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Optimal monetary policy in a currency union with labour market heterogeneity

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Abstract

I construct a New Keynesian, two-country model with labour market frictions in the search and matching process and real wage rigidity. Following a linear-quadratic approach, I analyse quantitatively the welfare-based optimal monetary policy in a currency union. I allow for labour market heterogeneity among the member states captured by an index based on the real wage rigidity differential. I show that when the optimal monetary policy is conducted, in the presence of productivity shocks, the welfare loss in the currency union increases monotonically with the value of the labour market heterogeneity index. That is based on the key role of the terms of trade which intensify the effects of the shocks. I also draw the implications of labour market heterogeneity for the optimal regime choice by the central bank.

JEL Classification: E24; E31; E52; F41

Keywords: Currency union; Optimal monetary policy; Labour market heterogeneity

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1 Introduction

Following the financial crisis of $2008-2009$, unemployment in many euro area countries has dramatically increased. According to the European Union (EU) Labour Force Survey (LFS) data, the average unemployment of the euro area at the beginning of 2009 was 7.6% while at the end of 2012 was 11.4%. Despite the social and political pressures caused by unemployment persistence, inflation stabilisation is the main policy objective of the euro area monetary policy-maker, the European Central Bank (ECB). The average CPI inflation in the euro area in 2009 declined from 3.3% to 0.3%, but in 2012 was 2.5%. It is worth pointing out that there is a homogenous adjustment of inflation among the member states, while the unemployment adjustment is heterogeneous, as there are substantial differences in the volatility and the persistence.[1]

The unemployment adjustment is shown in figure 1. After 2008, the unemployment in Ireland, Greece, Spain and Portugal has increased and exhibited some persistence, while in Germany, France, Austria and Belgium the adjustment was smoother.

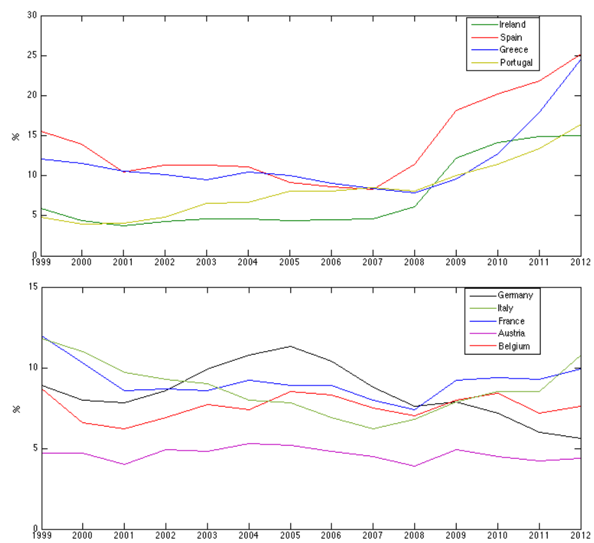

Figure 1: *Unemployment of euro area member states, % annually, Eurostat LFS.*

The issue of unemployment heterogeneity is highly concerned from the ECB. Considering the well-known theory by Mundell (1961) for an optimum currency union area, a heterogeneous unemployment adjustment raises questions regarding the flexibility of the labour markets which is considered as a necessary criterion for the desirability of a currency union.

In terms of a macroeconomic, theoretical framework there are questions not addressed yet. Indeed, these questions have become of high importance after the recent crisis. For example, in a currency union how could heterogeneous labour markets affect monetary policy? Can the observed unemployment heterogeneity be considered as an outcome of an optimal monetary policy?

In order to answer these questions, I provide a normative analysis of the monetary pol-

[1]See the EU LFS data for the HCPI inflation in the technical Appendix which is available on request from the author.

icy in a currency union framework. The labour markets of the member states are assumed to be heterogeneous beforehand. Particularly, I build on Benigno (2004) a New Keynesian (NK), two-country model where the two countries form a currency union. In each member state, I introduce Diamond-Mortensen-Pissarides (DMP) labour market frictions in the search and matching process in order to invite involuntary unemployment.[2] I also allow for a form of real wage rigidity (RWR) close to the wage norm introduced by Hall (2005). The assumption of RWR in the euro area is supported by empirical evidence. Examples include a firm-level survey by Babecky (2009) and a micro-study by Messina (2010).[3] Then, I use the RWR assumption to construct a labour market heterogeneity index based on the degree of RWR differential between the two countries.

The novelty of this paper is that it provides a theoretical framework in which the implications of labour market heterogeneity for monetary policy are drawn. Particularly, in the presence of a country-specific (domestic) or an aggregate (union-wide) productivity shock, the welfare consequences of labour market heterogeneity are explored and assessed quantitatively. As a consequence and not surprisingly, it comes out that labour market heterogeneity can explain the heterogeneous unemployment adjustment. That is consistent with Estrada, Gali, and Lopez-Salido (2013) who examine empirically the factors of divergence of unemployment in the euro area. They highlight the presence of asymmetric shocks and transmission mechanisms as potential reasons of the unemployment heterogeneity among the member states.

The primary objective of the paper is to analyse the welfare-based optimal monetary policy in a currency union area with heterogeneous labour markets. For this reason, I follow a Linear-Quadratic (L-Q) approach pioneered by Rotemberg and Woodford (1997) and Woodford (2003) to derive a welfare criterion for the central bank which clarifies the policy objectives. Deriving the policy objectives of the central bank from the micro-foundations of the model, contributes to the New Keynesian (NK) literature of the currency union models. Particularly, I show that in the current framework the policy objectives are: the domestic inflation and unemployment of each member state, and their terms of trade. The fact that the two member states are linked through the terms of trade is a main implication of a two-country model with small open economies. In this case, the terms of trade act as a transmission mechanism of the domestic productivity shock from one member state to the other and also intensify the asymmetric effects of an aggregate productivity shock. The results of the paper are derived by quantitative comparisons based on adjustments of the value of the labour market heterogeneity index. In a currency union with sticky prices and real wages and labour market frictions it is expected that the monetary policymaker is not able to eliminate all distortions with one instrument and trade-offs between stabilising unemployment and stabilising inflation arise. Therefore, it is expected that there will be welfare losses even if the central bank implements an optimal regime.

The main result of the paper is that, in the presence of a domestic or an aggregate productivity shock, when the central bank conducts an optimal monetary policy regime; the timeless perspective optimal commitment or the optimal discretion, the welfare loss in the currency union increases monotonically with the value of the labour market heterogeneity index. Therefore

[2]For an early contribution of a Real Business Cycle (RBC) model with search and matching frictions, see Merz (1995) and Andolfatto (1996). For an early contribution of NK models with unemployment see Cheron and Langot (2000), Walsh (2005), Trigari (2006).

[3]For a discussion about some controversial empirical evidence for RWR, see the section 2.4.2.

labour market heterogeneity has a distortionary effect in the currency union and it is inefficient.

This result is based on the key implication of the model that the volatility of the terms of trade increases monotonically with the value of the labour market heterogeneity index. In the domestic productivity shock scenario, the terms of trade act as a transmission mechanism of the shock and add a ”cost-push” component on inflation of the member state which is not hit by the shock. Therefore, higher labour market heterogeneity increases the volatility of the domestic inflation and the domestic unemployment of that member state. In the aggregate productivity shock scenario, it is expected that the asymmetric effects arise when the labour markets are heterogeneous. However, the more heterogeneous is the inflation adjustment between the two countries the more volatile the terms of trade become. Consequently, the volatility of inflation and unemployment of the more rigid country increases more and the welfare loss increases as well. I also compare the timeless perspective optimal commitment with the optimal discretion. I show that when the value of the labour market heterogeneity index increases optimal discretion becomes a more desirable regime for the country hit by a domestic shock. That is as the index increases the inefficient unemployment fluctuation decreases more under discretion. Nevertheless, the opposite is true for the country which is not hit by the shock. In this case the optimal commitment becomes more desirable. This outcome is based on the sensitivity of inflation with respect to unemployment changes and on the role of the public's expectations.

Existing work in currency union models which follows the L-Q approach includes Benigno (2004) and Gali and Monacelli (2008). Particularly, Benigno (2004) focuses on the implications of the nominal price rigidity for monetary policy, while Gali and Monacelli (2008) examine the implications of the coordination between the single monetary authority with the fiscal policymakers. This paper adds to this literature by providing the domestic unemployment rates as policy objectives of the central bank and by exploring the labour markets of the member states.

This paper also contributes to the rest of the literature of the open economy NK models. There are a few papers which incorporate wage rigidity. The closest are by Andersen and Seneca (2010) and Fahr and Smets (2010). Andersen and Seneca (2010) study the implications of country size and nominal wage rigidity heterogeneity on the inflation and output adjustments to country-specific and aggregate productivity shocks. Fahr and Smets (2010) incorporate nominal and real wage rigidity to study the effects of asymmetric productivity shocks on inflation. Both papers find that there is an effect of the terms of trade on the transmission of the shock which strengthens with the asymmetry of the degree of wage rigidity. This paper adds to this literature by providing a normative analysis of monetary policy and by deriving unemployment as a policy objective. Some other papers incorporate labour market frictions and RWR in the NK currency union model but they do not focus on the optimal monetary policy analysis. Examples include Campolmi and Faia (2011) who study the changes of inflation volatility of euro area countries when there is heterogeneous unemployment insurance and Abbritti and Mueller (2013) who focus on the implications of heterogeneous degree of real wage rigidity on the inflation and unemployment differentials of the member states.

Comparing the annual European Union (EU) Labour Force Survey (LFS) data for the period $1997-2014$, the model can replicate the fast and homogeneous inflation adjustment across member states and the slow, heterogeneous unemployment adjustment. Despite the fact that in the model only a simple labour market heterogeneity index is used, this paper is useful to provide some suggestive ideas and some guidance regarding the design of the optimal monetary policy in a currency union.

The rest of the paper is organised as follows. The currency union framework is described in Section 2. The reduced form log-linear equilibrium conditions of the model are presented in Section 3. Section 4 describes the L-Q approach and the welfare based optimal monetary policy for the currency union. Section 5 describes the numerical solution of the model and presents the main results and implications. Section 6 concludes with a further discussion.

2 The model

The simplest form of a currency union model is presented with two countries or member states, A and B. There is a single monetary policy-maker and for simplicity fiscal authorities are absent. The currency union is populated by a continuum of households and firms on the interval [0,1]. I assume for tractability that in each country the number of households is equal to the number of firms. The population of households and firms on $[0, \zeta)$ live in country A and the ones on $[\zeta, 1]$ live in country B. I assume that there is not on-the-job search, so labour is immobile within and across countries. Finally, I assume for simplicity that unemployed workers cannot cross borders. Hence, no migration can take place across countries at any point of time.

2.1 Preferences

Households have homogeneous preferences within country. Therefore, it can be assumed the existence of one representative infinitely-lived household of country $j \in [A, B]$. At any period of time, a fraction $u_t^j \in [0, 1]$ of a household's members are unemployed. Being unemployed entails the loss of labour income. However, assuming that employed household members pool their income and distribute it equally across all members before making the optimal consumption decision, guarantees perfect consumption insurance across all of the household's members.[4] Assuming identical households implies that u_t^j is considered as the unemployment rate of country j.

I follow a similar framework with Benigno (2004). A representative household living in country j $\in [A, B]$, has preferences described by the additively separable, intertemporal utility function

$$U^j\left(C_t^j, N_t^j\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(C_t^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d\frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

At any discrete time interval the household members obtain utility from the consumption of C_t^j units of final private goods and disutility from labour, $N_t^j \in [0, 1]$. The constant component, $d > 0$, accounts for the disutility associated with labour and $\beta \in (0, 1)$ is the intertemporal discount factor. Households form rational expectations. The utility function is strictly increasing and strictly concave on C_t^j and strictly increasing and strictly convex on N_t^j. The inverse of the Frisch elasticity of labor supply is $\varphi > 0$ and $\sigma > 0$ is a measure of a household's risk aversion. C_t^j is the index of composite consumption of final private goods consumed in country j $\in [A, B]$

[4]Perfect consumption insurance was initially presented by Merz (1995) and adopted by Thomas (2008) and Gertler

which is defined as:

$$C_t^A \equiv \frac{(C_{At}^A)^\zeta (C_{Bt}^A)^{1-\zeta}}{\zeta^\zeta (1-\zeta)^{1-\zeta}} \tag{2}$$

and

$$C_t^B \equiv \frac{(C_{Bt}^B)^\zeta (C_{At}^B)^{1-\zeta}}{\zeta^\zeta (1-\zeta)^{1-\zeta}} \tag{3}$$

for countries A, B respectively. For country A, ζ is the weight that households put on the domestic goods (home bias), thus, following Gali and Monacelli (2008) the term $(1-\zeta)$ reflects a natural index of openness.[5]

C_{At}^j and C_{Bt}^j are the Dixit-Stiglitz indexes (baskets) of consumption across the continuum of differentiated final goods, consumed in country j and produced in countries A and B respectively. They are given by the CES functions:

$$C_{At}^j \equiv \left[\left(\frac{1}{\zeta}\right) \int_0^\zeta c_t^j(a)^{\frac{\gamma-1}{\gamma}} da \right]^{\frac{\gamma}{\gamma-1}} \tag{4}$$

and

$$C_{Bt}^j \equiv \left[\left(\frac{1}{1-\zeta}\right) \int_\zeta^1 \left(c_t^j(b)^{\frac{\gamma-1}{\gamma}} db \right]^{\frac{\gamma}{\gamma-1}} \tag{5}$$

for $a \in [0, \zeta)$, $b \in [\zeta, 1]$ and $c_t(a)$, $c_t(b)$ denoting the variety of final goods produced in countries A and B respectively and $\gamma > 1$ is the within country elasticity of substitution of final goods assumed to be the same for both countries.

Given this framework, the representative household takes an intertemporal consumption/savings decision and two intratemporal decisions: The optimal allocation of nominal spending between domestic and imported final goods, and the optimal allocation of the shares of nominal spending among the differentiated final goods produced in each country.

2.1.1 Households' decision

The optimal allocation of nominal spending between domestic and imported final goods requires the minimisation of total nominal spending $P_{ct}^j C_t^j$ given equations (2) and (3), where

$$P_{ct}^A \equiv P_{At}^\zeta P_{Bt}^{(1-\zeta)} \tag{6}$$

and

$$P_{ct}^B \equiv P_{Bt}^\zeta P_{At}^{(1-\zeta)} \tag{7}$$

is the Consumer Price Index (CPI) for countries A and B respectively, and P_{jt} is the Dixit-Stiglitz domestic price index for country j given by:

$$P_{At} \equiv \left[\frac{1}{\zeta} \int_0^\zeta \left(p_t(a)^{1-\gamma} da \right) \right]^{\frac{1}{1-\gamma}} \tag{8}$$

and Trigari (2009), amongst many others.

[5]Following Benigno (2004), I have assumed the across countries elasticity of substitution of final goods to be equal to one. See Benigno and Benigno (2003) for the case which the across country elasticity is higher than 1, and the implications for the monetary policy.

$$P_{Bt} \equiv \left[\frac{1}{1-\zeta} \int_{\zeta}^{1} \left(p_t(b)^{1-\gamma} db \right) \right]^{\frac{1}{1-\gamma}} \tag{9}$$

The solution of the problem yields the optimal shares for country A:

$$P_{At} C_{At}^{A} = \zeta P_{ct}^{A} C_t^{A}$$

and

$$P_{Bt} C_{Bt}^{A} = (1-\zeta) P_{ct}^{A} C_t^{A}$$

while similar conditions hold for country B.[6]

The optimal allocation of shares of nominal spending among the differentiated final goods requires household to maximize the Dixit- Stiglitz indices given by eq. (4) and (5), for any given level of nominal spending. This yields the system of demand equations:

$$c_t^j(a) = \left(\frac{p_t(a)}{P_{At}} \right)^{-\gamma} S_t^{(1-\zeta)} C_t^j \tag{10}$$

for final goods $a \in [0, \zeta)$ produced in country A and

$$c_t^j(b) = \left(\frac{p_t(b)}{P_{Bt}} \right)^{-\gamma} S_t^{(-\zeta)} C_t^j \tag{11}$$

for final goods $b \in [\zeta, 1]$ produced in country B where I define,

$$S_t \equiv \frac{P_{Bt}}{P_{At}} \tag{12}$$

as the terms of trade of country B.[7]

The intertemporal consumption/savings decision requires the household to chooses the set of processes $\{C_t^j, B_t^j\}$ in order to maximize eq. (1), subject to a sequence of budget constraints. The budget constraint can take the form:

$$P_{ct}^j C_t^j + (1+q_t) B_t^j \leq B_{t-1}^j + N_t^j W_t^j \tag{13}$$

where the set of processes of $\{q_t\}$, $\{P_{ct}^j\}$, $\{W_t^j\}$ are given. B_t^j is the one-period riskless bond and W_t^j is the nominal wage in country j, while q_t is the gross nominal interest rate which is common in the currency union. Given the solvency condition, $\lim_{T\to\infty} E_t B_t \geq 0$, the solution to the above problem yields:

$$\beta E_t \left\{ \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\frac{1}{\sigma}} \frac{P_{ct}^j}{P_{ct+1}^j} \right\} = \frac{1}{(1+q_t)} \tag{14}$$

which is the standard consumption Euler equation.[8]

[6]See technical Appendix 1.1.
[7]See technical Appendix 1.2
[8]See technical Appendix 1.3.

2.2 International trade and risk sharing

Given that the two economies are open, following Gali and Monacelli (2008), I provide a definition of CPI of each country in terms of their terms of trade. In the absence of transaction costs, the price of a good produced in a country is the same for both countries, i.e. $P_{At} = P_{Bt}$. Therefore the law of one price holds across the currency union. However CPI may be different due to different home biased preferences. Combining eq.(12) with eq.(6) and (7) the CPI of each country is given by:

$$P^A_{ct} = P_{At} S_t^{(1-\zeta)} \tag{15}$$

and

$$P^B_{ct} = P_{Bt} S_t^{(\zeta-1)} \tag{16}$$

In addition, in a currency union area there is a single gross nominal interest rate, q_t. Assuming symmetric households' preferences and initial conditions across countries implies that the Euler condition, eq.(14), is symmetric.[9] In technical Appendix 1.4, I show that using the definitions above and the definition of the terms of trade, the consumption indices of both countries are linked:

$$E_t\left(\frac{C^A_t}{C^A_{t+1}}\right) = E_t\left\{\left(\frac{S_t}{S_{t+1}}\right)^{\sigma(2\zeta-1)}\left(\frac{C^B_t}{C^B_{t+1}}\right)\right\} \tag{17}$$

2.3 Technology

In each country there are two types of production. Production for intermediate and for final goods. I distinguish technology between intermediate and final goods production for tractability as in Blanchard and Gali (2010). The representative intermediate good firm has a simple technology constituted by the input of labour N^j_t. In any country j $\in [A, B]$, the labour market is subject to frictions in the search and matching process, as for any firm posting a vacancy v^j_t is costly. In each country there are also final good firms which buy the homogeneous intermediate good in a perfectly competitive market and use it as the only input to produce a differentiated final good. The final good is sold in a monopolistically competitive market. Final good producers are subject to nominal price rigidity a la Calvo (1983).

2.3.1 The labour market

A job match is formed when a posted vacancy is filled by an unemployed worker. The total number of new job matches in country j at any period is given by the matching function:

$$m^j_t = m^j(u^j_t, v^j_t) = {v^j_t}^{\kappa} {u^j_t}^{(1-\kappa)} \tag{18}$$

where u^j_t is the total number of unemployed workers and v^j_t is the total number of posted vacancies in country j. The specification of the matching function satisfies some standard properties: It is strictly increasing and strictly concave in both arguments, it exhibits constant returns

[9] This is a remark from the standard assumption of complete securities markets. Assuming a constant of proportionality, i.e. country level initial conditions, equal to 1 leads to link countries' consumption. For this, I follow Gali and Monacelli (2008) and Abbritti and Mueller (2013).

to scale and it is homogeneous of degree 1.[10] In this case, $\kappa \in (0,1)$ is considered as the elasticity of matching function with respect to the number of vacancies. Given the properties of the matching function, I define labour market tightness as

$$\theta_t^j \equiv \frac{v_t^j}{u_t^j} \tag{19}$$

where the rate that job seekers find a job is given by the job-finding rate $p(\theta_t^j) \equiv \frac{m(u_t^j, v_t^j)}{u_t^j}$, while the rate that firms fill a vacancy is given by $q(\theta_t^j) \equiv \frac{m(u_t^j, v_t^j)}{v_t^j}$. Combining with eq.(18), I can write the job-finding rate as:

$$p(\theta_t^j) = (\theta_t^j)^\kappa \tag{20}$$

and I can write the vacancy-matching rate as

$$q(\theta_t^j) = (\theta_t^j)^{\kappa-1} \tag{21}$$

From (20), $p(\theta_t^j)$ is increasing in θ_t^j, as from a job seeker's point of view there is higher probability to find a job as θ_t^j increases. Similarly, by (21), $q(\theta_t^j)$ is decreasing in θ_t^j.

A job match ends for exogenous reasons, at a constant separation rate δ. Given this and the definition of the matching function, I can define the evolution of employment over time in each country as:

$$N_t^j = (1-\delta)N_{t-1}^j + m(u_t^j, v_t^j) \tag{22}$$

At every period the employment in country j is given by the total number of those who continue to work plus the total number of new job matches. Unemployment evolves over time according to:

$$u_t^j = 1 - N_{t-1}^j + \delta N_{t-1}^j = 1 - (1-\delta)N_{t-1}^j \tag{23}$$

which implies that unemployment is a predetermined variable at time t. (22)and (23) with (19) determine the Beveridge curve of the DMP model, i.e. the negative relationship between the number of vacancies and unemployment.

2.3.2 The intermediate good producers

The intermediate good, X_t^j, is sold in a perfectly competitive market in a real price $\phi_t^j \equiv \frac{(P_t^j)^I}{P_{ct}^j}$. That is intermediate good producers deflate their income with CPI and not with the domestic price index, as in order to post vacancies they buy units of final good.[11] The technology is described by the production function:

$$X_t^j = Z_t^j N_t^j$$

where Z_t^j is the technology and it can be different between countries. Letting $\log Z_t^j \equiv z_t^j$, I write technology directly as a log deviation from the steady state, the technology follows an

[10] See empirical evidence in Pissarides and Petrongolo (2001) that the matching function exhibits constant returns to scales. See in Pissarides (2000) ch. 1, that it can be approximated by a log-linear Cobb-Douglas function.

[11] This is different with Abbritti and Mueller (2013) and Campolmi and Faia (2011), who suggest that intermediate good firms deflate their nominal income with the domestic price index. Here this is not the case as, following Ravenna and Walsh (2011), I assume that the intermediate good producers buy the final good which is exported.

AR (1) process:

$$\hat{z}_t^j = \rho \hat{z}_{t-1}^j + \epsilon_t^j \tag{24}$$

where $\rho \in (0,1)$ and $\epsilon_t \sim NID(0, \sigma^2)$.[12]

2.3.3 Intermediate good producers' decision

Following Ravenna and Walsh (2011), I assume that each intermediate good producer buys $v_t^j(\iota)$ units of the final good in order to post vacancies, facing a management cost per vacancy, ψ. Hence, the total nominal spending on posting vacancies for each firm is given by:

$$\psi \left(\int_0^\zeta p_t(a) v_t^j(a) da + \int_\zeta^1 p_t(b) v_t^j(b) db \right)$$

In that sense, the total number of vacancies v_t^j demanded by firms in terms of final goods is given by the Dixit-Stiglitz indices, which are equivalent with those given from (4) and (5). That means:

$$v_{At}^j \equiv \left[\left(\frac{1}{\zeta} \right) \int_0^\zeta v_t^j(a)^{\frac{\gamma-1}{\gamma}} da \right]^{\frac{\gamma}{\gamma-1}} \tag{25}$$

and

$$v_{Bt}^j \equiv \left[\left(\frac{1}{1-\zeta} \right) \int_\zeta^1 \left(v_t^j(b)^{\frac{\gamma-1}{\gamma}} db \right]^{\frac{\gamma}{\gamma-1}} \tag{26}$$

Hence, the intermediate good producer faces similar intratemporal problems with consumers. That is the optimal choice of nominal spending and the optimal allocation of the shares. Hence, this yields a system of demand equations for final goods used for posting vacancies, which is equivalent with (10) and (11):

$$v_t^j(a) = \left(\frac{p_t(a)}{P_{At}} \right)^{-\gamma} S_t^{(1-\zeta)} v_t^j \tag{27}$$

$$v_t^j(b) = \left(\frac{p_t(b)}{P_{Bt}} \right)^{-\gamma} S_t^{(-\zeta)} v_t^j \tag{28}$$

Combining the demand equations, i.e (10) with (27) and (11) with(28), I can write the total demand for the final good of country A and B respectively:

$$\left(y_t^j\right)^d(a) = \left(\frac{p_t(a)}{P_{At}} \right)^{-\gamma} S_t^{(1-\zeta)} \left\{ C_t^j + \psi v_t^j \right\} \tag{29}$$

$$\left(y_t^j\right)^d(b) = \left(\frac{p_t(b)}{P_{Bt}} \right)^{-\gamma} S_t^{(-\zeta)} \left\{ C_t^j + \psi v_t^j \right\} \tag{30}$$

The intertemporal problem of the representative intermediate good producer is, given the law of motion of employment, equation (22), to choose the number of vacancies, v_t^j which maximises the expected present discounted sum of real profits. In Appendix 1.5 I show that

[12] Technology is defined as $Z_t^j \equiv (Z^j)_{t-1}^{\rho} (Z^j)^{(1-\rho)} e^{\epsilon_t^j}$.

for country A the solution of the problem yields:

$$\frac{\psi}{q(\theta_t^A)} = \left[\frac{(P_t^A)^I}{P_{ct}^A} Z_t^A - \frac{W_t^A}{P_{ct}^A} + (1-\delta)E_t\beta_{t,t+1}\left\{\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \frac{\psi}{q(\theta_{t+1}^A)}\right]\right\} \tag{31}$$

where I have used a definition for the stochastic discount factor $\beta_{t,t+1} \equiv \beta\left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\sigma^{-1}}\left(\frac{S_t}{S_{t+1}}\right)^{(1-\zeta)}$ for an open economy. A similar condition holds for country B for which the relative terms of trade between two periods is given by $\left(\frac{S_{t+1}}{S_t}\right)^{-\zeta}$.

Equation (31) implies that the optimal hiring decision requires the average cost per vacancy $\frac{\psi}{q(\theta_t^j)}$ to depend on the value of the current filled job. That is the difference between the real marginal product and real marginal cost of labour, $\frac{(P_t^A)^I}{P_{ct}^A} Z_t^A - \frac{W_t^A}{P_{ct}^A}$, plus the expected continuation value of the job, $(1-\delta)\frac{\psi}{q(\theta_{t+1}^A)}$. As intermediate good producers buy units of the final good, the optimal hiring decision requires firm to consider the changes of the relative terms of trade between two periods, $\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}$.

2.4 Wage determination

2.4.1 Flexible real wages

The equilibrium in the labour market model is concluded with the wage determination. As it is discussed in Pissarides (2000), the economic rent created from the search cost is typically shared through a Nash bargaining of the real wage. Bargaining parties renegotiate at any time period taking into account the economic conditions, like the productivity changes. This implies a real wage flexibility.

The problem is formalised in terms of country A. I follow a procedure similar to Thomas (2008). Let $\xi \in (0,1)$ be the bargaining power of firms.[13] Firm's surplus, Λ_{at}^f, is given by the marginal value of an additional employment relationship. That is:

$$\Lambda_{at}^f = \phi_t^A Z_t^A - \frac{W_{at}^A}{P_{ct}^A} + (1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \Lambda_{at+1}^f \tag{32}$$

where recall that $\phi_t^A \equiv \frac{(P_t^A)^I}{P_{ct}^A}$ is the real price for intermediate good firms. The worker's surplus, Λ_{at}^w, is given by the marginal value of the household's welfare criterion from an additional employment relationship. In order to express this in utility terms, I divide by the marginal utility with respect to consumption. That is:

$$\begin{aligned}\Lambda_{at}^w = \frac{W_{at}^A}{P_{ct}^A} - d\left(N_t^A\right)^{\varphi}\left(C_t^A\right)^{\sigma^{-1}} - (1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} p(\theta_{t+1})\Lambda_{at+1}^w \\ +(1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \Lambda_{at+1}^w\end{aligned} \tag{33}$$

[13] Symmetry implies that in equilibrium all firms behave in the same way hence I have dropped the subscript i.

In Appendix (1.6), I show that the solution of the Nash bargaining problem determines the real wage in country A:

$$\left(\frac{W_{at}^{A}}{P_{ct}^{A}}\right)^{Nash} = (1-\xi)\Big(\phi_t^A Z_t^A + (1-\delta)E_t\beta_{t,t+1}\Big(\frac{S_{t+1}}{S_t}\Big)^{(1-\zeta)}\psi\theta_{t+1}\Big) + \xi d\Big(N_t^A\Big)^{\varphi}\Big(C_t^A\Big)^{\sigma^{-1}} \quad (34)$$

A similar condition holds for country B. Equation (34) implies that the Nash bargaining real wage is determined by the weighted average of the higher wage that firms are willing to offer, and the lower wage that workers are willing to accept (reservation wage). The weights are given by the bargaining power of both the workers and firms. Firms use their bargaining power, ξ, to push the real wage down to the worker's reservation wage, $d\Big(N_t^A\Big)^{\varphi}\Big(C_t^A\Big)^{\sigma^{-1}}$, which is the workers' marginal rate of substitution between labour and consumption. Workers use their bargaining power, $(1-\xi)$, to push the real wage up to the higher level of the wage that firms are keen to offer. That is the sum of firm's real marginal product of labour, $\phi_t^A Z_t^A$, and the savings from not posting a vacancy the next period, i.e. the continuation value of the job, $(1-\delta)E_t\beta_{t,t+1}\Big(\frac{S_{t+1}}{S_t}\Big)^{(1-\zeta)}\psi\theta_{t+1}$.

2.4.2 Real wage rigidity

I build on the Nash bargaining real wage specification, to create a form of RWR. The RWR assumption is based on the critique by Shimer (2004, 2005) and Hall (2005) on the DMP model. Shimer (2004) suggests that the Nash bargaining wage determination in the DMP model implies a real wage flexibility. Firms and workers renegotiate the wage every period, therefore the real wage adjusts to economic changes. Thus, the number of posting vacancies by firms do not vary substantially and unemployment remains almost constant.

Assuming RWR serves two purposes. Besides the convenience of a construction of a labour market heterogeneity index, it allows to observe stabilisation trade-offs between the policy objectives. The NK literature which combines wage rigidity is exhaustive. Examples include standard NK models extended with wage rigidity like Erceg, Henderson, and Levin (2000) and Blanchard and Gali (2007) and NK models extended with involuntary unemployment and wage rigidity like Krause and Lubik (2007), Thomas (2008), Christoffel, Kuester, and Linzert (2009) and Blanchard and Gali (2010). [14]

I follow a similar notion of the wage norm suggested by Hall (2005) and I define the rigid real wage as a weighted average between the Nash and the last period real wage. For the later, I assume for simplicity that it is the steady-state value. The RWR scheme is:

$$\left(\frac{W_t^j}{P_{ct}^j}\right)^{index} = (1-\mu^j)\left(\frac{W_t^j}{P_{ct}^j}\right)^{Nash} + \mu^j W^j \quad (35)$$

where $W^j = \left(\frac{W^j}{P_c^j}\right)^{index}$. Higher weight to the last period (steady-state value) real wage implies more rigid wage as this makes the real wage inelastic to current productivity changes. When $\mu^j = 1$ the real wage is determined by its steady-state, while when $\mu^j = 0$, the real wage

[14]Some empirical evidence challenge the assumption of real wage rigidity. While the aggregate data are supportive, there is some ambiguity which lies on the composition bias of real wages which time-series data cannot capture. This highlighted first by Solon, Barsky, and Parker (1994). In addition, Pissarides (2009) provides a survey of mixed evidence from panel data which do not support the real wage rigidity assumption for some euro area countries, the US and the

is reduced to the Nash equilibrium outcome.

2.4.3 Equilibrium under RWR

Under RWR, the equilibrium in labour market is given by the Beveridge curve and (31), (35). The last two, if combined with (34), give the optimal hiring decision under RWR, which is a key equation for the model. For country A, I obtain:

$$\frac{\psi}{q(\theta_t^A)} = \phi_t^A Z_t^A - \Bigg\{(1-\mu^A)\Big[(1-\xi^A)(\phi_t^A Z_t^A + (1-\delta)E_t\beta_{t,t+1}\Big(\frac{S_{t+1}}{S_t}\Big)^{(1-\zeta)}\psi\theta_{t+1}) + \xi^A d\Big(N_t^A\Big)^{\varphi}\Big(C_t^A\Big)^{\sigma^{-1}}\Big] + \mu^A W^A\Bigg\} + (1-\delta)E_t\beta_{t,t+1}\Big(\frac{S_{t+1}}{S_t}\Big)^{(1-\zeta)}\frac{\psi}{q(\theta_{t+1}^A)} \tag{36}$$

while a similar condition holds for country B. In the presence of real wage rigidity firms' optimal number of posted vacancies requires a look backwards to the agreed real wage of the previous period.

2.5 Final good producers

In each country, the final good producers $a \in [0,\zeta)$, for $j = A$ and $b \in [\zeta, 1]$ for $j = B$, produce a differentiated final good which sell it under monopolistic competition. In order to produce, they use the intermediate good which is bought from the domestic intermediate good producers at a price $\phi_{t+s}S_{t+s}^{(1-\zeta)}$ in country A and $\phi_{t+s}S_{t+s}^{-\zeta}$ in country B. This price is the real marginal cost.[15] The production technology is given by:

$$y_t^j = X_t^j$$

The NK element of monopolistic competition creates a proper environment to invite the other NK element of nominal price rigidity which is crucial for monetary policy non-neutrality. For this reason, I assume that the price setting decision is subject to nominal price rigidity a la Calvo (1983). Every time period each producer faces a probability $(1-\omega)$ of resetting her own price, which is independent of the time since the last reset. Each producer chooses a price $p_t(j)$ for $j = a, b$ to maximise her expected discounted profits considering that her choice will be optimal at time $t+s$ with probability ω^s, subject to the demand equations (29) and (30) given by the consumers' problem. The optimal price setting for a final good firm in country A solves the problem:

$$\max_{p_t^*(\iota)} E_t \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s}\Big\{(1+\tau)\frac{p_t(a)}{P_{At+s}}y_t^A(a) - \phi_{t+s}S_{t+s}^{(1-\zeta)}y_t^A(a)\Big\}$$

for $s = 0, 1, 2, 3...$, subject to

$$(y_t^A)^d(a) = \Big(\frac{p_t(a)}{P_{At}}\Big)^{-\gamma} S_t^{(1-\zeta)}\Big\{C_t^j + \psi v_t^j\Big\}$$

UK. Also, Haefke, Sonntag, and van Rens (2013) recently provide evidence of real wage flexibility for the US

[15] The reason that the final good producers deflate their nominal cost with the domestic price index is because in the setup of the model they face a real marginal cost expressed in domestic prices. The terms of trade in the real marginal cost links the CPI with the domestic price index.

Following the standard NK literature, I assume that firm's final output is subsidised by a constant rate τ to guarantee that the output inefficiency implied by the monopolistic competition will be eliminated.

The first-order condition associated with the problem above, using the definition of the stochastic discount factor yields:

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left\{ \left(\frac{C^j_{t+s}}{C^j_t} \right)^{-\sigma^{-1}} \left(\frac{S_t}{S_{t+s}} \right)^{(1-\zeta)} \left[(1+\tau)(1-\gamma) \frac{p_t(a)^*}{P_{At+s}} + \gamma \phi_{t+s} S_{t+s}^{(1-\zeta)} \right] \left(\frac{p^*_t(a)}{P_{At+s}} \right)^{-\gamma} \frac{1}{p^*_t(a)} y^A_{t+s} \right\} = 0$$

Rearranging and dividing by the domestic price index P_{At}, i can write the above equation as:

$$\frac{p^*_t(a)}{P_{At}} = \frac{\gamma}{(\gamma-1)(1+\tau)} \frac{E_t \sum_{s=0}^{\infty} (\omega\beta)^s {C^j_{t+s}}^{-\sigma^{-1}} \left(S_{t+s} \right)^{(\zeta-1)} \phi_{t+s} S_{t+s}^{(1-\zeta)} \left(\frac{P_{At+s}}{P_{At}} \right)^{\gamma} y^A_{t+s}}{E_t \sum_{s=0}^{\infty} (\omega\beta)^s {C^j_{t+s}}^{-\sigma^{-1}} \left(S_{t+s} \right)^{(\zeta-1)} \left(\frac{P_{At+s}}{P_{At}} \right)^{\gamma-1} y^A_{t+s}} \tag{37}$$

A similar result holds for country B. Under the assumption of perfect price flexibility, $\omega = 0$, this result is reduced to

$$\frac{p^*_t(a)}{P_{At}} = \frac{\gamma}{(\gamma-1)(1+\tau)} S_{t+s}^{(1-\zeta)} \phi_{t+s}$$

i.e. firms set a price equal to a markup over the real marginal cost. Notice that the optimal price setting is affected from the terms of trade because in the model intermediate and final good producers use different price index to deflate their nominal cost.

2.6 Market clearing conditions

The total demand for the final good at country level is given by the Dixit- Stiglitz aggregators:

$$Y^A_t \equiv \left[\left(\frac{1}{\zeta} \right) \int_0^{\zeta} y^j_t(a)^{\frac{\gamma-1}{\gamma}} da \right]^{\frac{\gamma}{\gamma-1}} \tag{38}$$

$$Y^B_t \equiv \left[\left(\frac{1}{1-\zeta} \right) \int_{\zeta}^{1} \left(y^j_t(b)^{\frac{\gamma-1}{\gamma}} db \right] ^{\frac{\gamma}{\gamma-1}} \tag{39}$$

The market clearing condition for the final good requires at any period the total quantity of final good to be consumed by households or be purchased by intermediate good producers. Combining equation (29) with (38), and (30) with (39), this yields:

$$Y^A_t = S_t^{(1-\zeta)} \left\{ C^A_t + \psi v^A_t \right\} \tag{40}$$

$$Y^B_t = S_t^{(-\zeta)} \left\{ C^B_t + \psi v^B_t \right\} \tag{41}$$

2.7 The constrained-efficient allocation

In this section, I analyse the social planner's problem to examine under which conditions it is replicated by the decentralised equilibrium. I use these conditions as the benchmark of the monetary policy-maker because as in Ravenna and Walsh (2011) I assume that the central bank reacts to deviations from the efficient steady-state.

The social planner maximises households' utility in each country subject to the technology and the law of motions of employment and unemployment. This requires the choice of agents' control variables, C_t^j, v_t^j, and the choice of the state variables. The problem is formalised for economy A:

$$\max_{C_t^A, v_t^A, N_t^A, u_t^A} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\frac{(C_t^A)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d\frac{(N_t^A)^{1+\varphi}}{1+\varphi} \right) + \lambda_{1t}^A \Big(Z_t^A N_t^A - S_t^{(1-\zeta)} \Big\{ C_t^A + \psi v_t^A \Big\} \Big)$$
$$+\lambda_{2t}^A \Big((1-\delta) N_{t-1}^A + (v_t^A)^{\kappa} (u_t^A)^{(1-\kappa)} - N_t^A \Big) + \lambda_{3t}^A \Big(u_t^A - 1 + (1-\delta) N_{t-1}^A \Big) \Bigg\}$$

In Appendix 1.7, I show that the combination of the first-order conditions yields the social planner's outcome:

$$\frac{\psi}{q(\theta_t^A)} = \kappa \Big(S_t^{(\zeta-1)} Z_t^A - d\Big(N_t^A\Big)^{\varphi} \Big(C_t^A\Big)^{\sigma^{-1}} \Big) + (1-\delta) \Big(E_t \beta_{t,t+1} \Big(\frac{S_{t+1}}{S_t} \Big)^{(1-\zeta)} \frac{\psi}{q(\theta_{t+1}^A)} \Big)$$
$$-(1-\delta) \Big((1-\kappa) E_t \beta_{t,t+1} \Big(\frac{S_{t+1}}{S_t} \Big)^{(1-\zeta)} \psi \theta_{t+1} \Big) \quad (42)$$

A similar condition holds for country B. Now, I compare the constrained-efficient outcome, equation (42), with the decentralised outcome, equation (36). From (42), efficiency associates the real marginal cost with the terms of trade. Particularly, for country A, efficiency requires $\phi_t^A = S_t^{(\zeta-1)}$ and for country B $\phi_t^B = S_t^{(1-\zeta)}$. From (37), this requires prices to be flexible and particularly, the policy-maker of each country to eliminate the inverse of the mark-up, by imposing a tax-financed subsidy to the final good sales equal to $\tau = \frac{1}{\gamma - 1}$.[16] Moreover, comparing the two outcomes, efficiency requires $\kappa = \xi$. That is, the elasticity of the matching function with respect to vacancies must be equal to the firm's bargaining power. This is the Hosios (1990) condition which satisfies that job creation is efficient. Finally, comparing the two outcomes any degree of RWR is undesirable, so it must be $\mu = 0$. These conditions are similar to the conditions in Abbritti and Mueller (2013). In summary:

Proposition 1 *In a currency union NK model with labour market frictions and RWR, the decentralised outcome can replicate the efficient flexible-price equilibrium under three conditions for each country: i) The policy-maker imposes a tax-financed subsidy to the final good sales equal to $\tau = \frac{1}{\gamma-1}$. ii) The Hosios (1990) condition for efficient job creation which is, $\kappa = \xi$, holds and iii) The degree of RWR is equal to zero.*

3 The log-linearised model of the currency union

The linear representation of the model requires some extra notation. For any generic variable X_t, a small letter with a hat denotes the log deviation from its steady-state value. That is:

[16] The importance of assuming tax-financed subsidies has been stressed extensively. See, among others, Rotemberg

$\hat{x}_t = \log X_t - \log X$. The model is log-linearised by applying some standard techniques by Uhlig (1997).

The final good market clearing condition for both countries, equations (40) and (41), are log-linearised according to:

$$\hat{y}_t^A = (1-\zeta)\hat{s}_t^A + \frac{S^{(1-\zeta)}C^A}{Y^A}\hat{c}_t^A + \psi\frac{S^{(1-\zeta)}v^A}{Y^A}\hat{v}_t^A \tag{43}$$

$$\hat{y}_t^B = -\zeta\hat{s}_t^B + \frac{S^{-\zeta}C^B}{Y^B}\hat{c}_t^B + \psi\frac{S^{-\zeta}v^B}{Y^B}\hat{v}_t^B \tag{44}$$

The intermediate good production function is given by $\hat{x}_t^j = \hat{z}_t^j + \hat{n}_t^j$ for $j = A, B$. Given the final good production function, $\hat{y}_t^j = \hat{x}_t^j$, I can write:

$$\hat{y}_t^j = \hat{z}_t^j + \hat{n}_t^j \tag{45}$$

while the shock is already given in log-linear form from equation (24): $\hat{z}_t^j = \rho\hat{z}_{t-1}^j + \epsilon_t^j$. The Euler equation in log-linear form is:

$$\hat{c}_t^j = E_t\hat{c}_{t+1}^j + \sigma E_t\pi_{ct+1}^j - \sigma\hat{q}_t$$

Notice here that $\hat{q}_t$ is the absolute deviation of the gross nominal interest rate from its steady-state, as it is already expressed in percentages. [17] The superscript j is missing in the nominal interest rate as it is common for currency union member states. By using the log-linear form of the standard Fischer equation:

$$\hat{r}_t^j = \hat{q}_t - E_t\pi_{ct+1}^j \tag{46}$$

I can rewrite the Euler equation in terms of the real interest rate $\hat{r}_t$:

$$\hat{c}_t^j = E_t\hat{c}_{t+1}^j - \sigma\hat{r}_t^j \tag{47}$$

The equations (15) and (16) which link CPI with the domestic price index, are log-linearised according to:

$$\hat{p}_{ct}^A \approx \hat{p}_{At} + (1-\zeta)\hat{s}_t, \hat{p}_{ct}^B \approx \hat{p}_{Bt} + (\zeta-1)\hat{s}_t$$

Subtracting their own one period lag and defining inflation as $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$, I get a link of CPI inflation with the domestic inflation and the terms of trade:

$$\pi_{ct}^A = \pi_{At} + (1-\zeta)\Delta\hat{s}_t \tag{48}$$

$$\pi_{ct}^B = \pi_{Bt} + (\zeta-1)\Delta\hat{s}_t \tag{49}$$

where inflation is expressed as the log-deviation of a zero steady-state and the log-linear form of terms of trade is given by $\hat{s}_t = \hat{p}_{Bt} - \hat{p}_{At}$, which implies that:

$$\Delta\hat{s}_t = \pi_{Bt} - \pi_{At} \tag{50}$$

and Woodford (1997) and Thomas (2008).

[17] See Appendix of chapter 2 of Walsh (2010).

The labour market tightness relationship:

$$\hat{\theta}_t^j = \hat{v}_t^j - \hat{u}_t^j \tag{51}$$

The law of motion of employment:

$$\hat{n}_t^j = (1-\delta)\hat{n}_{t-1}^j + \delta\Bigg((\kappa - 1)\hat{\theta}_t^j + \hat{v}_t^j\Bigg) \tag{52}$$

The unemployment relationship:

$$\hat{u}_t^j = -\alpha_0^j \hat{n}_{t-1}^j \tag{53}$$

where $\alpha_0^j = (1-\delta)\frac{N^j}{u^j}$

The intermediate good firm's optimal hiring decision in country A:

$$\begin{aligned}\psi(\theta^A)^{(1-\kappa^A)}(1-\kappa^A)\hat{\theta}_t^A &= \gamma_1^A(\hat{\phi}_t^A + \hat{z}_t^A) + (1-\delta)\beta\psi\frac{\theta}{\sigma}(\gamma_0 - (\theta^A)^{-\kappa^A})(\hat{c}_{t+1}^A - \hat{c}_t^A) \\ &- (1-\delta)\beta\psi\theta^A(\gamma_0^A - (1-\kappa^A)(\theta^A)^{-\kappa})\hat{\theta}_{t+1}^A \\ &- (1-\mu)\xi^A\gamma_2^A(\varphi\hat{n}_t^A + \frac{1}{\sigma}\hat{c}_t^A)\end{aligned} \tag{54}$$

where I have already substituted the log-linearised version of the real wage with RWR and $\gamma_0^A = (1-\mu^A)(1-\xi^A)$, $\gamma_1^A = (1-\gamma_0^A)\phi^A$, $\gamma_2^A = d(N^A)^{\varphi}(C^A)^{\sigma^{-1}}$. A similar condition holds for country B. Finally, in the section 1.8 of the technical Appendix, I show that a first-order Taylor expansion of (37) combined with the average domestic price at time t, gives the New Keynesian Phillips Curve (NKPC) for an open economy. For country A this is:

$$\pi_{At} = \delta_p^A\hat{\phi}_t^A + \delta_p^A(1-\zeta)\hat{s}_t + \beta E_t\pi_{At+1} \tag{55}$$

while for country B this is given by:

$$\pi_{Bt} = \delta_p^B\hat{\phi}_t^B + \delta_p^B(\zeta-1)\hat{s}_t + \beta E_t\pi_{Bt+1} \tag{56}$$

where $\delta_p^j = \frac{(1-\omega^j\beta)(1-\omega^j)}{\omega^j}$ is the elasticity of domestic inflation with respect to the real marginal cost of intermediate good firms, $\hat{\phi}_t^j$.

Domestic inflation is a forward looking variable and has two driving forces, the real marginal cost and the terms of trade which create an extra cost channel. Therefore there is an international spillover on domestic inflation through the real marginal cost.

3.1 The reduced-form of the model

The log-linearised model can be reduced to a system of a dynamic IS curve and a New-Keynesian Philips curve (NKPC) with unemployment for each member state. For this exercise, I have found the work by Ravenna and Walsh (2011) to be very useful.

3.1.1 The dynamic IS for small open economies

Focusing on country A, by combining the production function, equation (45), and the market clearing condition, equation (43), I can solve for consumption:

$$\frac{S^{(1-\zeta)}C^A}{Y^A}\hat{c}_t^A = \hat{z}_t^A + \hat{n}_t^A - (1-\zeta)\hat{s}_t - \psi\frac{S^{(1-\zeta)}v^A}{Y^A}(\hat{\theta}_t^A + \hat{u}_t^A) \tag{57}$$

Taking the unemployment relationship, equation (53), one period forward, and then using the law of motion of employment, equation (52), to substitute for $\hat{n}_t^A$, it yields:

$$\hat{u}_{t+1} = (1-\delta)\left(1 - \frac{\delta N^A}{u^A}\right)\hat{u}_t^A - \alpha_0\delta\kappa\hat{\theta}_t^A \tag{58}$$

Using (58) to solve for $\hat{\theta}_t^A$ and substitute in (57), I can also substitute for $\hat{n}_t^A$ from equation (52). Doing this, I get an expression for consumption in terms of unemployment, the terms of trade and the productivity shock. Taking this expression one period forward, I can use the Euler equation, (47), and substitute for $\hat{c}_t^A$ and $\hat{c}_{t+1}^A$ to get the dynamic IS expressed in terms of unemployment. For country A this is:

$$\begin{aligned}\hat{u}_{t+1}^A = \frac{\eta_1^A}{\eta_1^A + \eta_2^A}\hat{u}_t^A + \frac{\eta_2^A}{\eta_1^A + \eta_2^A}E_t\hat{u}_{t+2}^A + \frac{\sigma^A}{\eta_1^A + \eta_2^A}\hat{r}_t^A + \frac{\alpha_1^A}{\eta_1^A + \eta_2^A}(1-\zeta)\left(E_t\hat{s}_{t+1} - \hat{s}_t\right)\\ - \frac{\alpha_1^A}{\eta_1^A + \eta_2^A}\left(E_t\hat{z}_{t+1}^A - \hat{z}_t^A\right)\end{aligned} \tag{59}$$

and for country B:

$$\begin{aligned}\hat{u}_{t+1}^B = \frac{\eta_1^B}{\eta_1^B + \eta_2^B}\hat{u}_t^B + \frac{\eta_2^B}{\eta_1^B + \eta_2^B}E_t\hat{u}_{t+2}^B + \frac{\sigma^B}{\eta_1^B + \eta_2^B}\hat{r}_t^B - \frac{\alpha_1^B}{\eta_1^B + \eta_2^B}\zeta\left(E_t\hat{s}_{t+1} - \hat{s}_t\right)\\ - \frac{\alpha_1^B}{\eta_1^B + \eta_2^B}\left(E_t\hat{z}_{t+1}^B - \hat{z}_t^B\right)\end{aligned} \tag{60}$$

where $\eta_1^j = -\alpha_2(\alpha_0\delta\kappa + \delta_u^j)$, $\eta_2^j = (\alpha_2^j - \alpha_3^j)$ and $\alpha_1^A = \frac{Y^A}{S^{(1-\zeta)}C^A}$, $\alpha_1^B = \frac{Y^B}{S^{-\zeta}C^B}$, , $\alpha_2^j = \frac{\psi^j v^j}{\alpha_0^j\delta\kappa^j C^j}$, $\alpha_3^j = \frac{\alpha_1^j}{\alpha_0^j}$ and $\delta_u^j = ((1-\delta) - \alpha_0^j\delta)$

Like in Ravenna and Walsh (2011), the unemployment is predetermined at time t and has a forward looking and a backward looking component.

3.1.2 The NKPC for small open economies

In order to get an NKPC with unemployment, I rearrange equation (54) and solve for the real marginal cost $\hat{\phi}_t^j$. Then, I use the Euler equation to eliminate $E_t\hat{c}_{t+1}^j - \hat{c}_t^j$ and then I use the market clearing condition to express $\hat{c}_t^j$ in terms of employment. Using equation (58), I eliminate θ_t^j and express it in terms of unemployment. Finally, I use equation (52) to express n_t^j in terms of unemployment. I get an expression of the NKPC:[18]

[18] In the technical Appendix 1.9 and 1.10 I show how we get the IS and the NKPC with variables expressed as deviations from their efficient steady-state. As in the Appendix of Ravenna and Walsh (2011), the productivity term appears even in the absence of real wage rigidity.

$$\pi_{At} = \beta E_t \pi_{At+1} + \delta_p^A \rho_0 \hat{u}_t^A + \delta_p^A \rho_1^A \hat{u}_{t+1}^A - \delta_p^A \rho_2^A E_t \hat{u}_{t+2}^A + \delta_p^A \rho_3^A ((1-\zeta)\hat{s}_t - \hat{z}_t^A) + \delta_p^A \rho_4^A \hat{r}_t^A \quad (61)$$

$$\pi_{Bt} = \beta E_t \pi_{Bt+1} + \delta_p^B \rho_0^B \hat{u}_t^B + \delta_p^B \rho_1^B \hat{u}_{t+1}^B - \delta_p^B \rho_2^B E_t \hat{u}_{t+2}^B + \delta_p^B \rho_3^B \zeta \hat{s}_t + \delta_p^B \Big(\rho_3^B + \frac{1-2\zeta}{\zeta}\Big) \hat{z}_t^B + \delta_p^B \rho_4^B \hat{r}_t^B \quad (62)$$

where the ρ^j coefficients are given in the technical Appendix 1.10. Notice in equations (61) and (62) the presence of the domestic productivity term,$\hat{z}^j$, which acts as a cost-push shock on domestic inflation.

The equilibrium in the currency union is described by the linear difference system of equations (59), (60), (61), (62), (50), (48), (49), (46), the AR process of the domestic productivity shocks and an interest-rate rule which satisfies the determinacy of equilibrium.[19]

4 Welfare-based optimal monetary policy in the currency union

In this section, I characterise the welfare-based optimal monetary policy for a currency union with labour market frictions. For this reason, I follow a Linear-Quadratic (L-Q) approach pioneered by Rotemberg and Woodford (1997) and Woodford (2003) to derive the welfare-based union's loss function. L-Q approach has been used in many closed economy NK models with unemployment like Thomas (2008), Blanchard and Gali (2010) and Ravenna and Walsh (2011) and it has been used in NK currency union models like Benigno (2004) and Gali and Monacelli (2008). The L-Q approach requires a second order approximation of households' utility function. Drawing insights from the general equilibrium framework, the monetary policy objectives of the monetary authority of the currency union are the domestic inflation and the domestic unemployment of the member states and their terms of trade.

The monetary policy-maker chooses the same efficient steady-state with the social planner. In the section 2 of the technical Appendix, I show that the currency union's welfare loss function, given as by the discounted weighted average of households' welfare criterion, will be:

$$\begin{aligned}\Omega_{t+i} = -\Bigg\{ & U'(C)C\Bigg(\zeta \frac{\gamma}{2\delta_p^A} \sum_{i=0}^{\infty} \beta^i \pi_{At+i}^2 + (1-\zeta)\frac{\gamma}{2\delta_p^B} \sum_{i=0}^{\infty} \beta^i \pi_{Bt+i}^2 \Bigg) \\ & + U'(C)N\Bigg(\zeta \frac{\delta_3^A}{2(\alpha_0^A)^2} \sum_{i=0}^{\infty} \beta^i (\hat{u}_{t+1+i}^A)^2 \\ & + (1-\zeta)\frac{\delta_3^B}{2(\alpha_0^B)^2} \sum_{i=0}^{\infty} \beta^i (\hat{u}_{t+1+i}^B)^2 \Bigg) \\ & + U'(C)C\Bigg(\zeta(1-\zeta)\frac{1+\sigma}{2\sigma} \sum_{i=0}^{\infty} \beta^i \hat{s}_{t+i}^2 \Bigg) \Bigg\} \end{aligned} \quad (63)$$

[19]See the technical Appendix 1.11 for a discussion regarding a monetary policy rule which satisfies the determinacy

where $\delta_3^j = \frac{N^j}{\sigma C^j} S + \frac{\varphi U'(N^j)}{U'(C^j)}$. Notice that if one of the two member states becomes very small in size i can be neglected, i.e. $\zeta \to 0$ or $(1-\zeta) \to 0$, the terms of trade terms are eliminated and the loss function takes a form similar to the closed economy case, like Blanchard and Gali (2010).

The currency union monetary policy-maker chooses the sequence of the variables:

$$\left\{ \pi_{Jt+i}, \hat{u}^J_{t+1+i}, \hat{s}_{t+i}, \pi^W_{t+i}, \hat{u}^W_{t+i} \right\}_{i=0}^{\infty}$$

to minimise the welfare loss of the currency union:

$$E_0 \sum_{i=0}^{\infty} \beta^t \left\{ \Omega_{t+i} \right\}$$

subject to the IS curves, eq. (59), (60); the NKPC, eq. (61),(62); the link of terms of trade with inflation, (50); and the union-wide constraints:

$$\pi^W_t = \zeta \pi_{At} + (1-\zeta)\pi_{Bt} \tag{64}$$

$$\hat{u}^W_t = \zeta \hat{u}^A_t + (1-\zeta)\hat{u}^B_t \tag{65}$$

Then, the optimality conditions derived for each monetary policy regime, together with the equations (50), (59) - (62), (64), (65) and the productivity AR(1) process, constitute the dynamical system which describes the optimal monetary policy.[20]

5 Quantitative analysis

The model is stochastic and it is solved numerically. I choose the benchmark parameter values which have been selected from estimations of the euro area and from standard parameter values used in other multi-country NK models or NK models with unemployment. Then I study the effects of a domestic and an aggregate productivity shock to the currency union when the central bank implements an optimal policy regime. The calibrated values are summarized in table 1.

5.1 Calibration

Following Gali and Monacelli (2008) and Abbritti and Mueller (2013), I assume that both countries are symmetric except for the labour market heterogeneity that I introduce.

Preferences: I assume a quarterly frequency for the variables of the model. The value of the discount factor β for quarterly time interval is set equal to 0.99. I assume that the member states are of equal size, so I choose $\zeta = 0.5$. I assume a relative risk aversion coefficient, $\sigma = 1$. Following Abbritti and Mueller (2013), I assume the labour supply elasticity to be $\varphi = 0$, while other studies like Blanchard and Gali (2010) choose $\varphi = 1$.

of equilibrium.

[20]For the derivation of the optimality conditions, see technical Appendix 2.1.1.

Table 1: Baseline Parameter Values

Description	Parameter	Value	Reference
Preferences			
Discount factor	β	.99	Quarterly time interval
Index of openness	ζ	.5	Member states of equal size
Relative risk aversion	σ	1	Log utility, Abbritti and Mueller (2013)
Labour supply elasticity	φ	0	Homogeneous pref. of leisure, Abbritti and Mueller (2013)
Labour market			
Prob. of filling a vacancy	$q(\theta^j)$	.97	Ravenna and Walsh (2011)
Exogenous job separation rate	δ^j	.10	Shimer (2005), Ravenna and Walsh (2011)
Elasticity of vacancies w.r.t matches	κ^j	.5	Pissarides and Petrongolo (2001), Campolmi and Faia (2011)
Bargaining power of firms	ξ^j	.5	Satisfy the Hosios (1990) condition
Posting vacancy cost	ψ^j	.097	$\frac{.01Y}{v^j}$, Walsh (2005), Thomas (2008), Blanchard and Gali (2010)
Union-average real wage rigidity	μ^j	.5	Blanchard and Gali (2010), Abbritti and Mueller (2013)
Technology			
Nominal price rigidity	ω^j	.75	Blanchard and Gali (2010)
Steady-state marginal cost	ϕ^j	.83	Inverse mark-up, Blanchard and Gali (2010), Ravenna and Walsh (2011)
Initial steady-state values			
Terms of trade	S	1	Symmetric member states, Gali and Monacelli (2008)
Unemployment	u^j	.10	European average unemployment rate, Blanchard and Gali (2010)
Productivity shock			
Autocorrelation	ρ	.95	Euro area estimates, Abbritti and Mueller (2013)
Std. deviation	σ_z^j	.00624	Smets and Wouters (2003), Abbritti and Mueller (2013)
Correlation btw shocks	ρ_z	.258	Abbritti and Mueller (2013)

Labour market: The probability for firms to fill a vacancy, $q(\theta^j)$, is set equal to 0.97, following Ravenna and Walsh (2011).[21] This value is relatively higher than $q(\theta) = 0.7$ which is used in other studies like Campolmi and Faia (2011) and Walsh (2005). The elasticity of vacancies with respect to the number of matches, κ^j , is set equal to 0.5, following the estimations by Pissarides and Petrongolo (2001), while the same value is used by Thomas (2008) and Campolmi and Faia (2011). For the calibration of the bargaining power of firms, ξ^j, I assume that the Hosios (1990) efficient steady-state condition holds, so I set $\xi^j = \kappa^j$. The exogenous job separation rate, δ^j is set equal to 0.08 following Ravenna and Walsh (2011). This value seems very high for the sclerotic labour market of the euro area where the separation rate is very low. In this direction, Blanchard and Gali (2010) set a value equal to 0.04, while Campolmi and Faia (2011) choose 0.06 and Abbritti and Mueller (2013) choose 0.071.[22] The posting vacancy cost is calculated as a fraction 0.01 of the GDP and is set equal to 0.097. The same strategy is used by Walsh (2005), Thomas (2008), Blanchard and Gali (2010) and Abbritti and Mueller (2013). The value of the posting vacancy cost varies with the specification of the posting cost function.[23] For the calibration of the degree of real wage rigidity, the empirical evidence is controversial. In the benchmark calibration I choose $\mu^j = 0.5$, following Blanchard and Gali (2010), Campolmi and Faia (2011) and Abbritti and Mueller (2013). However, when I study the labour market heterogeneity, I let RWR to vary between $0.2 - 0.8$ like in Abbritti and Mueller (2013). Finally, I set a steady-state of unemployment, $u^j = 0.10$ following, Blanchard and Gali (2010) definition for the sclerotic European labour market. Abbritti and Mueller (2013) choose a value equal to 0.08.

Final good production: The degree of nominal price rigidity is calibrated following estimates that find that prices change every three to four quarters. Hence, I set $\omega^j = 0.75$, following Thomas (2008), Blanchard and Gali (2010) and Ravenna and Walsh (2011). This implies an elasticity of inflation with respect to real marginal cost, $\delta_p = 0.086$. I set the elasticity of substitution among the differentiated final goods within country to $\gamma = 6$ like Ravenna and Walsh (2011). This implies a steady state real marginal cost of $\phi^j = 0.83$, where I have used that for symmetric countries $S = 1$ as does Gali and Monacelli (2008).

Productivity shock: I assume a persistent productivity shock by setting an autocorrelation coefficient, ρ, equal to 0.95 and a standard deviation productivity shock of σ_z^j equal to 0.624 following the euro area estimates by Smets and Wouters (2003) and adopted by Abbritti and Mueller (2013) as well.

5.2 Responses of the currency union under the optimal monetary policy

I solve the model using DYNARE and I evaluate two optimal monetary policy regimes: The timeless perspective optimal commitment and the optimal discretion.[24]

Under optimal commitment, the central bank makes credible announcements about her

[21] This is based on a calculation of a 5% daily probability times the average number of working days per month, times three, given that I treat time as quarters.

[22] These numbers are based on estimates and calculations for the European economy and they are very different than the more fluid US labour market in which the separation rate is set around $0.10 - 0.15$. For this reason, I calibrate the model by using a wide range for the value of the separation rate around $0.04 - 0.12$.

[23] For example, Thomas (2008) uses a convex posting vacancy cost following Gertler and Trigari (2009), while I use a simpler linear cost similar to Ravenna and Walsh (2011).

[24] For details about DYNARE visit http://www.dynare.org/. For details about the DYNARE code for optimal mon-

future actions. That is the central bank chooses the optimal path for the current and future objectives. The public's expectations can be affected as the central bank's announcements are considered as credible.

Under optimal discretion, the central bank resets her actions every period. Therefore, current period actions do not bind at the future and any announcement made are not be considered as credible by the public; so the later's expectations will not be affected.[25] The difference between the two regimes is that under discretion the current period's actions are not binding, therefore, the initial conditions are set to zero. [26]

The strength of the response of the central bank on the policy objectives depends on their relative weights, which are derived from the L-Q approach. A higher value implies higher welfare loss in the currency union, therefore more strong actions of the central bank are required. From the benchmark calibration, I obtain the parameter values reported in table 2.

Table 2: Weights of central bank objectives

Policy Objective*	Description	Policy Weight**
π_{At}	Domestic inflation of country A	17.48
π_{Bt}	Domestic inflation of country B	17.48
$\hat{u}_{t+1}^{A}$	Domestic unemployment of country A	.0031
$\hat{u}_{t+1}^{B}$	Domestic unemployment of country B	.0031
$\hat{s}_t$	Terms of trade	.25

*Expressed as gap from the efficient steady-state.

**The weights are expressed in absolute value.

From table 2 it becomes clear that the main source of welfare loss in the currency union is the variability of domestic inflation. Thus, the central bank has an incentive to be more aggressive in stabilising the domestic inflation. The loss from inefficient fluctuations of domestic unemployment is substantially less relative to the loss from inflation. Also there is a welfare loss from the deviation of the terms of trade which is relatively higher than the loss from unemployment.

5.2.1 Optimal response to domestic productivity shocks

In this section, I analyse the dynamic response of the policy objectives to a domestic productivity shock when the optimal monetary policy is implemented. This shock is referred as country-j specific. In this case the autocorrelation of the shocks, $\rho_{a,b}$, is set equal to zero. To keep the analysis tractable, only the optimal commitment regime is presented in the main content. The dynamic responses under optimal discretion are presented in the technical Appendix 2.4. The qualitative outcomes of the two regimes do not differ substantially. A comparison of moments between the two regimes is presented in the next section.

etary policy under commitment and/or discretion please see Adjemian et al. (2011).

[25]This discussion about optimal commitment and discretion has been inspired by Walsh (2010) chapter 8.4.3.

[26]That is $\lambda_{1t-1} = \lambda_{2t-1} = 0$, where λ_t are the Langrangian multipliers associated with the inflation adjustment constraints, the NKPC. Obviously, that is not the case for commitment where current period actions bind at the future

In order to characterise the role of labour market heterogeneity, I construct an index measured from the differential of the degree of RWR, $\Delta\mu = \mu^A - \mu^B$. Following Andersen and Seneca (2010), to avoid confusion from the aggregate level effects, I allow RWR to vary between $0.2 - 0.8$, but I keep the union-average RWR constant and equal to $\mu^W = \zeta\mu^A + (1-\zeta)\mu^B = 0.5$. This implies $\mu^B = 1 - \mu^A$. Therefore, $\Delta\mu$ can vary from 0 to 0.6. Finally, as the focus of the paper is on the labour market heterogeneity, I limit the analysis to the case where there is a symmetric degree of nominal price rigidity.[27]

Figure 2 displays the impulse response of the policy objectives and the real interest rates to a 0.624% standard deviation country A-specific shock when the central bank implements an optimal commitment regime. The case presented here is the one which μ^A varies between $0.5 - 0.8$ and μ^B varies between $0.2 - 0.5$. That is, I study the dynamic responses of the model when the country hit by the shock has equal or greater degree of RWR than the other.

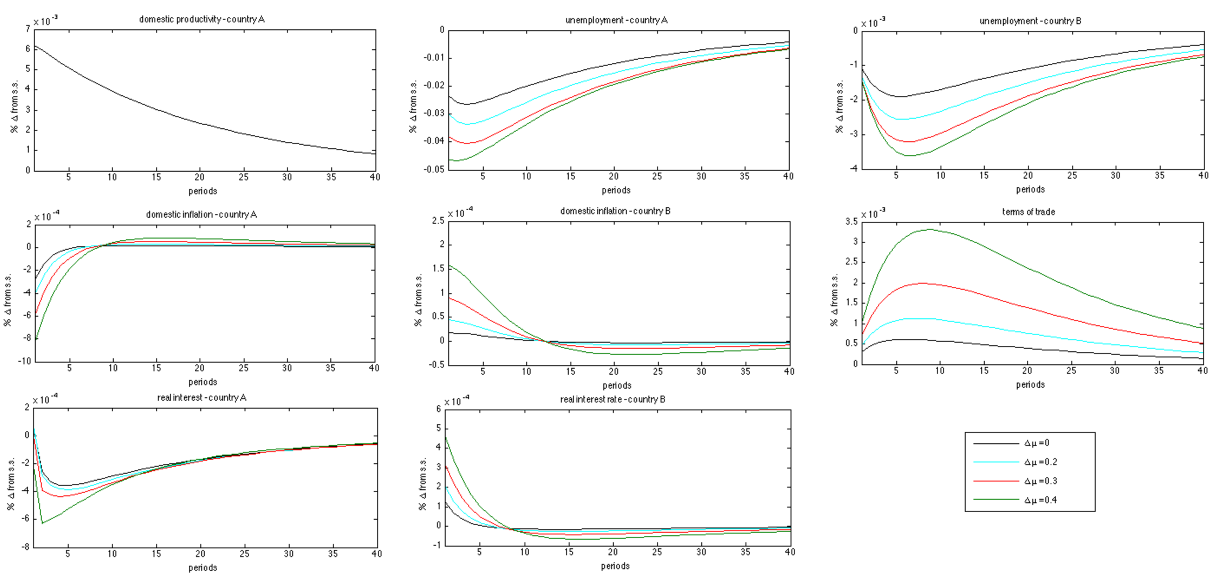

Figure 2: *Optimal commitment; country A-specific shock* ($\rho_{a,b} = 0$)

From figure 2, it is observed that the domestic inflation and unemployment of country A are not fully stabilised. A similar intuition like in Andersen and Seneca (2010) applies: The assumption of RWR implies that the real wage is not simultaneously adjusted to the new economic conditions. Therefore, firms, on average, change their optimal hiring decision, so unemployment decreases from the positive productivity shock and the final good production is increased. The firms, which are not constrained to reset their price, reduce the price of their good. The price dispersion makes domestic inflation volatile, i.e. the domestic inflation will decrease as well.[28] Hence, the central bank has an incentive to keep unemployment below the baseline level for several periods, in order to create inflationary pressure through expectations (under commitment only). The central bank has an incentive to let unemployment to be more volatile, as RWR in country A is increasing.

periods, therefore it can be $\lambda_{1t-1} \neq 0$, $\lambda_{2t-1} \neq 0$

[27] See Benigno (2004) for a normative analysis of a currency union when there is an asymmetric degree of nominal price rigidity.

[28] This follows from the Lemma 6.3 in Woodford (2003), which I show in technical Appendix 2.0.8.

Country B is not hit by a domestic shock. Nevertheless, the domestic inflation and unemployment of country B are not fully stabilised under the optimal commitment. The reason lies on the terms of trade effect. Households demand baskets of final goods from both countries. A country-A specific shock affects inflation of country A, so the relative competitiveness of the member states changes and this is translated into fluctuations of the terms of trade. Indeed, the terms of trade act as a transmission mechanism of the country A-specific shock and add a "cost-push" component on inflation of country B. As it is shown in the graph, the volatility of the terms of trade increases with $\Delta\mu$. The higher is the labour market heterogeneity the more volatile is inflation and unemployment of country B and consequently the welfare loss increases.

Table 3 displays a moment analysis. The standard deviations of the policy objectives are reported for different values of the index $\Delta\mu$. From the last column, the volatility of the terms of trade increases with $\Delta\mu$. That increases the standard deviation of the domestic inflation and unemployment of country B. From the second column, we observe that the welfare loss of the currency union increases monotonically with $\Delta\mu$.

Table 3: Welfare loss. country A-specific shock*

$\Delta\mu = \mu^A - \mu^B$	Welfare loss	$\sigma_{\pi_{At}}$	$\sigma_{\pi_{Bt}}$	$\sigma_{\hat{u}^A_{t+1}}$	$\sigma_{\hat{u}^B_{t+1}}$	$\sigma_{\hat{s}_t}$
0	.0030	.0003	.0000	.0968	.0077	.0026
.2	.0054	.0005	.0001	.1241	.0104	.0049
.4	.0093	.0008	.0002	.1503	.0131	.0088
.6	.0151	.0012	.0003	.1650	.0146	.0146

*Optimal Commitment.

From this quantitative exercise, we notice that labour market heterogeneity has a distortionary effect on the welfare of the currency union and it is inefficient. The single monetary authority of the currency union has a lack of sufficient number of instruments and this makes the stabilisation of all the policy objectives unfeasible. That is true even for the member states in which domestic inflation is not affected by a cost-push shock. I summarise in the following proposition:

Proposition 2 *In a currency union with labour market heterogeneity, stabilisation trade-offs arise for the central bank from the international linkages between the member states. Particularly, the terms of trade act as a transmission mechanism of domestic productivity shocks from the one member state to the other. The volatility of the terms of trade is monotonically increasing with the labour market heterogeneity index, $\Delta\mu$. Consequently, the welfare loss in the currency union is monotonically increasing with $\Delta\mu$.*

5.2.2 Optimal response to aggregate shocks

In this section, I examine the dynamic response of the policy objectives in the presence of an aggregate productivity shock. This is the case when the two country-specific productivity shocks are perfectly correlated ($\rho_{a,b} = 1$). The impulse responses under optimal commitment are displayed in figure 3, while the results under optimal discretion are given on the technical Appendix.

When there is a common positive productivity shock, the production is expanded in both countries and consequently unemployment falls because of the assumption of RWR which makes firms in both countries to adjust their optimal hiring decision. When $\mu^A = \mu^B = 0.5$, then $\Delta\mu = 0$, hence, the shock has symmetric effects. Figure 3 is key to understand the role of labour market asymmetries. Not surprisingly, the asymmetric effects of the shock arise when $\mu^A > \mu^B$, so $\Delta\mu > 0$. Unemployment in country A is more volatile as RWR is higher. The domestic inflation is more volatile as well. Consequently, the terms of trade deviate more the higher is the value of $\Delta\mu$. On the other hand, when $\mu^A = \mu^B = 0.5$, the domestic inflation varies symmetrically. Any effect of the terms of trade is offset and there is no welfare loss from the terms of trade. In the symmetric case, the welfare loss of the union is smaller than the asymmetric case. When there is perfect symmetry, the central bank reacts to the union-wide variables. The intuition is similar to the one in Benigno (2004) in which when the two countries have the same degree of nominal price rigidity, the central bank reacts to the union-wide/CPI stabilisation. Here, when the two countries have the same degree of RWR, the central bank reacts to the union-wide unemployment, as symmetry implies no distortion from the labour market heterogeneity and the gain of one degree of freedom.

Table 4: Welfare loss. Aggregate shock*

$\Delta\mu = \mu^A - \mu^B$	Welfare loss	$\sigma_{\pi_{At}}$	$\sigma_{\pi_{Bt}}$	$\sigma_{\tilde{u}^A_{t+1}}$	$\sigma_{\tilde{u}^B_{t+1}}$	$\sigma_{\tilde{s}_t}$
0	.0065	.0003	.0003	.1044	.1044	.0000
.2	.0074	.0005	.0002	.1293	.0827	.0036
.4	.0104	.0008	.0002	.1534	.0644	.0082
.6	.0157	.0012	.0003	.1665	.0484	.0144

*Optimal Commitment.

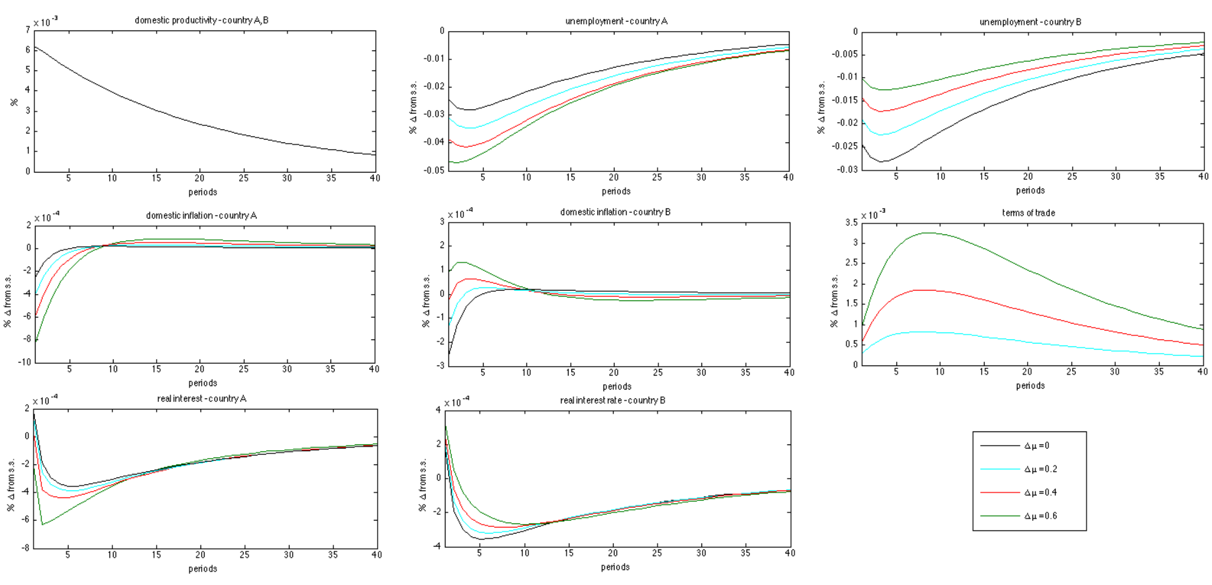

Figure 3: *Optimal commitment; aggregate productivity shock* ($\rho_{a,b} = 1$)

inflation and unemployment of country B in the fourth and sixth column respectively. In this quantitative example, country B is the more flexible country so it is expected as $\Delta\mu$ increases the standard deviation of unemployment to decline. Despite that fact the welfare loss in the currency union increases monotonically with $\Delta\mu$ because the volatility of the terms of trade increases monotonically as well. Proposition 3 summarises:

Proposition 3 *In a currency union with labour market heterogeneity, in the presence of aggregate productivity shocks, the terms of trade intensify the asymmetric effects of the shock. The standard deviation of the terms of trade is monotonically increasing with the value of the labour market heterogeneity index $\Delta\mu$. Consequently, the welfare loss of the currency union is monotonically increasing in $\Delta\mu$.*

5.3 Unemployment adjustment. Optimal commitment vs optimal discretion.

In this section, I compare two optimal monetary policy regimes: the timeless perspective optimal commitment with the optimal discretion. The impulses responses are available on the technical Appendix. Regarding the inflation adjustment, the analysis between commitment and discretion does not differ that much with that one of a closed economy model, so it is not presented in the main content of the paper. Concerning the inflation adjustment, labour market heterogeneity does not play an important role on the optimal regime choice as the optimal commitment is desirable for both member states. The second column of the table 5 displays the welfare loss in the currency union. The reason that is lower under commitment lies on the role of public's expectations. Under commitment the volatility of the domestic inflation for both member states is lower than the one under discretion, therefore the welfare loss is lower.

However, regarding the unemployment adjustment, there are implications of the labour market heterogeneity for the optimal monetary policy regime. A comparison of moments from the impulse responses of unemployment under commitment and discretion is suggestive. The last two columns of the table 5 report the unemployment differential between the two regimes, for each country. The commitment and discretion are denoted with a superscript c and d respectively. We observe that as $\Delta\mu$ increases, $(\sigma_{\tilde{u}^{Ac}_{t+1}} - \sigma_{\tilde{u}^{Ad}_{t+1}})$ increases, but $(\sigma_{\tilde{u}^{Bc}_{t+1}} - \sigma_{\tilde{u}^{Bd}_{t+1}})$ decreases. Therefore, when the value of the labour market heterogeneity index increases, optimal discretion becomes a more desirable regime for the country hit by the domestic shock, as the inefficient unemployment fluctuation becomes lower. For the country which is not hit by the domestic shock unemployment fluctuations are reduced more under an optimal commitment regime, so this is more desirable.

Table 5: Optimal Commitment vs Discretion. Country A-specific shock

$\Delta\mu = \mu^A - \mu^B$	Welfare loss *comm.*	Welfare loss *discr.*	$\sigma_{\tilde{u}^{Ac}_{t+1}} - \sigma_{\tilde{u}^{Ad}_{t+1}}$	$\sigma_{\tilde{u}^{Bc}_{t+1}} - \sigma_{\tilde{u}^{Bd}_{t+1}}$
0	.0030	.0042	.0085	-.0081
.2	.0054	.0074	.0157	-.0075
.4	.0093	.0121	.0234	-.0055
.6	.0151	.0184	.0233	-.0028

Proposition 4 *Increasing labour market heterogeneity in a currency union has important implications for the monetary policy optimal regime choice concerning the unemployment stabilisation. When the labour market heterogeneity index $\Delta\mu$ increases, the inefficient fluctuations of unemployment of the country hit by a domestic shock are reduced more under an optimal discretion, while the opposite is true for the other country.*

The reason behind this result lies on the sensitivity of inflation to unemployment changes (in the NKPC). Table 6 shows that as the degree of RWR increases the sensitivity of inflation to unemployment decreases. In the quantitative exercises of this paper, higher $\Delta\mu$ implies higher degree of RWR in country A. Under an optimal discretionary regime the central bank loses the instrument of public's expectations. In this case, unemployment acts as an instrument for inflation stabilisation. However, a lower sensitivity to unemployment requires a stronger reaction to the latter in order for inflation to be stabilised. However, for country B this is not the case. A higher $\Delta\mu$ implies a lower value of RWR and a high sensitivity of inflation to unemployment changes. Therefore, the central bank has not a strong incentive to adjust unemployment of country B fast under discretion. In this case, unemployment of country B fluctuates less under the optimal commitment, so it is a more desirable regime for country B.

Table 6: RWR and sensitivity of domestic inflation

RWR	$\tilde{u}^j_{t+1}$	$\tilde{s}_t$	$\tilde{s}_{t+1}$	$\tilde{z}^j_t$
0.2	-.2321	.1665	.0418	-.4124
0.3	-.192	.1205	.1439	-.5144
0.4	-.1576	.081	.2314	-.6018
0.5	-.1279	.0469	.3073	-.6776
0.6	-.1018	.017	.3736	-.7439
0.7	-.0788	-.0094	.4322	-.8023
0.8	-.084	-.0329	.4843	-.8543

5.4 Further sensitivity analysis

In this study, I examine all cases in which the degree of RWR varies from 0 to 1. However, like in Abbritti and Mueller (2013), I present the results where the degree of RWR varies from 0.2 to 0.8 as they are more plausible values. I also allow for different values of the exogenous job separation rate, δ^j which, in the literature is found to vary between $0.10 - 0.15$. The value for $q(\theta^j)$ is controversial and varies a lot in the literature, from 0.7 to 0.97. However, as Ravenna and Walsh (2011) suggest, using a similar calibration strategy, the value of $q(\theta^j)$ is not crucial for the results. In addition, I find no substantial change when I allow for elastic labour supply, i.e. $\varphi = 1$, which is the value chosen by Blanchard and Gali (2010). Moreover, given that this paper has been motivated by the euro area currency union, the labour market tightness is constructed by using only the average rate of unemployment for the euro area. Finally, the degree of nominal price rigidity is chosen to be fixed at $\omega^j = 0.75$, as this is consistent with the most studies in the NK literature.

6 Summary and further discussion

To the best of my knowledge this paper is the first which follows a L-Q approach to provide an analysis of the welfare-based optimal monetary policy in a currency union framework with a source of labour market heterogeneity. The labour market frictions are merged to a standard NK currency union model in order to "generate" involuntary unemployment. A simple index of labour market heterogeneity is constructed based on the differential of the degree of RWR. The main results of the paper are derived by quantitative comparisons based on different values of the RWR differential between two member states. The dynamic response of the economy under productivity shocks is examined given that the central bank conducts an optimal monetary policy regime; optimal commitment or optimal discretion.

The terms of trade play a crucial role for the main remarks of the paper as it act as a transmission mechanism of a domestic shock from one member state to the other and intensify the asymmetric effects of an aggregate shock. The paper highlights the role of the labour market heterogeneity on the optimal monetary policy and explore its welfare consequences. Higher value of the labour market heterogeneity index implies higher welfare loss for the currency union. Therefore, the labour market heterogeneity described in the paper has distortionary effects.

The comparison between the two optimal regimes could be useful for monetary policy in practice. The ECB has as a primary goal the maintenance of the price stability across the euro area. This objective is publicly announced. If this announcement acts as a commitment device, it could affect public's expectations. In this case, it could be summarised that ECB follows an optimal commitment regime. The paper highlights a case of labour market heterogeneity where the optimal commitment is not desirable by the member states with more rigid real wages than others. It should be considered though that the paper deals with a particular case of labour market heterogeneity, so consideration is needed before any use of policy in practice.

The model at a certain extent confirms the theory of optimum currency union areas by Mundell (1961). It is a realisation, in the presence of too many frictions, a currency union area is far from being optimal. The frictions of the real wage rigidity and the perfect labour immobility seem to play a very important role on this sub-optimality. Mundell (1961) highlights the importance of wage flexibility and labour mobility on the optimality of a currency union. In this model, real wage rigidity is important to realise the monetary policy trade-offs and construct a labour market heterogeneity index. The no-migration across countries assumption could make the model far more complicated as it would require the construction of migration strategies by the agents, which in a dynamic framework could add a lot of complexity. It is though the next step of further research. Based on a two country framework of migration like Ortega (2000), this model could be extended to a dynamic framework of migration. Then it would be interesting to study to what extent the increasing labour mobility could absorb the welfare loss coming from the real wage rigidity. Then, such a paper could be useful to provide some intuition regarding migration policies.

Finally, the construction of the current NK DSGE framework allows for other steps of further research. One of this is the estimation of the degree of RWR for the member states of the euro area using a Bayesian approach. As it is discussed in the paper, the empirical evidence of RWR are controversial, while the studies about the euro are are limited. Another extension could consider recent developments of NK models with financial frictions. However an interaction between the financial market with the labour market is incomplete. A macroeconomic framework which merges both markets would be useful for policy purposes but the interaction of capital with unemployment could add further complexity to the model.

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