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**Trade in value-added and the welfare gains
of global value**

by

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TRADE IN VALUE-ADDED AND THE WELFARE GAINS OF GLOBAL VALUE
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JOB MARKET PAPER

Abstract:

In the current era of global value chains, the manufactured goods exported by a given country embed the value-added of many different other countries that intervened at specific stages of the production process. Unlike what is suggested by traditional trade statistics, the trade relationship between two countries is thus more multilateral than bilateral. Classical theoretical models that explain international trade, most often based upon the aforementioned trade statistics, do not explicitly take into account the complexity of the international production process. We propose a model that reflects more the functioning of international trade today by relying upon value-added exports, a data that reports the value-added of a given country in the goods consumed by its trading partners. Our model allows us to calculate the net share of international fragmentation in the welfare gains of trade and show that it is not that high, at least compared to the gross share. We also show that the total welfare gains of trade are different than what could predict a classical trade model, especially for upstream countries which would lose less real wage from a move to autarky and for downstream countries which would lose more.

Keywords: Global value chains, Gravity model, trade costs, trade in value-added

JEL classification code: F110

1. INTRODUCTION

In 2017, the WTO reported that trade in manufactured goods represented 70 % of world trade, far ahead primary goods like agricultural and fuel and mining products. In the current era of global value chains, few countries perform the production process of these goods from the upstream to the downstream. Rather, the process is fragmented between a lot of countries so that each one is specialized on particular tasks that are realized with the highest degree of efficacy, conditionally to the fact that the gains from fragmentation exceed the costs. It means that before a good reaches its final form, it could cross the border of a country as many times as required for the completion of the production process. This situation has prompted the need of new kinds of trade statistics because the traditional strategy of recording exports as they cross borders renders difficult the identification of the true country of origin and the destination of final consumption of a good. A lot of statistical methods have thus been developed by different authors in order to solve this problem, among which we can cite Daudin et al (2011), Johnson and Noguera (2012), or Koopman, Wang and Wei (2014). These methods are used to determine the value-added of each country incorporated in the goods and services that are produced worldwide instead of the gross value of the goods. Does this change of paradigm has any implications in the way we apprehend trade between countries or regarding the gains from trade? This question is very important because most of the theoretical models that are used to explain trade between countries or even to determine the welfare gains of trade are based upon the aforementioned trade statistics. Among those, we can cite classical gravity models such as the one of Anderson and Van Wincoop (2003) or other variants such as Eaton Kortum (2002) or Chaney (2008) that are traditionally used to explain gross or final goods exports between countries. Contrarily to direct bilateral gross exports, a given country value-added exports to a particular destination depend on third countries final goods exports to this destination, and therefore, on third countries trade costs with it (Koopman, Wang and Wei 2014). The previous models that do not take into account this more complex structure of value-added exports appear to not be suitable to analyse this kind of trade flows.

To tackle this issue, Noguera (2012), proposed an approach that combines the gross trade equation with a log-linear Taylor approximation of bilateral value-added exports around a benchmark equilibrium. This gives an equation that relates bilateral value-added exports “in change” to gravity variables. Although taking into account the trade costs of third countries with the destination of final consumption, his method presents, however, the caveat to estimate

a log-linear gravity equation. In the presence of heteroscedasticity in the data as have shown Silva and Teneyro (2006), estimating this kind of log-linearized gravity models with ordinary least squares as it is done by Noguera (2012) could lead to biased parameter estimates. Besides, Noguera (2012) does not try to derive the welfare gains from trade with his model. To our knowledge, there are no papers that rely explicitly upon value-added exports to do so. We thus propose a framework that preserves the non-linearity of the gravity model and predicts the welfare gains of trade, following the exact hat algebra popularised by Dekle and al (2008). Our model is related to the work of Aichele and Heiland (2018) who derived a structural expression for value-added exports that they used to perform counterfactual analysis in general equilibrium. They do not formulate however a gravity like trade equation as we do, something that allows us to determine the net share of international fragmentation in the welfare gains of trade, but also to determine the real role played by trade costs in the process of integration into the global value chain.

These possibilities offered by our approach are valuable in two respects. On one hand, although it is acknowledged that there are gains associated with the international fragmentation of production, few authors in the literature estimate them explicitly. Ossa (2015) for instance establishes that the gains of trade are magnified by the sectoral heterogeneity in trade elasticities, the key parameters required to obtain these gains, but says nothing about fragmentation. Caliendo and Parro (2015) or Costinot and Rodriguez Clare (2014) do introduce input-output linkages in their models with gross exports as the variable of interest, something which allows them to capture in a certain manner the gross welfare gains of fragmentation¹. However, their approaches fail to explicit the net welfare gains from global value chains as it would require identifying the cost of fragmentation. Fally and Hillberry (2018) propose a sequential model of global value chains² which allows them to identify the net impact of fragmentation in the trade welfare gains of the participating countries. They show that fragmentation magnifies the trade welfare gains of downstream countries and reduces those of upstream countries in comparison to the results of a standard one stage Eaton Kortum model. In this respect, our approach is close to the work of the latter authors. Our identification of the

¹ We label gross welfare gains of fragmentation the share of the welfare gains from trade related to the introduction of tradable intermediate goods and input-output linkages in the model including the cost of fragmentation.

² Other authors like Antras and De Gortari (2017), Johnson and Moxnes (2016), Antras and Chor (2013), Yi (2003,2010) provide models that highlight the sequential arrangement of production activities across multiple countries.

net welfare gains of international fragmentation relies on the fact that we are able to identify all the trade costs required for it to occur, especially the trade costs of third countries through which the value-added of a given country passes to reach its final destination. More precisely, we calculate the trade cost of the goods indirectly exported by a given country to itself, a trade cost that appears in the welfare gains formula and attenuates the gains of trade. It is however worth to mention that contrary to the previous authors who just provide a welfare formula for a two-country case, we provide a closed form general formula for the welfare gains in the multi-country case.

On the other hand, our approach is also interesting because, as many countries anticipate that participating more in the global value chain will foster their exports and GDP growth, a lot of them are devising policies in order to stimulate their integration into the world production process. Among these policies, reducing the level of trade costs is one of their top priorities since it has been established that the role of trade costs on trade volumes is amplified in presence of vertical specialization. Yi (2003) for instance have demonstrated how a small reduction in tariffs can have magnified and non-linear effects on the growth of trade. As vertically specialized goods cross the borders of many countries or could cross the border of a single country several times, the impact of a one-percentage-point tariff reduction on trade is logically amplified. Although there is no debate on the importance of trade costs for a stronger integration into the global value chain, it is interesting to determine up to which extent or in which way these trade costs should decrease in order to induce a more significant participation in the global value chain, especially for countries facing an important level of trade costs with their bilateral partners.

Our results show that a reduction in the direct trade costs of a given country with each of its trading partners does not necessarily imply more forward participation in the global value chain. In fact, it appears that the relative trade costs of the directly exported value-added (the value-added that is sent directly to the destination of final consumption) become lower than that of the indirectly exported value-added (the value-added that is sent through third countries to the destination of final consumption), which implies an increase in value-added exports biased towards the directly exported value-added. A higher backward participation is however attested because the countries source more inputs from foreign suppliers and export relatively more final goods. Moreover, we show that the change in welfare that would imply a move from 2011 level of trade openness to a hypothetical situation of complete autarky is different when

estimated using our approach with value-added exports rather than gross exports. More specifically, it appears that these welfare losses are slightly lower for the countries that are more open when our approach is used compared to what they are with the gross trade model and higher for the less open countries. In addition, we also find as Fally and Hillberry (2018) that downstream countries feature higher welfare gains than upstream countries compared to what predicts a classical model. Finally, we show that the net share of international fragmentation in the welfare gains from trade is not as high as one could expect. In fact, turning off trade in intermediate goods would reduce welfare by 3 percentage points on average only, a figure way lower than what would suggest a classical trade model. The structure of the paper is as follows. The second section describes the model, the third and fourth sections present respectively the data with their different sources and the results of our estimations, and the last section concludes.

2. The model

In order to determine the implications of using value-added exports as the variable of interest instead of gross exports to model trade relationships between countries and to infer the welfare gains from trade, we follow a four steps approach. We begin by deriving the gravity model for value-added exports in the first sub-section. In the following, we derive the welfare formula that is used to infer the gains from trade against autarky. The third sub-section presents the method used to infer the change in the welfare gains related to any trade costs shock other than a move to autarky, and the last one the method to obtain the trade elasticity, a key parameter required to calculate the welfare gains of trade.

2.1 A gravity model for value-added exports

2.1.1 Supply side

To produce a unit of output, each country combines its value-added with inputs coming from other countries. We assume that the production function takes the form of the following Cobb-Douglas function:

$$G_i = A_i Y_i^{\alpha_i} I_i^{1-\alpha_i} \quad (1)$$

Where G_i represents total output in nominal value and where Y_i, I_i, α_i and A_i represent respectively the GDP of country “i”, the sum of the local and imported inputs used in order to produce, the share of GDP in the total output and the technology parameter of the country. The GDP of each country is by definition the sum of the value-added created by the country’s firms at each step of the production process. To create their value-added, we assume that firms use only labor. The sum of wages paid to workers thus represents the GDP. We have:

$$Y_i = w_i L_i \quad (2)$$

Where “ L_i ” represents the labor supply in country “i” and “ w_i ” is the unitary wage. As said earlier, the inputs used to obtain total output are either imported or sourced locally. I_i is therefore country “i” total demand for inputs inclusive of tax. In a perfectly competitive international market, the price of a given variety of inputs is set by the lowest cost supplier.³

We define the price of producing variety b as in Eaton and Kortum (2002):

$$p_{ij}(b) = \left(\frac{c_i}{z_i(b)} \right) t_{ij} \quad (3)$$

³ We choose to model how the inputs required to obtain total output are traded between countries using a Ricardian framework because we believe that unlike final consumers, firms are less likely to value variety in the acquisition of their inputs. Rather, a most credible assumption is that they search for the most qualitative inputs at the minimum price, and this with respect to their budget constraints.

$\frac{c_i}{z_i(b)}$ represents the unitary cost for producing variety b in country “ i ”. We assume that the production factors are mobile across activities within a country. Therefore, the cost of a bundle of factors c_i is the same for each of them. We also assume that a country’s efficiency at producing different varieties of inputs is heterogeneous, and country i efficiency at producing variety b is given by $z_i(b)$. This stems from the fact that by assumption, countries do not have the same access to technology, so that efficiency vary by commodity and country. t_{ij} represents the bilateral trade costs between country i and country j . Assuming iceberg trade costs and that the bilateral barriers obey the triangle inequality, we have $t_{ii} = 1$ and for any three countries i, j, s , $t_{ij} \leq t_{is}t_{sj}$.

The price of a given variety is therefore:

$$p_j(b) = \min\{p_{ij}(b); i = 1, \dots, N\} \quad (4)$$

With N being the number of countries. We use the same probabilistic representation of technologies as proposed by Eaton and Kortum (2002). More precisely, we assume that country i efficiency in producing variety b $z_i(b)$ is the realization of a random variable Z_i drawn for each variety independently from its country-specific probability distribution. This probability distribution is $F_i(z) = Pr[Z_i \leq z]$ which is also the fraction of varieties for which country i ’s efficiency is below z . Assuming a Fréchet distribution for the distribution of efficiencies, we have:

$$F_i(z) = e^{-Y_i z^{-\theta}} \quad (5)$$

Where $Y_i > 0$ is the country-specific state of technology parameter whose value indicates the likeliness of a good efficiency draw for any variety b . The bigger its value, the higher the likeliness of a good efficiency draw. $\theta > 0$ is not country specific. As explained by Eaton and Kortum (2002), the higher its value the lesser is the variability of efficiency draws within the countries. This parameter therefore regulates the heterogeneity of efficiencies across varieties in the countries. It follows that the probability that country i provides a variety at the lowest price in country j is⁴:

$$\pi_{ij} = \frac{Y_i(c_i t_{ij})^{-\theta}}{\Phi_j} \quad (6)$$

With $\Phi_j = \sum_{i=1}^n Y_i(c_i t_{ij})^{-\theta}$ being an index of price in country j .

⁴ For more details, see Eaton, Kortum (2002)

π_{ij} represents the share of country i in the total value of inputs demanded by country j exclusive of tax H_j ⁵. The value of country i bilateral supply of inputs or intermediate goods to country j is therefore:

$$h_{ij} = \pi_{ij} H_j \quad (7)$$

These inputs are combined with the importing country own value added to obtain total output. The latter will be used either as intermediate inputs or final goods to be exported to other countries or consumed locally. The supply price of intermediate or final goods net of trade costs defined above as c_i is given by:

$$c_i = w_i^{\alpha_i} p_i^{1-\alpha_i} \quad (8)$$

Where p_i represents the price index of imported intermediate goods. This price index is obtained from the utility function that maximize the producers $= \left[\int_0^1 h(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega \right]^{\frac{\varepsilon}{\varepsilon-1}}$ with ω a variety of intermediate good and ε the elasticity of substitution among varieties. Assuming as Eaton and Kortum (2002) that $\varepsilon < 1 + \theta$, the exact price index is:

$$p_i = \gamma \left(\sum_{j=i}^n \Upsilon_i (c_j t_{ji})^{-\theta} \right)^{-\frac{1}{\theta}} \quad (9)$$

where $\gamma = \left[\Gamma \left(\frac{1-\varepsilon+\theta}{\theta} \right) \right]^{\frac{1}{1-\varepsilon}}$ with Γ the gamma function. Unlike the previous authors however, we do not assume that consumers maximize the same utility function as firms. Rather, we suppose that consumers have a “love of variety-like” utility function which has different implications in terms of final goods price indexes. This assumption is not devoid of sense because as Antras and De Gortari (2017) have shown, the trade shares of a given country regarding bilateral intermediate goods and final goods imports are in general not the same. At this stage, we just explained how the intermediate goods required for the production of total output are traded between countries. However, our assumption regarding the utility function of final consumers leads to a different model for final goods, a model that is more similar to the standard Anderson and Van Wincoop (2003) gravity model.

2.1.2 Demand side

Specifically, we assume that region j consumers maximize the following function:

$$\left(\sum_i \beta_i^{\frac{1-\sigma}{\sigma}} f_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (10)$$

⁵ We therefore have $H_j = I_j - T_j^I$ where T_j^I is the total amount of tax and tariff revenues collected regarding intermediate goods.

Subject to the budget constraint:

$$\sum_i p_{ij} f_{ij} = D_j \quad (11)$$

Where D_j is the total demand of final goods from region j .

f_{ij} represents the consumption of the composite⁶ final goods of country i by country j , p_{ij} the price of country i composite final goods for region j consumers and β_i a positive distribution parameter. As in Anderson and Van Wincoop (2003), $p_{ij} = c_i t_{ij}$ where c_i is as before the exporter's supply price for its composite final goods net of trade costs and t_{ij} the trade cost factor between i and j . The nominal value of composite final goods exports from i to j is therefore $x_{ij} = p_{ij} f_{ij}$. A simple maximization of the utility function under the budget constraint yields:

$$x_{ij} = \frac{(\beta_i c_i t_{ij})^{1-\sigma} D_j}{P_j^{1-\sigma}} \quad (12)$$

$$\text{with } P_j^{1-\sigma} = \sum_i (\beta_i c_i t_{ij})^{1-\sigma} \quad (13)$$

and where country "i" total supply of final goods is given by $F_i = \sum_j p_{ij} f_{ij}$. It follows that:

$$F_i = \sum_j \frac{(\beta_i c_i t_{ij})^{1-\sigma} D_j}{P_j^{1-\sigma}} \quad (14)$$

As AVW, we solve for the scaled price $\beta_i c_i$ as follows:

$$(\beta_i c_i)^{1-\sigma} \frac{F_i}{D_w \sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \frac{D_j}{D_w}} \quad (15)$$

with $D_w = \sum_j D_j$ the world income.

$$\text{By defining } \Pi_i^{1-\sigma} = \sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \frac{D_j}{D_w} \quad (16)$$

We get: $(\beta_i c_i)^{1-\sigma} = \frac{F_i}{D_w \Pi_i^{1-\sigma}}$ and thus,

$$P_j^{1-\sigma} = \sum_i \left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma} \frac{F_i}{D_w} \quad (17)$$

⁶ These composite final goods are composed of value-added from different origins, including the exporting country.

Finally, we obtain:

$$X_{ij} = \frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \quad (18)$$

As it should be clear now, this supply of final goods is as total output composed of value-added from different origins, be it local or foreign. We can determine the amount of value-added that a given country exports to all its trading partners including itself as a function of this total supply of final goods. This amount is equivalent to its GDP if we define value-added in a narrow sense⁷. Let us define α_{is} as the fraction of GDP sourced by country “s” from country “i” in order to produce a unit of final good. The GDP of country “i” is equal to the sum of the value-added that it provides to each country “s”. We have:

$$Y_i = \left(\sum_{s=i}^S \alpha_{is} F_s \right) \quad (19)$$

As each country “s” exports its final goods to the countries of final consumption including itself, we can also determine the value-added exported by a given origin country “i” to a given destination of final consumption “j” as the sum of the value-added from the said origin country that is exported through final goods exports from third countries “s ∈ S”⁸ to this destination. It means that the third countries should use the value-added from the origin country to produce their composite final goods. This sum is represented in the following equation which is similar to Koopman and Al (2014):

$$v_{ij} = \left(\sum_{s=i}^S \alpha_{is} X_{sj} \right) \quad (20)$$

Where X_{sj} is defined as in equation 18.

By combining equation 18 and equation 20, it follows that:

$$\begin{aligned} v_{ij} &= \sum_{s=i}^S \frac{F_s D_j}{D_w} \left(\frac{t_{sj}}{\Pi_s P_j} \right)^{1-\sigma} \alpha_{is} \\ &= \left(\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \alpha_{ii} \right) + \left(\sum_{s \neq i}^S \frac{F_s D_j}{D_w} \left(\frac{t_{sj}}{\Pi_s P_j} \right)^{1-\sigma} \alpha_{is} \right) \end{aligned}$$

⁷ In a broader sense, value-added also include net taxes on intermediate inputs. See Timmer et al (2015) or Koopman et al (2014).

⁸ Note that unless indications are given like $s \neq i$, the set S include the origin country “i” and the destination country “j”. It means that for the case where $s = j$, the origin country exports its intermediate value added to the destination of final consumption, but this intermediate value-added is combined with the intermediate value-added of the destination country before consumption.

$$\begin{aligned}
\Rightarrow v_{ij} &= \left(\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \alpha_{ii} \right) \left(\frac{\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \alpha_{ii} + \sum_{s \neq i}^S \frac{F_s D_j}{D_w} \left(\frac{t_{sj}}{\Pi_s P_j} \right)^{1-\sigma} \alpha_{is}}{\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \alpha_{ii}} \right) \\
&= \left(\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \right) \left(\frac{\frac{F_s D_j}{D_w} \left(\frac{t_{sj}}{\Pi_s P_j} \right)^{1-\sigma} \alpha_{is}}{\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma}} \right) \\
&= \left(\frac{F_i D_j}{D_w} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \right) \left(\frac{\sum_{s=i}^S \alpha_{is} F_s \left(\frac{t_{sj}}{\Pi_s} \right)^{1-\sigma}}{F_i \left(\frac{t_{ij}}{\Pi_i} \right)^{1-\sigma}} \right) \\
\Rightarrow v_{ij} &= \left(\frac{F_i D_j}{D_w} \left(\frac{t_{ij} t_{isj}}{\Pi_i P_j} \right)^{1-\sigma} \right) \tag{21}
\end{aligned}$$

$$\text{Where } t_{isj} = \left(\frac{\sum_{s=i}^S \alpha_{is} F_s \left(\frac{t_{sj}}{\Pi_s} \right)^{1-\sigma}}{F_i \left(\frac{t_{ij}}{\Pi_i} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

This term t_{isj} is very similar to AVW multilateral resistance, except for the fact that it is associated to a bilateral relationship whereas the AVW variable is idiosyncratic. It represents a ratio between the sum of weighted relative trade costs related to the value-added indirectly exported by the origin country “i” to the destination country “j” through third countries “ $s \in S$ ” and the weighted relative trade costs of the directly exported value-added. The weights are respectively the volumes of each third country imports of intermediate value-added produced in “i” ($\alpha_{is} F_s$) for the numerator and the origin country supply of final goods for the denominator (F_i). Besides, we can see that equation 21 is nothing more than the Anderson and Van Wincoop’s gravity equation scaled by this term t_{isj} that we label “the cost of fragmentation”. As bilateral trade costs, this term exerts a negative effect on bilateral value-added exports. However, it decreases with the amount of indirectly exported value added by the origin country, that is to say $\frac{\partial t_{isj}}{\partial \alpha_{is}} < 0$ ⁹. It means that the more connected a country is to a given destination main providers of final goods via its intermediate input exports, the lesser its cost of fragmentation will be, and the higher will be its exported value-added to a given partner comparatively to a less connected country. Consequently, upstream countries, that is to say countries that have a high forward participation in a given chain of production pay a low cost of fragmentation, whereas downstream countries that have a low forward participation in

⁹ This result is quite intuitive. The closer an origin country is to a given destination country main providers of final goods, the closer this origin country is to the given destination of final consumption, and therefore, the lower are the indirect trade costs of the origin country to the destination countries through the third countries which are the providers of final goods to the destination country.

comparison to the previous, but a higher backward participation pay a higher cost of fragmentation. This term therefore measures the proximity a given country to the final consumers of another one.¹⁰ The lower it is, the further the origin country is from the final consumer and the higher its exported value-added to the country of final consumption. On the contrary, the higher it is the closer the origin country is from the final consumer, and the lesser is its exported value-added because of the foreign inputs imported from upstream countries embedded in its exports of final goods. As equation 21 shows, this term depends critically on α_{is} which is the fraction of value-added sourced by country “s” from country “i” in order to produce a unit of final good. The latter is obtained using input-output analysis. More precisely, we have:

$$\alpha_{is} = \alpha_i * B_{is} \quad (22)$$

Where α_i is as said earlier the fraction of GDP in total output, and where B_{is} is the fraction of output sourced by country “s” from country “i” in order to produce a unit of final good. It is thus an element of the input requirements matrix also known as the Leontief inverse matrix. Let A be the input-coefficient matrix obtained from an input-output table with $\frac{h_{ij}}{G_j}$ as elements; h_{ij} being the value of country i bilateral supply of intermediate goods to country j and G_j the total output of country j . The Leontief inverse is given by $B = (ID - A)^{-1}$ with I being an identity matrix. As a unit of final good is composed of value-added from different origins, the sum of the shares of value-added imported from different countries should be equal to one.

$$\sum_{i=1}^n (\alpha_i + \tau_i) B_{is} = 1 \quad (23)$$

Here country “i” value-added is defined in its broader sense and include the share of taxes on intermediate inputs in total output τ_i .

2.1.3 Total expenditures and trade balance

In equilibrium, the total expenditures of a country, also equivalent to its total output in value are equal to the sum of expenditures by firms and households. Specifically, they are given by:

$$G_j = H_j + T_j^I + E_j + T_j^E + (X_j - M_j) \quad (24)$$

Where H_j represents the total expenditures on intermediate goods (local or foreign) net of taxes and tariffs with T_j^I the tariffs revenues, E_j the total expenditures on final goods (local or foreign) net of taxes and tariffs with T_j^E the tariffs revenues, and $(X_j - M_j)$ the trade balance. $E_j + T_j^E$

¹⁰ Fally (2012) and Antras et al. (2012) also proposed indexes to measure the distance of industries to final demand or the average position of countries in global supply chains.

thus, represents the domestic absorption of country “i” and $E_j + T_j^E + (X_j - M_j)$ is equal to its GDP Y_j . Rewriting equation 24, we get:

$$G_j = \sum_i \frac{\Upsilon_i (c_i t_{ij})^{-\theta} H_j}{\Phi_j} + T_j^I + Y_j \quad (25)$$

$$\Rightarrow \sum_i \frac{\Upsilon_i (c_i t_{ij})^{-\theta} H_j}{\Phi_j} = G_j (1 - \alpha_j - \tau_j)$$

Recalling that $\alpha_j = \frac{Y_j}{G_j}$ is the share of GDP in total output and $\tau_j = \frac{T_j^I}{G_j}$ the shares of taxes on intermediate inputs in total output.

2.2 Welfare predictions

Our welfare expression is given by the real wage level in each country. To determine this real wage, we combine the trade equation for the internal flows in a given region (equation 6), with equations 8 and 9 representing respectively the unit cost of production and the price index for intermediate goods to obtain the nominal wage. The final price index which is the price of final goods is either obtained by rearranging equation 21 or equation 18 respectively for the value-added trade equation or the final goods trade equation in order to express it in terms of trade data. We have with $t_{ii} = 1$:

$$\begin{aligned} \left(\frac{\pi_{ii} \Phi_i}{\Upsilon_i} \right)^{\frac{-1}{\theta}} &= c_i = w_i^{\alpha_i} p_i^{1-\alpha_i} \\ \Rightarrow w_i^{\alpha_i} &= \left(\gamma \Phi_i^{\frac{-1}{\theta}} \right)^{\alpha_i - 1} \left(\frac{\pi_{ii} \Phi_i}{\Upsilon_i} \right)^{\frac{-1}{\theta}} \\ \Rightarrow w_i &= (\gamma)^{\frac{\alpha_i - 1}{\alpha_i}} \left(\frac{\pi_{ii} \Phi_i}{\Upsilon_i} \right)^{\frac{-1}{\theta \alpha_i}} \end{aligned} \quad (26)$$

We also have from equation 21:

$$P_i = \frac{\beta_i c_i t_{isi}}{\lambda_{ii va}^{\frac{1}{1-\sigma}}} = \frac{\beta_i t_{isi}}{\lambda_{ii va}^{\frac{1}{1-\sigma}}} \left(\frac{\pi_{ii} \Phi_i}{\Upsilon_i} \right)^{\frac{-1}{\theta}} \quad (27)$$

Where we used $(\beta_i c_i)^{1-\sigma} = \frac{F_i}{D_w \Pi_i^{1-\sigma}}$ and $\lambda_{ii va} = \frac{v_{ii}}{D_i}$

Combining equation 26 and 27, we get the following real wage equation:

$$\frac{w_i}{P_i} = (\gamma)^{\frac{\alpha_i - 1}{\alpha_i}} \left(\frac{\pi_{ii}}{\Upsilon_i} \right)^{\frac{-1}{\theta} \left(\frac{1-\alpha_i}{\alpha_i} \right)} \left(\frac{\lambda_{ii va}}{\beta_i^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \frac{1}{t_{isi}} \quad (28)$$

As we can see, the real wage decreases with internal trade be it internal trade in intermediate goods π_{ii} or internal trade in value-added $\lambda_{ii va}$ and increases with technology Υ_i . We can also see that it decreases with the trade cost of fragmentation t_{isi} . As said earlier t_{isi} summarizes

the production linkages of the origin country with all the indirect exporters “ $s \in S$ ” of its value-added. In this case, the value-added is exported as intermediate inputs in the first step by the origin country to the third countries and exported back to the origin country embedded in these third countries final goods. From this formula, we see which factors can drive a given country welfare gains from a change in trade costs for example. Expressing 28 in relative change for a change in trade costs gives:

$$\ln \frac{\widehat{w}_i}{\widehat{p}_i} = \frac{-1}{\theta} \left(\frac{1-\alpha_i}{\alpha_i} \right) \ln \widehat{\pi}_{ii} - \frac{1}{\sigma-1} \ln \widehat{\lambda}_{uva} - \ln \widehat{t}_{isi} \quad (29)$$

Where $\widehat{X} = \frac{X'}{X}$ with X' the value after the shock.

Our result is in many regards similar to that of Caliendo and Parro (2015). They have built a Ricardian model with sectoral linkages and trade in intermediate goods to determine the welfare effects associated to tariff changes. Despite the fact that their model is designed for gross exports instead of value-added exports as in our case, they find a real wage formula that exhibits 3 sources of welfare, the final goods, the intermediate goods, and sectoral linkages in the market of intermediate goods. Our real wage formula also exhibits 3 sources of welfare, notably one which is associated to the intermediate inputs $\ln \widehat{\pi}_{ii}$, another associated to the exported value-added $\ln \widehat{\lambda}_{uva}$ which depends on final goods exports as shown earlier and the linkages with the third countries ($s \in S$) that export indirectly the value-added of the origin country $\ln \widehat{t}_{isi}$. Consider for example a reduction in the level of a given country bilateral trade costs with its trading partners. This shock would reduce the import price of its intermediate inputs, which is the source of the first gain. In this case, the share of internal trade in intermediate goods π_{ii} decreases between the initial and the counterfactual equilibrium because more intermediate inputs are exported to other countries as a result of the decrease in bilateral trade costs. $\frac{-1}{\theta} \ln \widehat{\pi}_{ii}$ which represents the first source of gains is thus positive. The second source of gains $\frac{1}{1-\sigma} \ln \widehat{\lambda}_{uva}$ is also affected positively by the decrease in the level of bilateral trade costs. In fact, the share of value-added exported to itself by the given country decreases between the initial and the counterfactual equilibrium, because more value-added is exported to other countries. This implies a positive value of $\frac{1}{\sigma-1} \ln \widehat{\lambda}_{uva}$ and therefore a positive change of the given country real wage. The last source of gains $\ln \widehat{t}_{isi}$ is the change in the trade costs paid by the given origin country “i” for the inputs exported to its partners or third countries ($s \in S$), and that are exported back by them to “i” embedded in their final goods. In autarky, this term is equal to one, the lower bound trade cost when a country trade with itself. It means that decreasing the level of bilateral trade costs (a move toward free trade) should have a positive

impact on this variable and therefore, exert a negative impact on the change in the welfare gains. In spite of the mentioned similarities, our model has different welfare predictions than Caliendo and Parro's model. This is partly due to the fact that the two models lead to different predictions of the trade shares and of the trade elasticities, the two key parameters necessary to determine the welfare gains from trade. Moreover, our approach allows us to determine the net welfare gains attributed to international fragmentation ($\ln \frac{\widehat{w}_i^F}{\widehat{P}_i}$). This share is equal to:

$$\ln \frac{\widehat{w}_i^F}{\widehat{P}_i} = \frac{-\frac{1}{\theta} \ln \widehat{\pi}_i - \ln \widehat{t}_{isi}}{\ln \frac{\widehat{w}_i}{\widehat{P}_i}} \quad (30)$$

Consider for example the hypothetical situation of a move to autarky for country "i". $\frac{-1}{\theta} \ln \widehat{\pi}_i$ on one hand that would be negative represents the log change in real wage related to the fact that the country of interest could not anymore source cheap inputs from other countries in order to produce its final goods. On the other hand, $-\ln \widehat{t}_{isi}$ which would be positive represents the log change in real wage related to the trade costs that the country of interest would not have to pay anymore to send its inputs to third countries before reimporting them embedded in final goods or intermediate inputs used in the production of its final goods. With a one stage production process the log change in real wage would simply be $-\frac{1}{\sigma-1} \ln \widehat{\lambda}_{uva}$ where λ_{uva} , the share of domestic expenditures on value-added would be equal to the share of domestic expenditures on final goods (equation 18). In this regard, our results share similarities with the model of Fally and Hillberry (2018) who proposed a sequential model of global value chains. More precisely, they proposed a welfare formula for a two-country case with one country upstream, the other one downstream, and they showed that the welfare gains in presence of fragmentation are lower than without for the upstream country and higher for the downstream country. This is due to the fact the upstream country reimports its previously exported inputs to the downstream one embedded in the latter final goods exports. As this amounts to an indirect export to oneself and that welfare decreases with internal trade, this result is totally sensical. The downstream country however does not export inputs whatsoever in their framework, but sources some of its inputs from the upstream one, everything that increases its welfare. Their welfare formula is therefore suitable to analyze the net welfare gains of fragmentation, but ours is more general because it works also for a "more than two country-case" where both upstream and downstream countries import and export intermediate inputs.

2.3 Counterfactual analysis

In this section, we describe how we can use the model presented above to predict the impact of any given change in trade costs on the different variables required to infer the welfare gains, notably π_{ii} the share of domestic expenditures on intermediate goods, $\lambda_{ii_{va}}$ the share of domestic expenditures on value-added and t_{isi} the cost of fragmentation. The approach used is based upon exact-hat algebra, popularized by Dekle and al. (2008).

We have $G_i = A_i Y_i^{\alpha_i} I_i^{1-\alpha_i}$ from equation 1. I_i is also equal to $(1 - \alpha_i)G_i$ since it is the amount of intermediate inputs in total output. Expressing this equation in logarithmic form gives:

$$\begin{aligned} \ln G_i &= \ln A_i + \alpha_i \ln Y_i + (1 - \alpha_i) \ln((1 - \alpha_i)G_i) \\ \Rightarrow \ln \hat{G}_i &= \ln \hat{A}_i + \alpha_i \ln \hat{Y}_i + (1 - \alpha_i)(\ln \hat{G}_i - \ln \hat{\alpha}_i) \\ \Rightarrow \ln \hat{G}_i &= \ln \hat{Y}_i \end{aligned} \quad (31)$$

Where $\hat{X} = \frac{X'}{X}$ with X' the value after the shock. We assume that the level of technology A_i and the share of value-added in total output are fixed across equilibria as Caliendo and Parro (2015). These assumptions imply that the change in GDP is equivalent to the change in total output. As GDP is the sum of wages paid to employees as shown in equation 2, $Y_i = w_i L_i$, the change in GDP is therefore given by:

$$\ln \hat{Y}_i = \ln \hat{w}_i \quad (32)$$

From equation 6 and 7, we know that $h_{ij} = \frac{Y_i (c_i t_{ij})^{-\theta} H_j}{\Phi_j}$, where $H_j = I_j - T_j^I$ is the total demand of intermediate inputs exclusive of tax in country j. H_j is therefore also equal to $(1 - \alpha_j - \tau_j)G_j$ with $\tau_j = \frac{T_j^I}{G_j}$ as said earlier. In relative change, we would get:

$$\begin{aligned} \ln h_{ij} &= \ln Y_i - \theta \ln(c_i) - \theta \ln(t_{ij}) + \ln((1 - \alpha_j - \tau_j)G_j) - \ln \Phi_j \\ \Rightarrow d \ln h_{ij} &= \theta d \ln(c_i) - \theta d \ln(t_{ij}) + d \ln(G_j) - d \ln \Phi_j \\ \Rightarrow \ln \hat{h}_{ij} &= \ln(\hat{c}_i)^{-\theta} + \ln(\hat{t}_{ij})^{-\theta} + \ln \hat{G}_j - \ln \hat{\Phi}_j \end{aligned} \quad (33)$$

We know from equation 6 that $\Phi_j = \sum_{i=1}^n Y_i (c_i t_{ij})^{-\theta}$. It implies that:

$$\begin{aligned} \ln(\Phi_j) &= \ln \sum_{i=1}^n Y_i (c_i t_{ij})^{-\theta} \\ \Rightarrow d \ln(\Phi_j) &= \frac{\sum_{i=1}^n Y_i (-\theta) (c_i t_{ij})^{-\theta-1} d(c_i t_{ij})}{\sum_{i=1}^n Y_i (c_i t_{ij})^{-\theta}} \\ \Rightarrow \ln(\hat{\Phi}_j) &= \sum_{i=1}^n \frac{h_{ij}}{(1 - \alpha_j - \tau_j)G_j} \ln(\hat{c}_i \hat{t}_{ij})^{-\theta} \end{aligned} \quad (34)$$

$\ln \widehat{c}_i$ is the change in the unit cost of production associated to a trade costs shock. Equation 8 states that this cost is equal to $c_i = w_i^{\alpha_i} p_i^{1-\alpha_i}$. Using the same approach, we find that this change is equal to:

$$\ln \widehat{c}_i = \alpha_i \ln \widehat{w}_i + (1 - \alpha_i) \ln \widehat{p}_i \quad (35)$$

With $\ln \widehat{p}_i$ the change in the intermediate inputs price index given by:

$$\ln(\widehat{p}_i) = \frac{-1}{\theta} \ln(\widehat{\Phi}_i) \quad (36)$$

Now we can turn to the determination of the change in the input requirements following a trade costs shock. As previously mentioned, the input requirements matrix is $B = (ID - A)^{-1}$ with I an identity matrix, A the matrix of input coefficients with elements $\frac{h_{ij}}{G_j}$ and B_{ij} the elements of the input requirements matrix. From matrix algebra, we know that $ID = (ID - A)^{-1}(ID - A)$. If we define by ID_{ij} the elements of the identity matrix, it follows that the Leontief inverse can be obtained by solving $ID_{ij} = \sum_{s=1}^n B_{is} \left(ID_{sj} - \frac{h_{sj}}{G_j} \right)$. Expressing this equation in change gives:

$$\begin{aligned} dID_{ij} &= \sum_{s=1}^n \left(ID_{sj} - \frac{h_{sj}}{G_j} \right) d B_{is} + B_{is} d ID_{sj} - \frac{B_{is}}{G_j} d h_{sj} + \frac{B_{is} h_{sj}}{G_j^2} d G_j \\ \Rightarrow 0 &= \sum_{s=1}^n \left(ID_{sj} - \frac{h_{sj}}{G_j} \right) d B_{is} - \frac{B_{is}}{G_j} d h_{sj} + \frac{B_{is} h_{sj}}{G_j^2} d G_j \\ &= \sum_{s=1}^n \left(B_{is} ID_{sj} - \frac{B_{is} h_{sj}}{G_j} \right) d \ln B_{is} - \frac{B_{is} h_{sj}}{G_j} (d \ln h_{sj} - d \ln G_j) \\ \Rightarrow 0 &= \sum_{s=1}^n (B_{is} ID_{sj}) \ln \widehat{B}_{is} - \frac{B_{is} h_{sj}}{G_j} (\ln \widehat{h}_{sj} - \ln \widehat{G}_j + \ln \widehat{B}_{is}) \end{aligned} \quad (37)$$

From this variable, depends critically the change in the cost of fragmentation $\ln \widehat{t}_{isj}$ and consequently the change in bilateral value-added exports $\ln \widehat{\lambda}_{ij va}$. From equation 21, we know

that $\lambda_{ij va} = \left(\frac{\beta_i c_i t_{ij} t_{isj}}{P_j} \right)^{1-\sigma}$ with $\lambda_{ij va} = \frac{v_{ij}}{D_j}$ and $(\beta_i c_i)^{1-\sigma} = \frac{F_i}{D_w \Pi_i^{1-\sigma}}$; it follows that:

$$\begin{aligned} \ln \lambda_{ij va} &= (1 - \sigma) [\ln(\beta_i c_i) + \ln t_{ij} + \ln t_{isj} - \ln P_j] \\ \Rightarrow d \ln \lambda_{ij va} &= (1 - \sigma) [d \ln(\beta_i c_i) + d \ln t_{ij} + d \ln t_{isj} - d \ln P_j] \\ \Rightarrow \ln \widehat{\lambda}_{ij va} &= (1 - \sigma) \left[\frac{-1}{\theta} \ln(\widehat{c}_i) + \ln \widehat{t}_{ij} + \ln \widehat{t}_{isj} - \ln \widehat{P}_j \right] \end{aligned} \quad (38)$$

The change in the final goods price index $\ln \widehat{P}_j$ follows from equation 13. We therefore have:

$$\begin{aligned} 1 - \sigma(\ln P_j) &= \ln \sum_i (\beta_i c_i t_{ij})^{1-\sigma} \\ \Rightarrow d \ln P_j &= \frac{\sum_i (\beta_i c_i t_{ij})^{-\sigma} d(\beta_i c_i t_{ij})}{\sum_i (\beta_i c_i t_{ij})^{1-\sigma}} = \sum_i \frac{(\beta_i c_i t_{ij})^{-\sigma}}{P_j^{1-\sigma}} d(\beta_i c_i t_{ij}) \end{aligned}$$

By using, $\frac{\lambda_{ijva}}{t_{isj}^{1-\sigma}} = \left(\frac{\beta_i c_i t_{ij}}{P_j}\right)^{1-\sigma}$, we get

$$\ln \widehat{P}_j = \sum_i \frac{\lambda_{ijva}}{t_{isj}^{1-\sigma}} \ln(\widehat{c}_i \widehat{t}_{ij}) \quad (39)$$

Finally, the change in the cost of fragmentation $\ln \widehat{t}_{isj}$ comes from equation 21. We have:

$$\begin{aligned} t_{isj} &= \left(\frac{\sum_{s=i}^S \alpha_i B_{is} (\beta_s c_s t_{sj})^{1-\sigma}}{(\beta_i c_i t_{ij})^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \text{ with } (\beta_i c_i)^{1-\sigma} = \frac{F_i}{D_w \Pi_i^{1-\sigma}} \text{ and } \alpha_{is} = \alpha_i * B_{is} \\ \Rightarrow \ln t_{isj} &= \frac{1}{1-\sigma} \ln \left(\frac{\sum_{s=i}^S \alpha_i B_{is} (\beta_s c_s t_{sj})^{1-\sigma}}{(\beta_i c_i t_{ij})^{1-\sigma}} \right) \\ \Rightarrow d \ln t_{isj} &= \frac{1}{1-\sigma} \frac{\sum_{s=i}^S \alpha_i B_{is} \left[\frac{(\beta_s c_s t_{sj})^{-\sigma} d(\beta_s c_s t_{sj})}{(\beta_i c_i t_{ij})^{1-\sigma}} - \frac{(\beta_s c_s t_{sj})^{1-\sigma} d(\beta_i c_i t_{ij})}{(\beta_i c_i t_{ij})^{2-\sigma}} + \frac{(\beta_s c_s t_{sj})^{1-\sigma} d(\alpha_i B_{is})}{(\beta_i c_i t_{ij})^{1-\sigma} (1-\sigma) \alpha_i B_{is}} \right]}{t_{isj}^{1-\sigma}} \end{aligned}$$

We know from equation 21 that $\frac{1}{t_{isj}^{1-\sigma}} = \frac{1}{\lambda_{ijva}} \left(\frac{\beta_i c_i t_{ij}}{P_j}\right)^{1-\sigma}$

$$\Rightarrow \frac{1}{t_{isj}^{1-\sigma}} = \frac{\lambda_{sjva}}{\lambda_{ijva} t_{sSj}^{1-\sigma}} \left(\frac{\beta_i c_i t_{ij}}{\beta_s c_s t_{sj}}\right)^{1-\sigma}$$

where $P_j^{1-\sigma}$ is given by : $P_j^{1-\sigma} = \frac{t_{sSj}^{1-\sigma} (\beta_s c_s t_{sj})^{1-\sigma}}{\lambda_{sjva}}$

$$\begin{aligned} \Rightarrow d \ln t_{isj} &= \sum_{s=i}^S \frac{\alpha_i B_{is} \lambda_{sjva}}{\lambda_{ijva} t_{sSj}^{1-\sigma}} \left[\frac{d(\beta_s c_s t_{sj})}{(\beta_s c_s t_{sj})} - \frac{d(\beta_i c_i t_{ij})}{(\beta_i c_i t_{ij})} + \frac{d(\alpha_i B_{is})}{(1-\sigma) \alpha_i B_{is}} \right] \\ \Rightarrow d \ln t_{isj} &= \sum_{s=i}^S \frac{\alpha_i B_{is} \lambda_{sjva}}{\lambda_{ijva} t_{sSj}^{1-\sigma}} \left[d \ln(c_s t_{sj}) - d \ln(c_i t_{ij}) + d \ln B_{is}^{\frac{1}{(1-\sigma)}} \right] \\ \Rightarrow \ln \widehat{t}_{isj} &= \sum_{s=i}^S \frac{\alpha_i B_{is} \lambda_{sjva}}{\lambda_{ijva} t_{sSj}^{1-\sigma}} \left[\ln(\widehat{c}_s \widehat{t}_{sj}) - \ln(\widehat{c}_i \widehat{t}_{ij}) + \ln \widehat{B}_{is}^{\frac{1}{(1-\sigma)}} \right] \quad (40) \end{aligned}$$

It is interesting to analyze the condition required for a decrease in the cost of fragmentation associated to a bilateral relationship following a decrease in the level of trade costs; which would mean for the exporting country a higher forward participation to the production network of the goods bought by the importing country. For this to occur, it is necessary that the impact of a decrease in the level of trade costs regarding the indirect relationship from the origin country “i” to the destination of final consumption “j” through third countries $s \neq i \in S$ which is represented by “ $\sum_{s \neq i}^S \frac{\alpha_i B_{is} \lambda_{sjva}}{\lambda_{ijva} t_{sSj}^{1-\sigma}} \left[\ln(\widehat{c}_s \widehat{t}_{sj}) + \ln \widehat{B}_{is}^{\frac{1}{(1-\sigma)}} \right] < 0$ ”, be higher than the impact of trade costs on the direct relationship from the origin country “i” to the destination country “j” represented by “ $\frac{\alpha_i B_{ii}}{t_{isj}^{1-\sigma}} \ln \widehat{B}_{ii}^{\frac{1}{(1-\sigma)}} + \sum_{s \neq i}^S \frac{\alpha_i B_{is} \lambda_{sjva}}{\lambda_{ijva} t_{sSj}^{1-\sigma}} \left[-\ln(\widehat{c}_i \widehat{t}_{ij}) \right] > 0$ ”.

To close the model, we use the equilibrium condition defined in equation 25. Writing this condition in change gives the following equation:

$$\sum_i \frac{h_{ij}}{(1-\alpha_j-\tau_j)G_j} \ln \widehat{h}_{ij} = \ln \widehat{G}_j \quad (41)$$

Equations 31 to 41 represent the set of 11 equations and 11 unknowns that describe our model in relative change between an initial and a counterfactual equilibrium. As we can see, solving it requires mostly data that are readily observables except the trade costs indexes and the trade elasticities. As for the trade elasticities specifically, $(1 - \sigma)$ and $-\theta$, a particular attention should be given to them because of their critical role in determining the results. In the next section, we provide a straightforward framework to estimate them.

2.4 Estimation of the trade elasticities

The elasticity of import with respect to variable trade costs generally referred in the literature as the trade elasticity is a key parameter required to infer the gains from trade. Hummels and Hillberry (2013) even go so far as to say that it is the most important parameter in modern trade theory. Estimating it does not come without difficulties regarding notably the identification assumptions, as well explained by the previous authors. This is why a lot of trade theory practitioners have relied upon off-the-shelf elasticities provided by the literature. However, there is a recent trend among researchers to use their own model to estimate the key structural parameters necessary for counterfactual analysis.¹¹ We follow the same approach, but instead of specifying a different model to obtain this crucial parameter, we propose a method that allows us to obtain it directly from the parameter estimates of the gravity model without additional transformations.

Consider a simple trade costs function as the one used by Anderson and Van Wincoop (2003):

$$t_{ij} = d_{ij}^{\rho} b^{1-\delta_{ij}} \quad (42)$$

Where t_{ij} represents the trade costs factor between countries “i” and “j”, d_{ij} the distance between the two countries, ρ the average cost of travelling 1 km between 2 countries, $b - 1 > 0$ the average tariff-equivalent associated to a bilateral border barrier and δ_{ij} a dummy equal to one if the importing country is also the exporting country and 0 otherwise. From this equation, it follows that $t_{ij}^{1-\sigma} = (d_{ij}^{\rho} b^{1-\delta_{ij}})^{1-\sigma}$, where $1 - \sigma$ is the elasticity of trade with respect to trade costs. We set the average tariff-equivalent to be equal to $e - 1$ where e is for the cipher exponential. By doing so, we impose that the log difference in the tariff paid between

¹¹ Eaton and Kortum (2002), Caliendo and Parro (2015) or Ossa (2015) estimate their own elasticities to infer the welfare gains from trade.

the bilateral relationships exposed to the border effect and those who are not exposed equals 100%. It is straightforward to see this. Consider the following equation:

$$t_{ij}^{1-\sigma} = (d_{ij}^0 e^{1-\delta_{ij}})^{1-\sigma} \quad (43)$$

If $\delta_{ij} = 1$, a situation which appears, when $i=j$, $e^{1-\delta_{ij}} = 1$. If $\delta_{ij} = 0$, the situation corresponding to the case where the exporting country is different from the importing country, $e^{1-\delta_{ij}} = e$. It follows that $(\ln 1 - \ln e) = -100\%$

Log-linearizing $t_{ij}^{1-\sigma}$, we get:

$$(1 - \sigma) \ln t_{ij} = (1 - \sigma) \rho \ln d_{ij} + (1 - \sigma)(1 - \delta_{ij}) \ln e \quad (44)$$

Consequently, a 100% decrease in $(1 - \delta_{ij}) \ln e$ increases trade by $(\sigma - 1)\%$, the absolute value of the elasticity of trade with respect to trade costs. Using this method, the elasticity is perfectly identified if b is effectively equal to e . Otherwise, we would nevertheless be able to obtain critical information regarding its value. This is important because it will help us to distinguish between the elasticity of trade with respect to trade costs regarding the final goods (see equation 18) and the intermediate goods respectively (see equation 6). These values are critical in order to perform the calculations regarding the welfare gains of trade.

3 Data

This work is based on the GTAP 9 database which is a multi-country input-output table. The table comprises 140 entities which are countries or aggregations of countries and 57 sectors that we aggregate into one. Released in 2015, it has 3 base years among which we choose 2011 to conduct our analysis. We obtained our measure of value-added exports using the methodology developed by Koopman, Wang and Wei (2014). As the table is a multi-country table, imports of intermediate consumptions are not broken down by countries of origin just as final demand imports. This poses a problem because we need the complete set of bilateral intermediate and final demand imports in order to calculate the bilateral value-added exports of each country. To solve this problem, two solutions are generally used in the literature. Applying a proportionality assumption which amounts to assume that the imports of intermediate and final goods of a given country from a particular source are proportional to its total imports from this source. The second solution is to use the UN BEC classification of products by end-use category along with the UN COMTRADE database which reports bilateral exports and imports between countries at the HS 6 digits level, in order to obtain the share of intermediate and final goods in the exports of a given country to a particular destination. These shares are then applied to the export data

from the GTAP database to disentangle bilateral exports between intermediate and final goods and calculate the value-added exports. We decided to choose the second option as it is done in the seminal work of Koopman, Wang and Wei (2014). This option requires a reconciliation exercise to ensure that the new database is consistent with the previous. The exercise has been done following the quadratic mathematical programming model formulated by Hertel, Tsigas and Wang (2012).¹² It is worth to mention that our value-added exports include both goods and services. We therefore use the comprehensive database on trade in services of Francois and Pindyuk (2013) along with a preliminary draft of the UN BEC revision 5 classification by broad economic categories to perform our calculations. This revision, unlike previous ones, does a better job at distinguishing goods and services and classifying them by end-use categories. As the level of aggregation of the input-output table is very high, we are obliged to make assumptions regarding some gravity variables such as “contiguity” or “common official language”. In fact, if one country in an aggregation of countries shares a border with another country outside of the aggregated entity, it does not mean that all the entity shares a border with the said outside country. We therefore need to take this into account, and we consider arbitrarily that an aggregation of countries shares a common border with a given state if at least 80% of its component countries share a border with it. Table 1 presents some of the variables used in our estimations, their sources and the rules or methodology applied to get them. As we can see, we obtained the geographic distance between each pair of countries by using a generator built by the Centre for Biodiversity and Conservation of the American Museum of Natural History (AMNH), except for the distance of a country to itself calculated via the formula in column 3. As for the cost of fragmentation, it is obtained as follows: We estimate equation 18 to obtain a proxy of bilateral final goods exports indexes of trade costs ($t_{ij}^{1-\sigma}$) and exporters multilateral resistances ($\Pi_i^{1-\sigma}$)¹³, then we solve for t_{isj} in equation 21 with the said indexes. It is important to note that with this procedure, we do not get t_{isj} , but rather $t_{isj}^{1-\sigma}$. This implies that the effect of the obtained term on value added-exports will not be $(1 - \sigma)$ as it would have been the case if we were able to calculate t_{isj} directly, but unity instead (at least theoretically). The econometric results will make it more explicit.

¹² The GAMS code is available upon request

¹³ The exporters multilateral resistances are obtained using the properties of the PPML estimator following Fally (2015)

Table 1: Presentation of different variables used in our estimations

Variables	Source	Methodology/rule/formula
Value-added exports	Author calculations	Koopman Wang and Wei (2014)
Distance	Geographic distance Matrix generator (American museum of natural history)	Distance of an entity to itself: $d_{ii} = 0.33\sqrt{area/\pi}$
Contiguity	Cepii/Author calculations	1 if 80% of the countries of an aggregated entity share the characteristic in the first column with a given country, zero otherwise.
Common official language	Cepii/Author calculations	
Colony	Cepii/Author calculations	
Common colonizer	Cepii/Author calculations	
Regional trade agreement	(Mario Larch's Regional Trade Agreements Database)	
Cost of fragmentation (t_{isj})	Author calculations	$\left(\frac{\sum_{s=i}^S \pi_{is} Y_s \left(\frac{t_{sj}}{\Pi_s} \right)^{1-\sigma}}{Y_i \left(\frac{t_{ij}}{\Pi_i} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$

4 Results

This section will be composed of two sub-parts. The first will present the econometric results regarding the estimation of the gravity model with value-added exports, and the second the results regarding the counterfactual analysis.

4.1 Econometric results

As equation 29 in section 2 shows, five parameters are required in order to compute the welfare gains from trade. These parameters are respectively the share of internal trade in intermediate goods π_{ii} , the share of internal trade in value-added $\lambda_{ii\,va}$, the domestic cost of fragmentation t_{isi} , and the two elasticities of imports with respect to trade costs $-\theta$ and $1 - \sigma$. The first two parameters are readily observable from the data, the three remaining requiring to be estimated. In order to calculate the cost of fragmentation as in equation 21, we need proxies for the indexes of trade costs regarding the trade in final goods, recalling that bilateral value-added exports from an origin country “i” to a destination country “j” are functions of final goods exports of third countries $s \in S$. To obtain these proxies, we run a regression with final goods exports as the dependent variable (equation 18) using the following trade costs function:

$$T_{ij}^{1-\sigma} = \left[d_{ij}^{\delta_1} \cdot \exp(\delta_2 cont_{ij} + \delta_3 lang_{ij} + \delta_4 ccol_{ij} + \delta_5 col_{ij} + \delta_6 rta_{ij} + a_i border_{ij}) \right]^{1-\sigma} \quad (45)$$

Where d_{ij} represents the bilateral distance and $cont_{ij}$, $lang_{ij}$, $ccol_{ij}$, col_{ij} , rta_{ij} , $border_{ii}$ representing dummies respectively for the presence of a common border, a common official language, a common colonizer, if the country is or has been one of its partner colonies, for a regional trade agreement between partners and for the country’s trade with itself. The results of

this regression are presented in the first column of table one. It is important to mention that all the regressions in this table are done using the Pseudo Poisson maximum likelihood estimator (PPML). It means that the empirical model as regards equation 18, for example, is therefore:

$$X_{ij} = \exp (a_0 + a_1 \ln Y_i + a_2 \ln D_j + (1 - \sigma) \ln T_{ij} - (1 - \sigma) \ln \Pi_i - (1 - \sigma) \ln P_j) + \varepsilon_{ij} \quad (46)$$

We control for the exporting country GDP Y_i , the importing country demand D_j and the respective multilateral resistance terms Π_i and P_j using exporter and importer fixed-effects for each regression.

Table 1: Estimates of trade costs coefficients for different types of trade flows

VARIABLES	(1) Final goods Exports	(2) Total Value- added	(3) Direct Value- added	(4) Intermediate goods	(5) Direct Value- added	(6) Intermediate goods
ld_{ij}	-0.705*** (0.0434)	-0.714*** (0.0128)	-0.705*** (0.0183)	-0.699*** (0.0331)	-0.519*** (0.0370)	-0.411*** (0.0362)
$cont_{ij}$	0.289*** (0.0949)	0.302*** (0.0350)	0.259*** (0.0434)	0.224*** (0.0720)	0.641*** (0.120)	0.574*** (0.102)
$lang_{ij}$	0.249*** (0.0793)	0.273*** (0.0264)	0.256*** (0.0320)	0.0986 (0.0736)	0.236*** (0.0826)	0.397*** (0.0930)
col_{ij}	0.0108 (0.113)	-0.00694 (0.0343)	-0.0486 (0.0510)	0.104 (0.0907)	0.427*** (0.130)	0.339*** (0.107)
$ccol_{ij}$	0.513*** (0.195)	0.549*** (0.0554)	0.517*** (0.0969)	0.253* (0.140)	-0.193 (0.172)	0.270* (0.159)
rta_{ij}	0.209*** (0.0729)	0.220*** (0.0205)	0.183*** (0.0282)	0.195*** (0.0572)	0.489*** (0.0714)	0.374*** (0.0599)
$It_{isj}^{1-\sigma}$		1.144*** (0.0170)				
$border_{ij}$					4.833*** (0.162)	4.227*** (0.151)
Constant	18.44*** (0.443)	18.40*** (0.134)	18.47*** (0.190)	18.83*** (0.322)	15.02*** (0.393)	14.69*** (0.379)
Observations	19,321	19,321	19,321	19,321	19,321	19,321
Exporter FE	YES	YES	YES	YES	YES	YES
Importer FE	YES	YES	YES	YES	YES	YES
Border effect	YES	YES	YES	YES		

Robust standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

The second column of this table presents the regression with bilateral value-added exports as the dependent variable (equation 21). The only difference in the econometric specification

compared to the first column is $t_{isj}^{1-\sigma}$, the cost of fragmentation raised to the power of $1 - \sigma$ and obtained as said earlier from the first regression (column 1). We can see that it is significant at the 1% threshold. However, contrary to what should be theoretically, its coefficient is higher than unity. As this could suggest that multicollinearity is a problem, we also perform an estimation with the cost of fragmentation in the left-hand side of the equation, which means that the dependent variable becomes $\frac{X_{ijva}}{t_{isi}^{1-\sigma}}$. This comes down to impose that the coefficient of this term be equal to one, and also allows us to avoid potential problems related to multicollinearity. The new variable also has an economic meaning. It represents the value-added that would be directly exported to the final consumer in the destination country if there were no indirect exports. The results are reported in column 3. As we can see, the trade costs coefficients of the three first columns are quite close and consistent with the literature. The distance coefficient for example is around -0.7 as it is standard with a PPML estimation (Silva and Teneyro, 2006). All the other coefficients, except for the colony dummy are positive and significant at the 1% threshold. The results are slightly different in column 4 which reports the regression estimates regarding the intermediate goods exports occurring in the preliminary stages of the production process. It appears that sharing a common language or a common colonizer exert a lower and less significant on intermediate goods exports compared to final goods or value-added exports. The distance and regional trade agreements coefficients are however similar. We should note that the first four columns all feature dummies for each country internal trade. We don't show the coefficients in this table to save space, but as we can expect the countries tend to trade more with their selves than with foreign countries be it for intermediate, final goods or value-added exports. In columns 5 and 6, instead of including a dummy for each country internal trade, we include a dummy which is equal to one whenever the exporter and the importer represent the same country and zero otherwise. This allows us to estimate an average coefficient for the border effect. In column 5 the dependent variable is as in column 3 $\frac{X_{ijva}}{t_{isi}^{1-\sigma}}$ the directly exported value-added, and in column 6, the dependent variable is the intermediate goods exports. Although some concerns can be legitimately raised regarding endogeneity in these estimations, it is worth to note that our main objective is not necessarily to estimate consistent trade costs coefficients, but rather to determine whether there is a difference between the border effects regarding intermediate goods and the directly exported value-added. Our estimates indicate that countries tend to export more value-added directly to their selves than intermediate goods since the border effect coefficient is 4.83 in column 5 and 4.22 in column 6. We can notice that the other trade costs coefficients are different from what

they are in the previous columns which suggests that there is a potential problem of endogeneity, however, the same regressions have been conducted using ordinary least squared on log-linearized versions of the models, and we observe the same pattern in the sense that the border effect for the directly exported value-added is significantly higher than for intermediate goods. As shown in appendix 1 the trade costs coefficients do not seem to be different regardless of the way we control for the border effect, be it a dummy equal to one for each country internal trade and zero otherwise, or a dummy equal to one for all internal trade flows and zero otherwise. Analysing the border effects coefficients is interesting because as we explained in section 2.4, they can be used to recover the absolute value of the trade elasticities. The lower border effect coefficient for intermediate goods thus means that a reduction in trade costs will have a lesser impact on these trade flows compared to the others. Our finding is consistent with the work of Antràs and De gortari (2017) who suggest that the trade elasticity should be lower on average for intermediate inputs than for final goods. As the directly exported value-added is a value-added that is consumed directly in the country of destination, it is similar to final goods that are obtained at the latest stage of the production process. These parameters will play a prominent role in the following section dedicated to the counterfactual analysis. To perform this analysis, we use the coefficients in the last two columns for the trade elasticities associated respectively to the directly exported value-added and intermediate goods imports. It is worth to mention that these coefficients are very close to what is observed in the recent literature. Simonowska and Waugh (2014) for example find a range between 2.79 and 4.46, a range similar to Caliendo and Parro (2015) which goes from 3.29 to 4.49, or Donaldson (2018) which is between 3.8 and 5.2. Another thing to note is that the trade costs coefficients reported in the previous table are average coefficients and that they could in fact vary by countries or group of countries. For instance, the distance coefficient in equation 45 " δ_1 " can be interpreted as we explained in section 2.4 as the average cost of travelling 1km between 2 countries. For some countries, this cost could be higher because of the bad quality of transport infrastructures or even the time required to clear customs. As Limao and Venables (2001) or Freund and Rocha (2011) have shown, African countries face this kind of problems. The idiosyncratic share of these trade costs coefficients is in principle captured by the fixed-effects. It is however possible to recover these idiosyncratic shares using interactions between the trade costs variables and the fixed-effects. We do something similar by firstly calculating the indexes of bilateral trade costs using equation 45 and parameter estimates from table 1 excluding the idiosyncratic border effects that we control for separately. These indexes are then interacted with dummy variables distinguishing countries sharing similar characteristics. The goal is to re-estimate the models in

columns 3 and 4 of table 1 in order to get the true trade costs of the concerned group of countries. We do so for African countries, and table 2 shows that the average level of trade costs regarding African trade is higher than for the rest of world for direct value-added exports but not for intermediate goods exports. Specifically, $lt_{ij_final}^{1-\sigma}$ represents the index of trade costs that we get using the trade costs coefficients from the regression in the column 3 of table 1, and $aflt_{ij_final}^{1-\sigma}$ an interaction variable between the index and a dummy which equals to one for African trade flows and zero otherwise. The same goes for $lt_{ij_intermediate}^{1-\sigma}$ and $aflt_{ij_intermediate}^{1-\sigma}$ which are associated with the column 4 of table 1.

Table 2: African countries' level of trade costs

VARIABLES	(1) Direct value-added exports	(2) Intermediate goods exports
$lt_{ij_intermediate}^{1-\sigma}$		1.018*** (0.0231)
$aflt_{ij_intermediate}^{1-\sigma}$		0.198 (0.166)
$lt_{ij_final}^{1-\sigma}$	0.997*** (0.0142)	
$aflt_{ij_final}^{1-\sigma}$	0.251*** (0.0529)	
Constant	18.44*** (0.112)	19.00*** (0.171)
Observations	19,321	19,321
Exporter FE	YES	YES
Importer FE	YES	YES
Border effect	YES	YES

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The dependent variables remain the same as in both columns and the regressions are done using the PPML estimator with importer and exporter fixed effects. It appears that the level of trade costs is on average 25% higher for African countries trade flows compared to the rest of the world as regards direct value-added exports, and not significantly higher as regards intermediate goods exports. We will use these results in the counterfactual analysis to determine whether reducing the level of trade costs in Africa to the average in the rest of the world could improve the participation of these countries in the global value chain.

4.2 Counterfactual analysis results

In order to determine the implications of estimating the welfare gains of trade using a model with value-added exports rather than gross exports, we performed two counterfactual exercises featuring two different trade costs shocks. The first trade costs shock is a move from 2011 levels of trade openness to autarky for all the countries in the world, and the second a 20% reduction of the trade costs indexes regarding African trade flows.¹⁴ We first analyse the differences in predictions between our model based upon value-added exports and a model with gross exports which takes into account trade in intermediate goods but not sectoral linkages. The welfare formula regarding the latter is a special case of equation 29 in Costinot and Rodriguez Clare (2014) or equation 7 in Ossa (2015) and is equal to:

$$\ln \frac{\widehat{w}_i}{\widehat{p}_i} = \frac{-1}{\alpha_i} \ln \widehat{\lambda}_{ug} - \frac{1}{\sigma-1} \ln \widehat{\lambda}_{ug} \quad (47)$$

Where $\lambda_{ii,g}$ represents the share of domestic expenditures on gross exports and α_i the value-added to gross output ratio. Table 3 presents summary statistics on the results regarding the move to autarky. The detailed results are available in appendix 2. This table is composed of 5 parts, the first presenting the results for the entire set, the two following respectively for the countries the less open of the sample with an imports of value-added penetration ratio of less than 23% (ratio of domestic value-added exports over GDP > 77%) and for the most open countries with an imports of value-added penetration ratio of more than 40 %, and the two last respectively for the most downstream countries and the most upstream countries in the production process. The first 2 rows in each part of the table represent respectively the welfare gains obtained using the model with gross exports (equation 47) and the welfare gains using our approach with value-added exports (equation 29). The 3 following rows represent the gross and net shares of the welfare gains related to international fragmentation. For the model with

gross exports (G), the share is obtained using $1 - \frac{\frac{1}{\sigma-1} \ln \widehat{\lambda}_{ug}}{\ln \frac{\widehat{w}_i}{\widehat{p}_i}}$. For value-added exports (VA), the

net share is obtained using $\frac{\frac{-1}{\theta} \left(\frac{1-\alpha_i}{\alpha_i} \right) \ln \widehat{\pi}_{ii} - \ln \widehat{\tau}_{isi}}{\ln \frac{\widehat{w}_i}{\widehat{p}_i}}$, and the gross share just $\frac{\frac{-1}{\theta} \left(\frac{1-\alpha_i}{\alpha_i} \right) \ln \widehat{\pi}_{ii}}{\ln \frac{\widehat{w}_i}{\widehat{p}_i}}$. Finally, the

last 2 rows represent respectively the share of domestic expenditures on gross exports, and the share of domestic expenditures on value-added exports. It is important to note that the latter share is equal to the ratio of domestic value-added exports with the total imports of final goods. It is this share that is used to infer the welfare gains. As we do not impose equilibrium for the

¹⁴ Unlike the shock related to autarky, the second shock requires to solve the system of equations presented in section 2.3. We do so by using GAMS. The code is available upon request.

trade balances, the total imports of value-added can be different from GDP. As regards the “upstreamness” indexes we follow Fally and Hillberry (2018) who calculate them as a measure of the distance of each country to final demand. More precisely, we have:

$$U_i = 1 + \varphi_{ii}U_i + \sum_{i \neq j} \varphi_{ij}U_j \quad (48)$$

Where U_i is the upstreamness index of country “i” and φ_{ij} denotes the share of output from country “i” that is needed to produce one unit of output in country “j”.

Table 3: The welfare gains of trade (Autarky)

Entire set				
Variable	Obs	Mean	Min	Max
W_gross exports	139	-11,56%	-43,31%	-1,88%
W_value-added exports	139	-13,59%	-56,70%	-2,76%
Gross share of fragmentation (G)	139	50,80%	21,06%	66,71%
Gross share of fragmentation (VA)	139	65,68%	30,64%	96,14%
Net share of fragmentation (VA)	139	21,81%	6,57%	55,70%
Ratio of domestic value-added exports over GDP > 0,77				
Variable	Obs	Mean	Min	Max
W_gross exports	38	-5,25%	-7,42%	-1,88%
W_value-added exports	38	-8,33%	-28,00%	-2,76%
Gross share of fragmentation (G)	38	47,20%	21,06%	66,59%
Gross share of fragmentation (VA)	38	65,88%	35,13%	96,14%
Net share of fragmentation (VA)	38	17,27%	7,45%	35,38%
Ratio of domestic value-added exports over GDP < 0,6				
Variable	Obs	Mean	Min	Max
W_gross exports	39	-20,08%	-43,31%	-12,78%
W_value-added exports	39	-19,71%	-56,70%	-5,02%
Gross share of fragmentation (G)	39	53,22%	29,07%	66,71%
Gross share of fragmentation (VA)	39	67,37%	30,64%	86,91%
Net share of fragmentation (VA)	39	27,85%	7,35%	55,70%
Upstreamness < 1,7				
Variable	Obs	Mean	Min	Max
W_gross exports	20	-8,46%	-21,05%	-1,88%
W_value-added exports	20	-17,04%	-36,27%	-4,72%
Gross share of fragmentation (G)	20	47,05%	21,06%	58,56%
Gross share of fragmentation (VA)	20	61,34%	35,13%	96,14%
Net share of fragmentation (VA)	20	22,10%	7,45%	37,16%
Upstreamness > 2,4				
Variable	Obs	Mean	Min	Max
W_gross exports	20	-17,94%	-35,22%	-6,41%
W_value-added exports	20	-12,94%	-31,54%	-5,02%
Gross share of fragmentation (G)	20	50,43%	29,07%	66,71%
Gross share of fragmentation (VA)	20	60,55%	30,64%	86,91%
Net share of fragmentation (VA)	20	19,85%	7,35%	44,09%

Fally and Hillberry (2018) calculate a sectoral “upstreamness” index and obtain the aggregate index by calculating a weighted average where the weights represent the share of each sector in the exports of the country. As said in section 3 that describe the data, we use an interregional input-output matrix that do not features sectoral linkages within and between countries but only aggregate trade linkages, since we aggregated the 57 original sectors of the GTAP database into a unique sector. Our index is therefore not sectoral, and we don’t need to apply a weighting scheme to get the aggregate index.

As the table shows, on average, a move to autarky would reduce real wage by 11,56 % if we follow the model with gross exports, and by 13,59 % if we follow the model with value-added exports. These results seem quite close, however, the correlation between the two models’ results is only 76 %, which means that there are differences. Among these differences, it appears that the welfare losses for the countries that are less open are higher by 3.08 percentage points on average with value-added exports than with gross exports, whereas they are just slightly lower, less than 0.5 percentage points on average for the more open countries. It means that the gains from trade are understated for the less open countries when the model used is based upon gross exports and very slightly overstated for the more open countries. A result that is also worth mentioning is that the welfare gains of trade are higher (8.58 percentage points on average) for downstream countries with the value-added exports model compared to the gross exports model. On the contrary, the gains for the most upstream countries are 5 percentage points lower. This result relates as said earlier to the work of Fally and Hillberry (2018) who built a sequential model of international fragmentation and also find that downstream countries feature higher welfare gains compared to the prediction of a standard model of trade. The difference is that the model that they use as a benchmark for comparison is a model of trade without intermediate goods flows. Contrarily to them, we compare our model predictions to a standard trade model featuring intermediate goods flows. This benchmark is the relevant one because our model allows back and forth trade in intermediate goods contrarily to theirs. As regards the net welfare gains of fragmentation, it appears that they are not as high as one could expect. In fact, our model predicts that turning off trade in intermediate goods would only reduce the average real wage by 3 percentage points, with 5.5 percentage points on average for the more open countries and 1.5 percentage point for the less open countries. This is because the net share related to fragmentation represents only 21.81 % on average of the welfare reduction as it is shown in the fifth row of the first part of the table. Paradoxically, the gross share of the welfare gains related to fragmentation is higher (14.88 percentage points on

average) with our approach compared to the gross exports approach as it is shown in the rows 3 and 4 of the first part of the table. It represents 65,68% of the total gains with our model compared to 50,80% with the standard model. There is anyway a striking difference between the gross and net shares of the welfare gains related to fragmentation. This substantiates the necessity to take into account the trade costs associated to international fragmentation that would disappear because of autarky in the calculation of the net welfare gains of global value chains, as our model allows us to do. More precisely, because of autarky, the trade costs that are paid when a given country exports its intermediate goods to third countries and imports them back embedded in the latter final goods exports “ t_{isi} ” would not be borne anymore, which would attenuate the welfare losses. The evidence that the gross welfare gains from international fragmentation are high but largely compensated by its costs has a trivial implication. Reducing significantly the cost of fragmentation could drastically improve the gains from trade. It is thus interesting to determine the conditions under which this reduction could be achieved.

Table 4: The welfare gains of trade (20% decrease of African trade costs)

Country	Welfare	Fragmentation cost	Internal trade
TGO	16,07%	4,63%	-10,91%
BEN	14,58%	4,88%	-10,08%
GIN	12,02%	4,07%	-10,88%
MUS	10,53%	4,56%	-10,66%
TUN	9,91%	5,46%	-12,92%
SEN	9,62%	4,83%	-13,06%
MOZ	9,58%	2,50%	-12,25%
ZWE	9,09%	4,17%	-13,16%
MAR	8,88%	5,77%	-14,80%
TZA	8,70%	4,15%	-9,37%
GHA	8,46%	4,24%	-10,59%
KEN	8,32%	5,37%	-13,74%
ZMB	8,18%	5,09%	-16,75%
MWI	7,46%	4,64%	-13,76%
CIV	7,20%	4,08%	-11,39%
EGY	6,39%	4,86%	-11,72%
NAM	6,33%	4,01%	-12,22%
MDG	6,32%	4,48%	-12,40%
ZAF	6,14%	5,00%	-12,45%
UGA	6,12%	3,40%	-10,01%
CMR	5,95%	3,84%	-10,45%
ETH	5,81%	4,05%	-9,32%
BFA	5,29%	3,06%	-8,39%
BWA	5,27%	2,68%	-8,18%
RWA	4,90%	3,08%	-8,11%
NGA	4,33%	1,74%	-3,98%

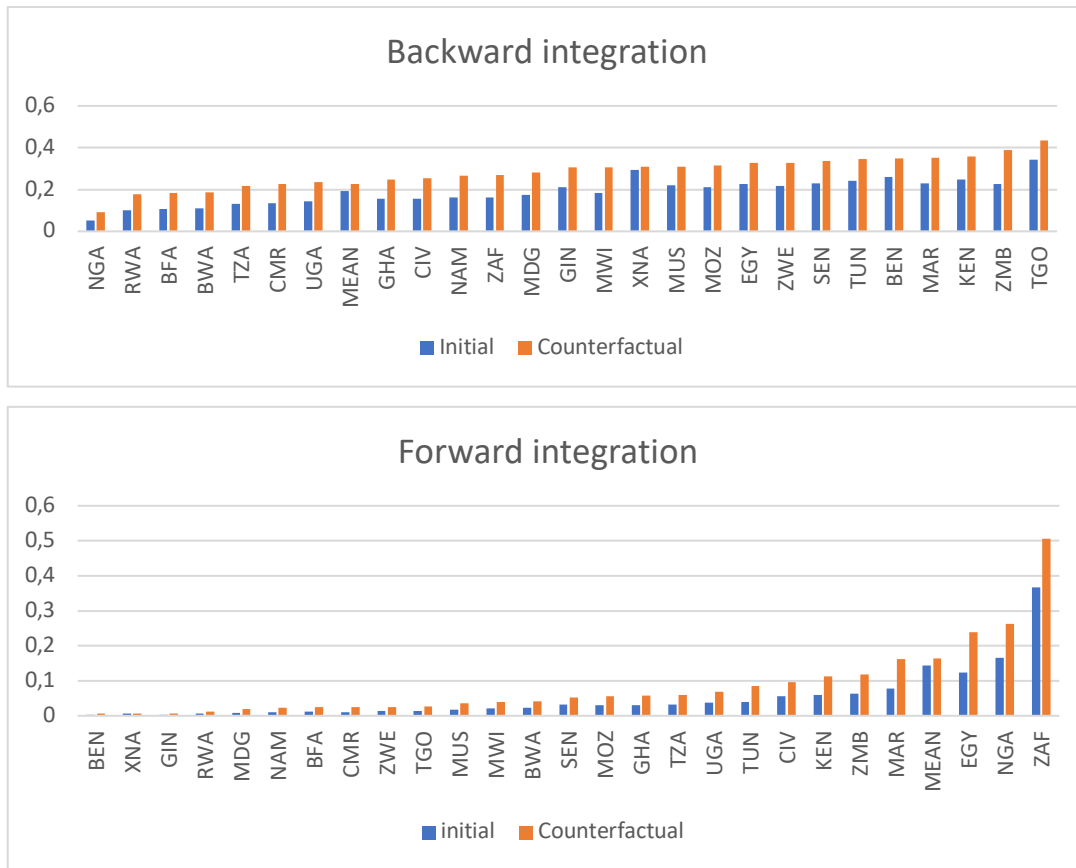
To do so, we simulate the impact of a decrease on the level of African countries' direct bilateral trade costs¹⁵ which are among the countries that participate the least to the global value chain. We showed in the econometric results that these trade costs are around 25 % higher for direct value-added exports compared to the rest of the world, even if for intermediate goods exports, no significant difference could be identified. We therefore simulate what a 20% reduction could mean for African countries in term of welfare, and specifically in term of participation to the global value chain. Table 4 presents the results, with the first column representing the change in real wage, the second column the average change in the cost of fragmentation, and the last column the change in the share of value added traded internally. As we can expect, a 20 % reduction in the level of African countries' direct bilateral trade costs would increase real wage by as much as 16% for small open economies like Togo or as much as 4 % for relatively closed large economies like Nigeria. What is surprising however is that the cost of fragmentation would increase on average. This seems paradoxical, but a closer look on the mathematical

expression of this term in equation 21 : $t_{isj} = \left(\frac{\sum_{s=i}^S \pi_{is} Y_s \left(\frac{t_{sj}}{\pi_s} \right)^{1-\sigma}}{Y_i \left(\frac{t_{ij}}{\pi_i} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$ shows that it can be

interpreted as a weighted average of the relative trade costs of the indirectly exported value-added with respect to the relative trade cost of the directly exported value-added. Thus, when direct bilateral trade costs decrease (t_{ij}), the trade costs of third countries " t_{sj} " which are not African remain constant. As a consequence, the relative trade cost of the indirectly exported value-added become higher than that of the directly exported value-added, and the increase in exports is biased towards the directly exported value-added. However, there are instances where this cost decreases, especially for intra-African trade. This is natural since the trade costs of third countries with them do decrease. It leads us to the conclusion that for a country to have a higher forward participation in the global value chain, the trade costs of the main buyers of its intermediate goods with the countries of final consumption should decrease because the only trade costs shock that could affect the indirectly exported value-added leaving unaffected the directly exported value-added is a decrease in the trade costs of these third countries. The last result that highlights table 4 is a reduction in the share of value-added exchanged inside a country following a decrease in trade costs. As shown in the table, the share of internal trade in value-added decreases for all the countries, which means that they trade more with foreign countries.

¹⁵ The trade costs that are paid when they export or import directly a good from a given country.

Chart 1: Participation of selected countries in the global value chain



The idea is confirmed by the chart above that presents the change in the share of value-added exported as inputs (forward integration) and imported (backward integration) caused by the trade costs shock. This chart suggests that the countries are in general more connected to the global production network compared to their initial situation. However, as shown earlier in table 4, the directly exported value-added increases more than the value-added exported as inputs. It means that forward integration is higher in absolute terms but less pronounced than before in relative terms. Backward integration, nevertheless, unambiguously grow.

5. CONCLUDING REMARKS

The goal of this paper was to determine the implications of using value-added exports as the trade statistic characterizing international trade instead of gross exports. We firstly proposed a gravity model that takes into account the current functioning of the world production process organized as a global value chain in order to fit the structure of value-added exports. This model has then been used to calculate the welfare gains of trade. Our findings show that using a gravity model based upon value-added exports to calculate the welfare gains of trade give different results than the model based upon gross exports. To summarize, we firstly show that the reduction in real wage that would imply a transition to autarky is lower using our approach than the traditional one for upstream countries, and higher for downstream countries and countries that are less open in terms of the imports in value-added penetration ratio. Secondly, we show that the net welfare gains from international fragmentation are not very high, at least compared to the gross gains that could be inferred from a traditional model. As our model allows us to deduct from the welfare losses that would imply the shutdown of international fragmentation the costs of fragmentation, the welfare losses are thus reduced. Finally, we show that reducing the level of a country's bilateral trade costs with its trading partners do not necessarily implies more forward participation in the global value chain. In fact, unless the reduction in trade costs affects more the indirectly exported value-added than the directly exported one, the increase in exports would be biased towards the latter, which implies a weaker forward participation in relative terms to the global production network. Backward integration, however, undoubtedly increase, and the countries are closer to the final consumers than before. This conclusion has interesting implications in term of trade policies since a lot of countries that barely participate in the global value chains are looking after the benefits of being more integrated into the global production network.

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7. APPENDICES

7.1 Appendix 1: Border effect coefficients, OLS regressions

VARIABLES	(1) Direct Value- added	(2) Intermediate goods	(3) Direct Value- added	(4) Intermediate goods
ld _{ij}	-0.725*** (0.00599)	-0.684*** (0.0147)	-0.696*** (0.00621)	-0.650*** (0.0145)
cont _{ij}	0.541*** (0.0253)	0.976*** (0.0591)	0.577*** (0.0268)	1.016*** (0.0596)
lang _{ij}	0.296*** (0.0128)	0.309*** (0.0314)	0.308*** (0.0136)	0.321*** (0.0316)
col _{ij}	0.134*** (0.0309)	0.366*** (0.0698)	0.146*** (0.0328)	0.377*** (0.0706)
ccol _{ij}	0.444*** (0.0153)	0.322*** (0.0381)	0.441*** (0.0162)	0.320*** (0.0385)
rta _{ij}	0.215*** (0.00976)	0.240*** (0.0240)	0.246*** (0.0103)	0.281*** (0.0241)
border _{ij}			6.106*** (0.0465)	5.454*** (0.102)
Constant	18.42*** (0.0774)	18.68*** (0.183)	18.09*** (0.0809)	18.30*** (0.182)
Observations	19,321	16,260	19,321	16,260
R-squared	0.975	0.894	0.972	0.890
Reporter FE	YES	YES	YES	YES
Partner FE	YES	YES	YES	YES
Border effect	YES	YES		

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

7.2 Appendix 2: Detailed results, trade and welfare

COUNTRY	Autarky		20% decrease African trade costs		
	Welfare Gross exports	Welfare Value-added exports	Welfare 20% (VA)	Fragmentation cost	Internal trade
NGA	-6,37%	-4,72%	4,33%	1,74%	-3,98%
RWA	-5,71%	-6,13%	4,90%	3,08%	-8,11%
BWA	-10,43%	-6,39%	5,27%	2,68%	-8,18%
BFA	-9,42%	-6,74%	5,29%	3,06%	-8,39%
ETH	-3,28%	-6,98%	5,81%	4,05%	-9,32%
CMR	-5,16%	-7,29%	5,95%	3,84%	-10,45%
ZAF	-7,27%	-7,78%	6,14%	5,00%	-12,45%
UGA	-7,14%	-8,11%	6,12%	3,40%	-10,01%
NAM	-11,40%	-8,47%	6,33%	4,01%	-12,22%
MDG	-7,89%	-8,61%	6,32%	4,48%	-12,40%
CIV	-11,48%	-9,13%	7,20%	4,08%	-11,39%
MWI	-8,67%	-10,58%	7,46%	4,64%	-13,76%

Appendix 2(continued)

COUNTRY	Autarky		20% decrease African trade costs		
	Welfare Gross exports	Welfare Value-added exports	Welfare 20% (VA)	Fragmentation cost	Internal trade
EGY	-6,17%	-10,69%	6,39%	4,86%	-11,72%
TZA	-6,43%	-11,00%	8,70%	4,15%	-9,37%
GHA	-7,15%	-11,08%	8,46%	4,24%	-10,59%
ZMB	-16,37%	-12,65%	8,18%	5,09%	-16,75%
MAR	-9,05%	-13,14%	8,88%	5,77%	-14,80%
KEN	-6,24%	-13,31%	8,32%	5,37%	-13,74%
SEN	-7,20%	-14,47%	9,62%	4,83%	-13,06%
MOZ	-11,96%	-14,79%	9,58%	2,50%	-12,25%
ZWE	-6,84%	-14,99%	9,09%	4,17%	-13,16%
TUN	-13,44%	-15,72%	9,91%	5,46%	-12,92%
MUS	-12,20%	-16,40%	10,53%	4,56%	-10,66%
GIN	-10,43%	-18,30%	12,02%	4,07%	-10,88%
BEN	-5,73%	-28,00%	14,58%	4,88%	-10,08%
TGO	-16,82%	-36,27%	16,07%	4,63%	-10,91%
BRA	-2,76%	-2,76%	0,14%	0,66%	-0,41%
ARG	-4,40%	-3,74%	0,08%	0,48%	-0,18%
JPN	-3,61%	-3,96%	0,10%	0,86%	-0,19%
USA	-2,84%	-4,05%	0,21%	1,00%	-0,59%
COL	-4,14%	-4,27%	0,07%	0,96%	-0,14%
VEN	-6,50%	-4,37%	0,08%	1,15%	-0,16%
PRI	-3,05%	-4,45%	0,20%	1,09%	-0,60%
AUS	-5,18%	-4,68%	0,14%	1,01%	-0,36%
QAT	-16,70%	-5,02%	0,13%	0,95%	-0,25%
IRN	-8,35%	-5,12%	0,11%	0,76%	-0,09%
RUS	-6,11%	-5,18%	0,10%	1,04%	-0,17%
XSE	-3,89%	-5,61%	0,07%	0,91%	-10,48%
PER	-7,13%	-5,64%	0,15%	0,95%	-0,25%
IDN	-6,14%	-5,91%	0,12%	0,89%	-0,19%
CHN	-6,51%	-5,98%	0,29%	0,88%	-0,71%
PAK	-3,46%	-6,36%	0,18%	0,52%	-0,35%
KAZ	-10,80%	-6,40%	0,16%	1,22%	-0,38%
XEC	-6,40%	-6,92%	5,49%	3,50%	-10,06%
NPL	-1,91%	-6,92%	0,13%	1,03%	-0,18%
XNF	-8,44%	-6,98%	5,72%	2,57%	-9,01%
CAN	-6,98%	-7,05%	0,17%	1,13%	-0,40%
NZL	-7,55%	-7,07%	0,16%	0,80%	-0,38%
URY	-6,15%	-7,14%	0,22%	0,98%	-0,53%
MEX	-7,90%	-7,36%	0,10%	1,02%	-0,18%
NOR	-9,62%	-7,46%	0,16%	1,11%	-0,34%
IND	-4,62%	-7,47%	0,49%	1,02%	-1,19%
AZE	-15,41%	-7,57%	0,13%	1,08%	-0,14%
ITA	-7,12%	-7,87%	0,37%	0,94%	-0,93%
ECU	-7,16%	-7,89%	0,11%	1,06%	-0,09%
BGD	-6,50%	-7,91%	0,16%	0,92%	-0,31%
FRA	-6,84%	-7,94%	0,36%	0,82%	-0,89%
BOL	-7,46%	-7,96%	0,11%	1,09%	-0,16%
XSU	-9,97%	-8,03%	0,12%	1,10%	-0,21%

Appendix 2(continued)

COUNTRY	Autarky		20% decrease African trade costs		
	Welfare Gross exports	Welfare Value-added exports	Welfare 20% (VA)	Fragmentation cost	Internal trade
XSC	-11,98%	-8,36%	6,76%	4,20%	-0,15%
KWT	-20,96%	-8,40%	0,26%	0,79%	-0,24%
ESP	-7,27%	-8,45%	0,50%	1,04%	-1,22%
TUR	-5,61%	-8,46%	0,49%	1,18%	-1,24%
XAC	-13,70%	-8,59%	6,61%	2,65%	-10,46%
CHL	-9,50%	-8,78%	0,16%	1,04%	-0,32%
GTM	-7,11%	-8,91%	0,09%	0,99%	-0,09%
SAU	-16,78%	-8,93%	0,31%	0,57%	-0,47%
XEA	-11,28%	-8,97%	0,16%	1,05%	-0,35%
DOM	-6,37%	-9,09%	0,10%	0,94%	-0,09%
GBR	-7,23%	-9,11%	0,27%	0,90%	-0,55%
LKA	-5,18%	-9,18%	0,14%	0,86%	-0,12%
XWS	-10,95%	-9,19%	0,33%	0,99%	-0,53%
XER	-5,94%	-9,24%	0,19%	0,97%	-0,34%
BRN	-16,29%	-9,35%	0,16%	1,09%	-0,19%
XCF	-15,43%	-9,36%	7,25%	3,33%	-9,75%
ARM	-4,73%	-9,96%	0,16%	1,15%	-0,24%
PRY	-8,45%	-10,20%	0,11%	0,76%	-0,16%
ISR	-8,71%	-10,28%	0,24%	1,13%	-0,39%
PHL	-7,77%	-10,63%	0,12%	1,00%	-0,18%
FIN	-10,00%	-10,79%	0,17%	0,89%	-0,32%
XCB	-7,92%	-10,85%	0,18%	0,91%	-0,32%
DEU	-11,66%	-10,96%	0,26%	0,90%	-0,41%
OMN	-18,79%	-11,01%	0,26%	0,79%	-0,16%
SWE	-11,73%	-11,04%	0,19%	0,86%	-0,33%
PRT	-9,22%	-11,13%	0,64%	1,02%	-1,57%
XSM	-10,75%	-11,61%	0,33%	1,18%	-0,64%
SLV	-7,68%	-11,64%	0,12%	0,99%	-0,08%
HRV	-10,55%	-11,67%	0,18%	0,90%	-0,30%
GRC	-7,54%	-12,06%	0,28%	1,09%	-0,40%
NLD	-12,99%	-12,06%	0,35%	1,01%	-0,69%
ROU	-9,78%	-12,11%	0,19%	0,88%	-0,31%
TTO	-23,75%	-12,26%	0,69%	0,60%	-1,66%
POL	-10,11%	-12,34%	0,18%	1,11%	-0,30%
CHE	-14,20%	-12,60%	0,51%	1,06%	-1,10%
XSA	-4,25%	-12,85%	0,28%	1,04%	-0,11%
LAO	-10,22%	-13,04%	0,14%	1,00%	-0,20%
CRI	-16,08%	-13,66%	0,13%	1,07%	-0,16%
DNK	-13,66%	-13,71%	0,22%	1,06%	-0,36%
BHR	-16,65%	-14,19%	0,25%	0,01%	-0,39%
KOR	-13,04%	-14,26%	0,22%	1,08%	-0,26%
GEO	-5,61%	-14,57%	0,21%	1,28%	-0,15%
AUT	-14,07%	-15,05%	0,20%	1,05%	-0,30%

Appendix 2(continued)

COUNTRY	Autarky		20% decrease African trade costs		
	Welfare Gross exports	Welfare Value-added exports	Welfare 20% (VA)	Fragmentation cost	Internal trade
ARE	-13,28%	-15,14%	0,33%	0,84%	-0,41%
UKR	-13,41%	-15,42%	0,23%	1,10%	-0,31%
HND	-13,94%	-15,46%	0,14%	1,11%	-0,20%
JAM	-9,42%	-15,72%	0,20%	1,02%	-0,28%
ALB	-9,29%	-15,95%	0,31%	1,17%	-0,42%
NIC	-15,54%	-16,81%	0,15%	1,04%	-0,09%
XEF	-14,39%	-16,88%	0,37%	1,06%	-0,68%
XOC	-11,49%	-18,40%	0,23%	1,13%	-0,28%
BLR	-11,59%	-18,50%	0,25%	1,20%	-0,36%
TWN	-22,11%	-18,50%	0,28%	1,03%	-0,45%
LVA	-12,26%	-18,85%	0,21%	1,18%	-0,25%
HKG	-18,11%	-19,55%	0,28%	1,19%	-0,42%
CZE	-20,78%	-20,37%	0,21%	1,03%	-0,30%
BGR	-17,57%	-21,41%	0,27%	0,98%	-0,35%
SVN	-18,08%	-21,45%	0,30%	1,15%	-0,43%
LTU	-16,13%	-21,56%	0,29%	1,12%	-0,32%
MYS	-23,59%	-21,58%	0,32%	1,10%	-0,52%
XCA	-26,44%	-21,59%	0,41%	0,95%	-0,78%
THA	-21,25%	-21,61%	0,26%	0,74%	-0,34%
XNA	-13,81%	-22,59%	0,53%	1,24%	-0,89%
CYP	-15,00%	-22,91%	0,38%	0,97%	-0,44%
JOR	-11,96%	-23,00%	0,64%	1,05%	-1,02%
MNG	-21,39%	-23,43%	0,19%	1,21%	-0,19%
SVK	-22,95%	-24,41%	0,23%	1,14%	-0,28%
HUN	-25,84%	-24,96%	0,21%	0,86%	-0,23%
IRL	-35,78%	-25,30%	0,32%	1,18%	-0,42%
XEE	-10,73%	-25,82%	0,35%	1,27%	-0,51%
KHM	-23,66%	-27,06%	0,20%	1,01%	-0,18%
EST	-21,77%	-27,24%	0,40%	1,25%	-0,57%
BEL	-23,13%	-28,66%	0,52%	1,11%	-0,67%
VNM	-22,34%	-29,64%	0,29%	0,78%	-0,30%
PAN	-14,85%	-30,60%	0,21%	1,00%	-0,16%
SGP	-31,27%	-31,54%	0,30%	1,13%	-0,25%
KGZ	-12,78%	-32,67%	0,28%	1,17%	-0,25%
XWF	-16,83%	-33,37%	17,43%	5,60%	-11,54%
MLT	-32,59%	-50,98%	0,38%	1,00%	-0,15%
LUX	-44,00%	-56,70%	0,35%	1,07%	-0,18%