_t^*|h_t) - \text{constant}.$$

Instead of replacing the above integral with a Monte Carlo average, the NAIS uses Gauss-Hermite (GH) quadrature to accurately calculate its value by noticing that

$$g(h_t|Y,X;\theta) \stackrel{\text{d.}}{=} N(E_g(h_t), V_g(h_t)),$$

where $E_g(h_t)$ and $V_g(h_t)$ are the smoothed mean and variance of h_t based on the importance model (B.2). Hence, the minimisation problem becomes

$$\min_{c_t, b_t} \sum_{j=1}^{S} \lambda_{t,j}(c_t, b_t)\omega_{\theta,t,j},$$

where S is the total number of GH nodes (we simply use $S = 10$) z_j with corresponding weights k_j, $j = 1, ..., S$, and where

$$\begin{aligned} \lambda_{t,j}(c_t, b_t) &= \log f_{H,t}(v_t(H^{(j)})) - \log g(y_t^*|h_t^{(j)}) - \text{constant}, \\ \omega_{\theta,t,j} &= \frac{f_{H,t}(v_t(H^{(j)}))}{g(y_t^*|h_t^{(j)})} k_j \exp(-\frac{1}{2}z_j^2). \end{aligned} \tag{B.3}$$

The above is a weighted least square (WLS) problem[24] with $h_t^{(j)}$ constructed using z_j, *i.e.*,

$$h_t^{(j)} = E_g(h_t) + \sqrt{V_g(h_t)}z_j, \quad j = 1, ..., S. \tag{B.4}$$

Thus, if one initialises the set of importance parameters c_t and b_t, the NAIS calculates first (B.4) and plugs it into the WLS problem (B.3), which has dependent variables

$$\Gamma_t = (\log f_{H^{(1)},t}, ..., \log f_{H^{(S)},t})',$$

a matrix Λ_t of regressors whose j-th row is

$$(1, h_t^{(j)}, -\frac{1}{2}h_t^{(j)2}),$$

and a diagonal weighting matrix Ω_t with the j-th diagonal element $\omega_{\theta,t,j}$.

Finally, the importance parameter can be updated by calculating

$$(\Lambda_t'\Omega_t\Lambda_t)^{-1}\Lambda_t'\Omega_t\Gamma_t.$$

Based on the updated value of c_t and b_t, GH nodes can be again used to construct $h_t^{(j)}$ for all t. This procedure iterates until convergence, and thus finishes the construction of importance density. The convergence is found to be fast and usually takes less than 5 iterations.

[24] Note that $\log f_{H,t}(v_t(H^{(j)}))$ is a constant and that $g(y_t^*|h_t^{(j)})$ is log-linear.

B.2 Particle filter for TVPM-SV

The inference given in Section 2 is about the smoothed estimate, which is based on the available information from $t = 1$ to T. One may be interested either in the filtered or in the one-sided estimate at time t, which is based on information up to t. To this end, we apply a particle filter to the TVPM-SV, which is a nonlinear state space model. Specifically, it is a *sampling importance resampling* (SIR) sequential Monte Carlo method based on the NAIS importance density.[25]

In the following, we suppress the dependence on the simulated ML estimate $\hat{\theta}$. From (B.2) it can be shown that a sequential sampler for h_t is given by the Gaussian density

$$g(h_t|h_{t-1},Y,X) \overset{\text{d.}}{=} N(\mu_t,\pi_t), \quad \pi_t = \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2 c_t}, \quad \mu_t = \pi_t\left(b_t + \frac{h_{t-1}}{\sigma_\varepsilon^2}\right), \quad t = 2,\dots,T,$$

with an obvious modification for initialisation $g(h_1|Y,X)$. This sequential sampler is highly efficient because it takes into account all the information in $\{Y,X\}$. Specifically, it is a *Rao-Blackwellisation*[26] version of the *particle efficient importance sampling* of Scharth and Kohn (2016). We summarise the filtering algorithm below:

1. At time $t = 1$, sample $h_1^{(j)} \sim g(h_1|Y,X)$, $j = 1,\dots,M$. Use diffuse initialisation to draw M samples of $\beta_{0,1}^{(j)}$ and $\beta_{1,1}^{(j)}$, $j = 1,\dots,M$.

   Compute the prediction errors $v_1^{(j)}$ and their variances $u_1^{(j)}$ via Kalman recursion.

   Compute the importance weight

   $$\omega_1^{(j)} = \frac{f_1^{(j)}(v_1^{(j)})p(h_1^{(j)})}{g(h_1^{(j)}|Y,X)}, \quad \text{where} \quad f_1^{(j)}(.) \overset{\text{d.}}{=} N(0,u_1^{(j)}).$$

   Record the log-likelihood contribution $\log\hat{l}_1 = \log(\sum_{j=1}^M \omega_1^{(j)}/M)$.

   Normalise the weights $W_1^{(j)} = \omega_1^{(j)}/\sum_{j=1}^M \omega_1^{(j)}$.

   Record the filtered (one-sided) estimate of the SV by

   $$\begin{aligned} \hat{E}_1(e^{\frac{h_1}{2}}) &= \sum_{j=1}^M W_1^{(j)} \exp h_1^{(j)}/2, \\ \hat{V}_1(e^{\frac{h_1}{2}}) &= \sum_{j=1}^M W_1^{(j)} \exp h_1^{(j)} - \left(\hat{E}_1(e^{\frac{h_1}{2}})\right)^2. \end{aligned} \tag{B.5}$$

   Record the filtered (one-sided) estimate of $\beta_{i,1}$, $i = 0,1$ by

   $$\begin{aligned} \hat{E}_1(\beta_{i,1}) &= \sum_{j=1}^M W_1^{(j)} \beta_{i,1}^{(j)}, \\ \hat{V}_1(\beta_{i,1}) &= \sum_{j=1}^M W_1^{(j)} V_{i,p}^{(j)} + \sum_{j=1}^M W_1^{(j)} {\beta_{i,1}^{(j)}}^2 - \left(\hat{E}_1(\beta_{i,1})\right)^2, \end{aligned} \tag{B.6}$$

[25] Readers may refer to Doucet et al. (2001) and the reference therein for general discussions on the sequential Monte Carlo method.

[26] Because, conditional on the propagation of the SV e^{h_t}, the Kalman filer is used to integrate out $\beta_{i,t}$, $i = 0,1$.

where $V_{i,p}^{(j)}$ is the filtered variance of $\beta_{i,1}$ derived from the Kalman recursion with $h_1^{(j)}$ given. Record the standardised residual

$$\hat{\varepsilon}_1 = (\sum_{j=1}^{M} W_1^{(j)} v_1^{(j)})/(\sum_{j=1}^{M} W_1^{(j)} \sqrt{u_1^{(j)}}). \tag{B.7}$$

Compute the effective sample size $ESS = 1/(\sum_{j=1}^{M} W_1^{(j)})^2$.

2. For $t = 2, ..., T$, propagate the particle system. If $ESS < 0.75M$, resample with replacement M particles $\{h_{t-1}^{(j)}, \beta_{0,t-1}^{(j)}, \beta_{1,t-1}^{(j)}, v_{t-1}^{(j)}, u_{t-1}^{(j)}\}_{j=1}^{M}$ with probability $\{W_{t-1}^{(j)}\}$ and set $W_{t-1}^{(j)} = 1/M$ for $j = 1, ..., M$.

   Sample $h_t^{(j)} \sim g(h_t|h_{t-1}^{(j)}, Y, X)$, $j = 1, ..., M$. Use Kalman recursion to compute $\beta_{i,t}^{(j)}$, $i = 0, 1$ with associated filtered variance $V_{i,p}^{(j)}$.

   Compute the prediction errors $v_t^{(j)}$ and their variances $u_t^{(j)}$.

   Compute the importance weight

$$\omega_t^{(j)} = W_{t-1}^{(j)} \times \frac{f_t^{(j)}(v_t^{(j)}) p(h_t^{(j)}|h_{t-1}^{(j)})}{g(h_t^{(j)}|h_{t-1}^{(j)}, Y, X)}.$$

   Record $\log \hat{l}_t = \log(\sum_{j=1}^{M} \omega_t^{(j)})$ and normalise the importance weight $W_t^{(j)}$.

   Record the filtered estimates in (B.5)-(B.7) and compute ESS.

3. After the recursion terminates, compute the log-likelihood $\hat{l} = \sum_{t=1}^{T} \hat{l}_t$ for TVPM-SV evaluated at the simulated ML estimate $\hat{\theta}$. This can be used to conduct likelihood-based tests and to calculate information criteria; and the standardised residuals can be used to test for model misspecification.

C Data and estimation results obtained from the time-varying parameter models without stochastic volatility

Table C1. Data and sources[a]

Country	Period	g_t[b]	Δu_t[c]	λ_t[d]	$\hat{h}_t$[e]
Canada	1961Q1-2016Q4	OECD	OECD	OECD	-
France[f]	1984Q1-2016Q4 and 1996Q1-2016Q4	OECD	OECD	FRED: 1996Q1-2011Q4 and OECD: 2012Q1-2016Q4	-
Germany	1963Q1-2016Q4	OECD	OECD	FRED: 1963Q1-2011Q4 and OECD: 2012Q1-2016Q4	-
Italy	1984Q1-2016Q4	OECD	OECD	FRED: 1984Q1-2011Q4 and OECD: 2012Q1-2016Q4	-
Japan	1961Q1-2016Q4	OECD	OECD	OECD	-
United Kingdom	1972Q1-2016Q4	OECD	FRED: 1972Q1-1983Q4 and OECD: 1984Q1-2016Q4	BE	-
USA	1951Q1-2016Q4	OECD	FRED	FRED	BLS

Notes: [a]OECD: Organization for Economic Cooperation and Development database. FRED: Federal Reserve Board of St. Louis database. BE: Bank of England's collection of historical macroeconomic and financial statistics, "A millennium of macroeconomic data for the UK", Version 3. BLS: Bureau of Labor Statistics; [b]Rate of growth of GDP (percent change from same quarter a year ago); [c]First differences of the unemployment rate (from same quarter a year ago). The unemployment rate in each country refers to the following indicators. Canada, France, Italy, Japan and the United Kingdom: Harmonised unemployment rate. Germany: Unemployment rate, aged 15 and over. USA: Civilian unemployment rate; [d]Rate of growth of the civilian labour force (percent change from same quarter a year ago); [e]Rate of growth of hours worked per worker (percent change from same quarter a year ago), which was only available for the US business sector; [f]1984Q1-2016Q4 refers to the estimation of the long-run output growth rate (model (15)) and 1996Q1-2016Q4 refers to the estimation of the long-run technical progress growth rate (model (16)) since the λ_t series is only available for the latter period.

Table C2. Long-run output growth rates (model (15)): estimation of the hyper-parameters for the time-varying parameter models without stochastic volatility

Hyper-parameters	Canada, 1961Q1-2016Q4	France, 1984Q1-2016Q4	Germany, 1963Q1-2016Q4	Italy, 1984Q1-2016Q4	Japan, 1961Q1-2016Q4	United Kingdom, 1972Q1-2016Q4	USA, 1951Q1-2016Q4
Models without bias correction terms[a]							
$\sigma_{\varepsilon,0}$	0.83**	0.53**	1.10**	0.94**	1.34**	0.84**	0.85**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.09)	(0.06)	(0.04)
$\sigma_{\varepsilon,1}$	0.00	0.08	0.23*	0.00	0.65*	0.41**	0.03
	(0.00)	(0.10)	(0.11)	(0.00)	(0.33)	(0.12)	(0.02)
ϕ	0.70**	0.88**	0.62**	0.88**	0.91**	0.45**	0.74**
	(0.12)	(0.10)	(0.06)	(0.10)	(0.07)	(0.12)	(0.06)
λ_0	0.09	0.07	0.09	0.04	0.06	0.12	0.03
L^{b}	-318.20	-109.69	-371.05	-197.21	-429.28	-270.95	-362.14
Models with bias correction terms[a,c]							
$\sigma_{\varepsilon,0}$	0.81**	0.51**	1.05**	0.92**	1.34**	0.85**	0.84**
	(0.04)	(0.05)	(0.05)	(0.06)	(0.07)	(0.06)	(0.04)
$\sigma_{\varepsilon,1}$	0.00	0.22	0.26**	0.11	0.59*	0.44**	0.03
	(0.03)	(0.13)	(0.10)	(0.08)	(0.26)	(0.12)	(0.02)
ϕ	0.68**	0.92**	0.57**	0.81**	0.91**	0.55**	0.87**
	(0.10)	(0.15)	(0.07)	(0.11)	(0.12)	(0.14)	(0.06)
ρ	0.02	-0.25**	-0.27**	-0.16	-0.08	-0.21*	-0.26**
	(0.07)	(0.09)	(0.07)	(0.11)	(0.06)	(0.08)	(0.06)
λ_0	0.08	0.08	0.08	0.07	0.07	0.04	0.04
L^{b}	-308.45	-104.04	-343.62	-187.25	-420.84	-246.42	-345.06

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood; [c]The combination of instruments employed for Δu_t in each country was the following:

1) Canada: $(g_{t-2}-\lambda_{t-2})$, $(g_{t-3}-\lambda_{t-3})$, and $(g_{t-4}-\lambda_{t-4})$.

2) France: Δu_{t-2}, λ_{t-3}, λ_{t-4}, λ_{t-5}, and λ_{t-6}.

3) Germany: Δu_{t-2}, λ_{t-4}, λ_{t-5} , λ_{t-6}, and λ_{t-7}.

4) Italy: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, $(g_{t-7}-\lambda_{t-7})$, and $(g_{t-8}-\lambda_{t-8})$.

5) Japan: Δu_{t-2}, Δu_{t-3}, and Δu_{t-4}.

6) United Kingdom: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, and $(g_{t-7}-\lambda_{t-7})$.

7) USA: $\hat{h}_{t-3}$, $\hat{h}_{t-4}$, $\hat{h}_{t-5}$, $\hat{h}_{t-6}$, and $\hat{h}_{t-7}$.

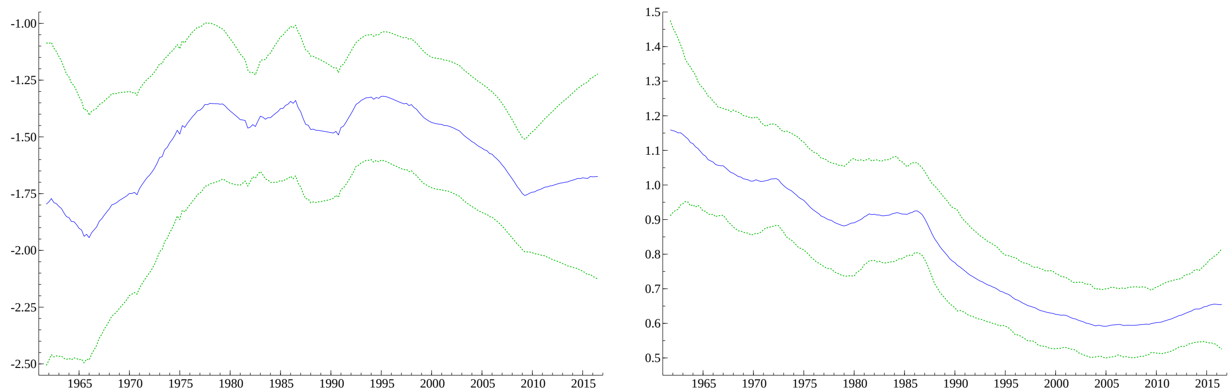
^,*, and ** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table C3. Long-run technical progress growth rates (model (16)): estimation of the hyper-parameters for the time-varying parameter models without stochastic volatility

Hyper-parameters	Canada, 1961Q1-2016Q4	France, 1996Q1-2016Q4	Germany, 1963Q1-2016Q4	Italy, 1984Q1-2016Q4	Japan, 1961Q1-2016Q4	United Kingdom, 1972Q1-2016Q4	USA 1951Q1-2016Q4
Models without bias correction terms[a]							
$\sigma_{\varepsilon,0}$	0.88**	0.72**	1.18**	1.00**	1.38**	0.91**	0.95**
	(0.04)	(0.06)	(0.06)	(0.06)	(0.09)	(0.07)	(0.04)
$\sigma_{\varepsilon,1}$	0.00	0.14	0.14*	0.00	0.53^	0.56**	0.05
	(0.00)	(0.18)	(0.07)	(0.00)	(0.32)	(0.13)	(0.03)
ϕ	0.52**	0.82**	0.62**	0.90**	0.95**	0.55**	0.81**
	(0.08)	(0.09)	(0.07)	(0.12)	(0.07)	(0.14)	(0.06)
λ_0	0.08	0.10	0.10	0.05	0.05	0.10	0.03
L^{b}	-330.32	-105.69	-388.39	-205.54	-428.15	-286.51	-390.11
Models with bias correction terms[a,c]							
$\sigma_{\varepsilon,0}$	0.94**	0.72**	1.13**	0.92**	1.35**	0.89**	0.94**
	(0.05)	(0.06)	(0.06)	(0.06)	(0.08)	(0.07)	(0.04)
$\sigma_{\varepsilon,1}$	0.05	0.11	0.22*	0.17*	0.54^	0.59**	0.04
	(0.04)	(0.10)	(0.11)	(0.08)	(0.28)	(0.13)	(0.02)
ϕ	0.61**	0.84**	0.59**	0.88**	0.93**	0.65**	0.88**
	(0.08)	(0.08)	(0.07)	(0.08)	(0.09)	(0.16)	(0.07)
ρ	-0.05	-0.35**	-0.31**	-0.15	-0.09	-0.20*	-0.32**
	(0.07)	(0.10)	(0.06)	(0.11)	(0.06)	(0.08)	(0.06)
λ_0	0.02	0.09	0.09	0.07	0.07	0.05	0.04
L^{b}	-319.66	-103.51	-357.37	-189.36	-420.59	-262.38	-366.76

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood; [c]The combination of instruments employed for Δu_t in each country was the following:
1) Canada: $(g_{t-2}-\lambda_{t-2})$, $(g_{t-3}-\lambda_{t-3})$, and $(g_{t-4}-\lambda_{t-4})$.
2) France: Δu_{t-2}, λ_{t-3}, λ_{t-4}, λ_{t-5}, and λ_{t-6}.
3) Germany: Δu_{t-2}, λ_{t-4}, λ_{t-5} , λ_{t-6}, and λ_{t-7}.
4) Italy: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, $(g_{t-7}-\lambda_{t-7})$, and $(g_{t-8}-\lambda_{t-8})$.
5) Japan: Δu_{t-2}, Δu_{t-3}, and Δu_{t-4}.
6) United Kingdom: Δu_{t-2}, $(g_{t-5}-\lambda_{t-5})$, $(g_{t-6}-\lambda_{t-6})$, and $(g_{t-7}-\lambda_{t-7})$.
7) USA: $\hat{h}_{t-3}$, $\hat{h}_{t-4}$, $\hat{h}_{t-5}$, $\hat{h}_{t-6}$, and $\hat{h}_{t-7}$.
^,*, and ** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

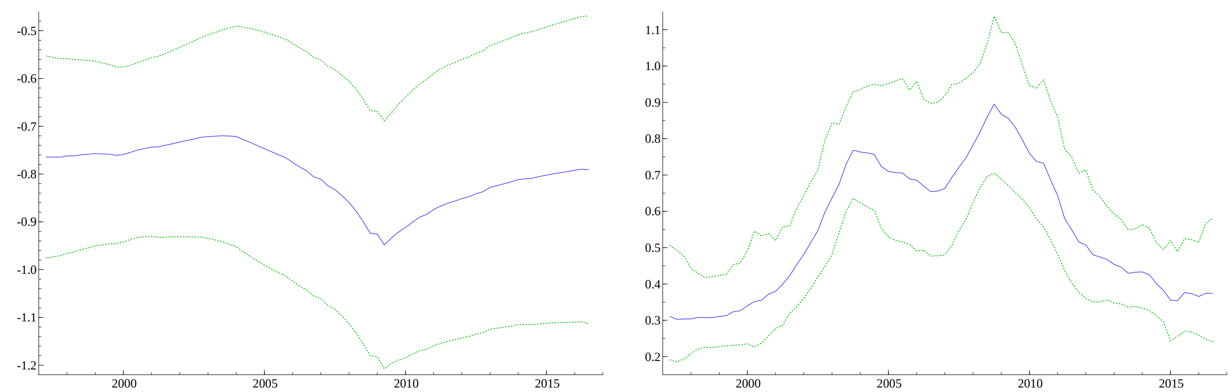
D Time-varying Okun coefficients on unemployment and stochastic volatility parameters obtained from model (15)



(a) Okun coefficient on unemployment

(b) Stochastic volatility parameter

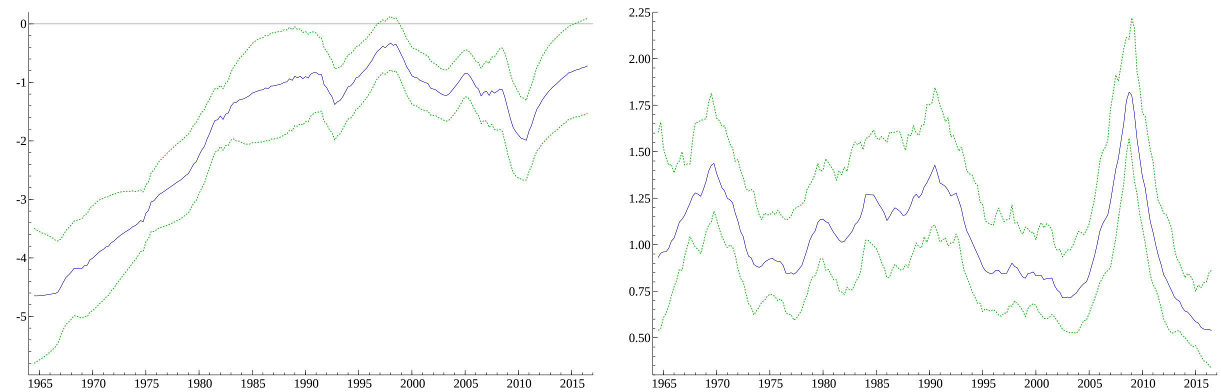
Figure D.1. Canada, 1961Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).



(a) Okun coefficient on unemployment

(b) Stochastic volatility parameter

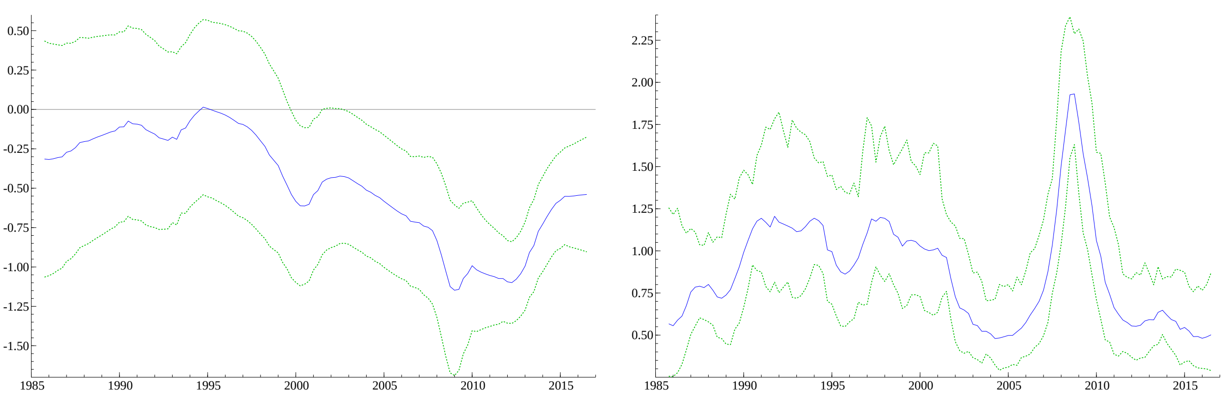
Figure D.2. France, 1984Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).



(a) Okun coefficient on unemployment

(b) Stochastic volatility parameter

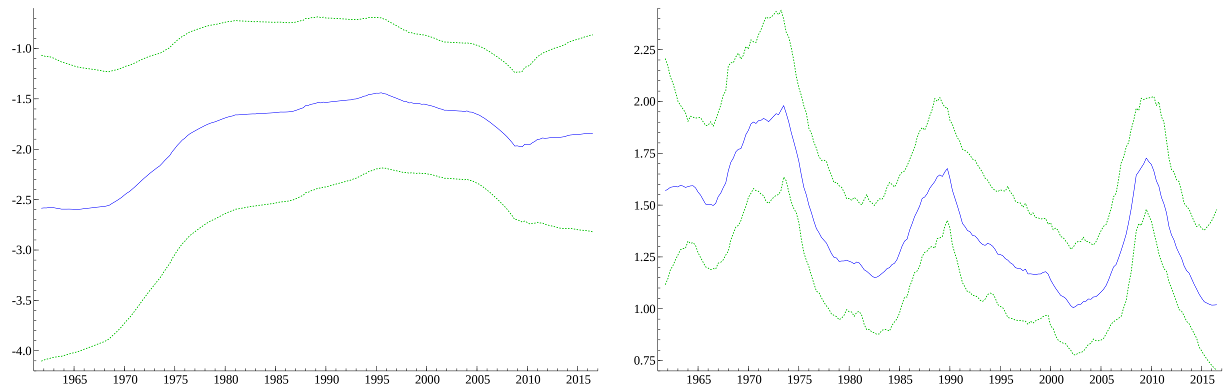
Figure D.3. Germany, 1963Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).



(a) Okun coefficient on unemployment

(b) Stochastic volatility parameter

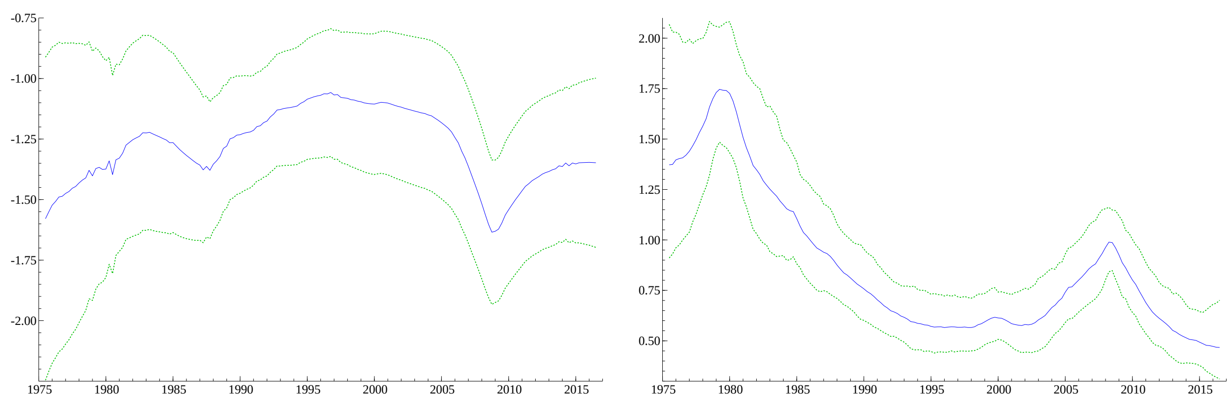
Figure D.4. Italy, 1984Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).



(a) Okun coefficient on unemployment

(b) Stochastic volatility parameter

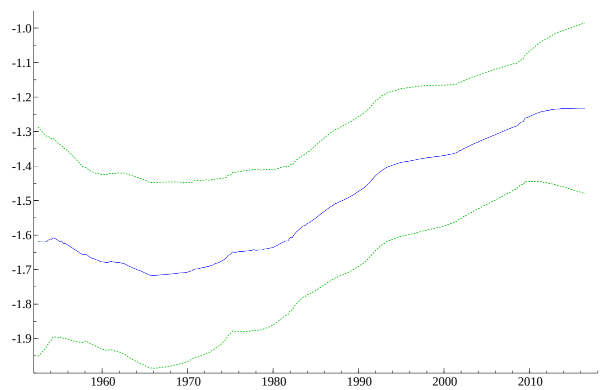
Figure D.5. Japan, 1961Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).



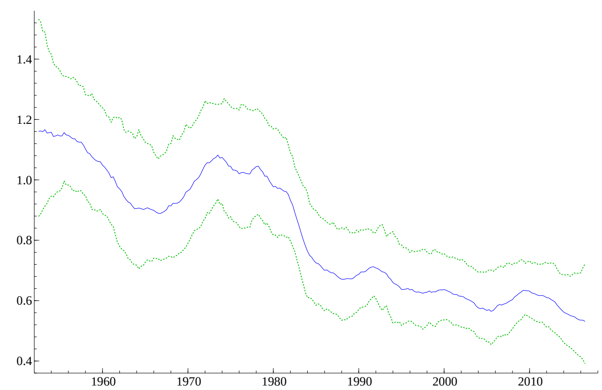
(a) Okun coefficient on unemployment

(b) Stochastic volatility parameter

Figure D.6. United Kingdom, 1972Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).



(a) Okun coefficient on unemployment



(b) Stochastic volatility parameter

Figure D.7. USA, 1951Q1-2016Q4. Smoothed estimates (blue straight lines) with 95% confidence intervals (green dotted lines).