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Systemic Sovereign Risk in Europe: an MES and CES Approach

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Abstract:

We transpose the concept of systemic risk measurement used in the financial literature to the sovereign debt crisis. We base our analysis on two systemic risk measures, the Marginal Expected Shortfall (MES) and the Component Expected Shortfall (CES), that are estimated by a Dynamic Conditional Correlation model (DCC) and by non parametric techniques. We use daily data on government bonds yields $10Y$ and quarterly sovereign debts over the period $2001-2013$ for eleven Eurozone countries. Our results allow us to identify the countries that have the highest contribution to systemic risk and to perform comparisons in terms of countries' riskiness within the Eurozone.

Keywords: Systemic risk, sovereign bonds, debt crisis.

JEL classification: G12, G62, H63.

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1 Introduction

With the set-up of the European Monetary Union (EMU), all member countries benefited from similar borrowing rates as their bonds were considered to be equally risky. This apparent convergence in interest rates went along with a competitiveness divergence. The financial crisis has further deepened the differences in competitiveness between countries and emphasized the difficulties encountered by some Eurozone members to raise funds in order to finance their increasing public spending.[1] As a consequence, the risk premia of these countries increased. The financial crisis, associated to these financing difficulties and to country-specific factors,[2] led to a sovereign debt crisis in Europe, rising borrowing costs for weaker countries.

In this paper, we focus on the sovereign debt crisis as it is specific to the European case (*i.e.* single currency and heterogeneous fiscal policies) in contrast to the financial crisis that had a world dimension. Moreover, "the sovereign debt crisis in the Eurozone has become systemic" (Tabellini, 2011), justifying the need for more careful analysis. In this context, we consider that identifying the countries that are more fragile to risk and that might default is crucial. In our analysis, this would imply first identifying countries' contribution to systemic risk, and then establishing a ranking able to reveal the systemically important Eurozone members.[3]

Our baseline assumption is that, in an integrated monetary area, such as the Eurozone, an initial crisis emerging at the area level, on the bonds market, due for example to a loss of confidence of investors or a rating downgrade of some euro area members, would create high pressure on the economic situation of individual countries leading to higher public debt. Within this framework, we focus on systemic risk caused by sovereign debt. We define it as *the failure of countries, in stress times, to meet their obligations to creditors that would have significant adverse consequences for the financial system − the bond market in particular − as well as negative externalities to the rest of the economy.* On this basis, we determine which countries contribute the most to systemic sovereign risk. A systemically important country is defined as *a country that needs to raise a large amount of funds and/or faces high bond yields when the system is already under stress.*

[1] Governments had to spend more public funds in order to stabilize the financial sector affected by the crisis.

[2] Several country-specific factors leading to unsustainable levels of public debt can be distinguished. In Ireland and Spain − countries that had low levels of public debt before the crisis − the *faulty mortgage loans* deepened the financial sector crisis, and consequently led to the sovereign debt crisis. For Greece and Italy, *public spending* has always been a matter of concern, as their debt-to-GDP ratio exceeded 100% at all times since the creation of the EMU. The situation in Portugal was different: its debt-to-GDP ratio has continuously increased since 2001 and exceeded the 60% (*i.e.* Maastricht public debt criterion) in 2006. The problems in Portugal were mainly due to *large public spending on economic stimuli* during the crisis.

[3] Our analysis is performed on data concerning 11 Eurozone countries, namely the EMU first members (except Luxemburg) plus Greece who joined the area in 2001.

Identifying these countries would allow authorities to detect the members whose eventual payment default might impose higher stress for the entire system. Furthermore, this ranking could be a useful tool for the regulators if the Stability and Growth Pact were to be revised. It could also help to establish a better surveillance mechanism and eventually to impose penalties on countries that infringe regulation.

We identify systemically important countries by employing an approach used in the financial literature. We choose to apply the same methodology as our research questions are similar to those faced when evaluating the systemic risk contribution of financial institutions. For the financial sector, a representative systemic risk measure is the Marginal Expected Shortfall (MES) proposed by Acharya et al. (2012). The MES is defined as the expected equity loss of a firm when the overall market declines beyond a given threshold over a given time horizon. Another measure based on the MES is the Component Expected Shortfall (CES) proposed by Banulescu and Dumitrescu (2014). The CES allows decomposing the risk of the aggregate financial system while accounting for institutions' characteristics. These two measures are complementary: when the system is already under stress, the MES captures the marginal contribution of a firm to the overall market decline, while the CES its absolute contribution. Their advantages are twofold: they can be computed in a simple way on publicly available data and their results can be easily interpreted.

We adapt these systemic risk measures to sovereign debt risk and transpose this methodology initially developed for market risk at country level. To the best of our knowledge, this is the first attempt to do so. In our case, the MES can be defined as *the expected increase in a particular government bond yield when the overall (European) government bonds market is under stress, capturing the marginal contribution of countries to the system distress*. The countries with the highest MES contribute the most to the market decline (*i.e.* to the overall risk of the Eurozone), being the greatest drivers of systemic risk. As regards the CES, it is used to assess *the absolute contribution of a country to systemic risk at a precise date*. We thus focus on the relation between the system and each country in particular and do not aim at studying contagion effects among countries.

The rest of the paper is structured as follows. Section 2 presents the literature review. In section 3, we describe the systemic risk measures and the econometric methodology employed to compute them. In section 4, based on the above two measures, we perform an analysis of systemic risk in the euro area and determine which countries impose the higher risk to the system. Section 5 concludes.

2 Literature Review

Our work is related to two strands of the literature: the financial literature on systemic risk and the sovereign debt literature. Within the vast literature on systemic risk, the measures constructed on market data that capture the relation between institutions and the market are of particular interest to us. In this literature, systemic risk measures have already been defined (Adrian and Brunnermeier, 2011; Acharya et al., 2012; Brownlees and Engle, 2012; Banulescu and Dumitrescu, 2014), analyzed (Drehmann and Tarashev, 2011; de Bandt et al., 2013) and compared (Gauthier et al., 2012; Rodriguez-Moreno and Pena, 2013).

Systemic risk measures can be defined in different ways. Acharya et al. (2012) introduce the Marginal Expected Shortfall (MES) as a measure of how a firm's risk-taking adds to the overall system risk. Using the MES developed by Acharya et al. (2012), Brownlees and Engle (2012) implement a multistep econometric model to determine the marginal contribution of financial firms to systemic risk[4]. The MES is also employed in the construction of the Component Expected Shortfall proposed by Banulescu and Dumitrescu (2014). It is obtained by calibrating the MES with the market capitalization of each financial institution. A different measure relying on the Value-at-Risk approach, is proposed by Adrian and Brunnermeier (2011). This measure, labeled CoVaR, captures the system's losses when a particular financial institution is in distress. They also define the contribution of the institution to systemic risk, ΔCoVaR, as the difference between its CoVaR and the CoVaR calculated at the median state.

According to Drehmann and Tarashev (2011), these measures can be classified either as top-down or bottom-up. The top-down measures "start with the risk of the system and allocate it to individual institutions". The MES and the CES belong to this group. The bottom-up measures "start with distress at a particular institution and then compute the associated level of system-wide distress". This category includes the CoVaR.

de Bandt et al. (2013) provide also a classification of systemic risk measures. They distinguish between indicators focusing on institutions, among which the MES, and indicators related to interconnectedness and to financial infrastructures and sectors.

In order to check the validity of systemic risk measures, several articles compare these measures or their performances. Gauthier et al. (2012) compare bank capital requirements issued from five systemic risk measures (component and incremental value-at-risk, Shapley values, the ΔCoVaR and the MES) with two benchmarks based on Basel II requirements. They find that a macroprudential approach to financial

[4] This index, called SRisk, is based on the MES, the leverage and the size of each financial institution. This measure initially developed for the US, has been expanded by to European financial institutions. They find that the riskiest financial companies are located in France and UK.

regulation, based on systemic risk measures, diminishes the default probabilities of banks. Benoit et al. (2013) compute the MES, SRISK and CoVaR using a common estimation framework, and compare, both theoretically and empirically, these measures. They find that these measures identify different financial firms as systemically important. Moreover, they reveal that using market risk measures or the level of liabilities would lead to the same rankings as the ones provided by these systemic risk measures, proving their limited capacity to capture the complex nature of systemic risk. A direct comparison of systemic risk measures is performed by Rodriguez-Moreno and Pena (2013). In their paper, market-based high-frequency macro and micro measures[5] are compared using three criteria: causality, Gonzalo and Granger metric and correlation with an index of systemic events. Their results show that the measures based on credit default swaps are the most reliable systemic risk indicators. Although they consider a large panel of systemic risk measures, they do not analyze other simple indicators like the MES.

In our paper, we propose a novel approach to address systemic risk at country level. For this we focus on two top-down measures, MES and CES. These indicators can provide a thorough analysis of systemic risk in Europe as they are complementary. Transposed to countries, they can lead to the construction of useful instruments for the regulators and help to establish eventual penalties for governments. Previous theoretical literature on sovereign debt concentrated on the incentives faced by sovereigns to repay their debt (Dooley and Svensson, 1994; Cole and Kehoe, 2000; Dooley, 2000). Empirical work on the sovereign debt issue usually searched to study the link between sovereign spreads and credit risk, liquidity risk and aggregate risk (Pan and Singleton, 2008; Longstaff et al., 2011). Credit and liquidity risk can be associated to a country risk perspective as in De Santis (2012). De Santis (2012) underlines that sovereign bonds yields dynamics can be linked to country-specific, aggregate and contagion risks. The latter has been extensively analyzed in the litterature (Afonso et al., 2014; Antonakakis and Vergos, 2013; Arghyrou and Kontonikas, 2012; Baum et al., 2014; Caceres et al., 2010). However, as previously argued, in this paper we choose not to focus on contagion effects between countries. Instead, we concentrate on aggregate risk and on country-specific risk, as the widening in EMU spreads is mainly driven by increased global risk and, to a certain extent, by country specific problems (Arghyrou and Kontonikas, 2012).

Aggregate risk can be linked to changes in monetary policy, global uncertainty and risk aversion (De Santis, 2012). In this process, the financial system can transform aggregate risk into sovereign risk through two channels (Gerlach et al., 2010). On the one hand, in periods of financial distress, governments might have to recapitalize

[5] These measures are: the LIBOR spreads, the principal component analysis (PCA) of portfolios of CDS spreads, the systemic factor extracted from the CDS indexes (CDX and iTraxx) and their tranches, the systemic risk index (SI) based on structural credit risk models, the multivariate densities and the aggregate of individual co-risk measures.

banks by using public funds which might worsen their fiscal situation. On the other hand, shortages in banking liquidity diminish credit to the private sector leading to a decrease in consumption and private investment, thus in production, and therefore to further fiscal imbalances. As national banking and financial sectors have different structures, sizes and degrees of exposure to global financial conditions, an increase in a common aggregate risk factor can cause a heterogeneous impact on countries bond yields. Moreover, Arghyrou and Kontonikas (2012) also put forward that during the crisis, markets penalized fiscal and other macroeconomic imbalances (*e.g.* excessive current accounts) much more heavily than they used to prior to the crisis. Attinasi et al. (2010), Sgherri and Zoli (2009), Mody (2009), Barrios et al. (2009), Gerlach et al. (2010) and Hagen et al. (2011) have all underlined the role of the aggregate risk during the crisis.

Country specific risk can be driven by changes in the default probability on the sovereign debt, in credit and liquidity factors (De Santis, 2012), as well as in competitiveness (Arghyrou and Kontonikas, 2012). Country risk and sovereign risk may mutually reinforce each other. A default on the sovereign debt can have a negative impact on a country's capital flows and its external private debt, leading to high country risk. Political and economic factors negatively influencing country risk may trigger rating downgrades impairing governments to obtain funds for repaying public debt. Over the long term, a convergence between country and sovereign risk can be expected (Canuto et al., 2012).

In our paper, both aggregate and country risk are treated. However, a stronger focus will be put on aggregate risk, as the systemic event appears at the area level. Economic difficulties characterizing a single country are not seen in our study as a systemic event *per se*. However, if these problems imply a high increase in the country's bond yield it might be possible that the bond index for the whole region rises, triggering eventually a systemic event. Consequently, a country risk might become an aggregate risk.

3 Methodology

In this section, definitions of the concepts used are first provided. Second, we present the econometric methodology employed to estimate the MES and to compute the CES: we briefly introduce the DCC model (Engle, 2002) and the nonparametric tail expectation estimators (Scaillet, 2005).

3.1 Definitions

The MES was conceived as a systemic risk measure for the financial sector and is defined as the marginal contribution of a financial institution to systemic risk. In our framework, we define the MES[6] as being the marginal contribution of a country to the overall risk of the Eurozone. To determine the formula for the MES, we follow the steps of Brownlees and Engle (2012) and make the necessary adjustments to transpose this measure to sovereigns. Consequently, we replace financial returns with nominal government bond yields for a 10-year maturity for each country in our sample. To derive the MES, first we need to construct a bond index, which also requires the definition of a weighting scheme, and then we define a measure for the aggregate risk of the system. The index allows us to specify a crisis event taking place at the area level. We build the virtual Eurobonds index r_{mt} as an average of each member state bond yield weighted by the level of its public debt in the total public debt of the Eurozone:

$$r_{mt} = \sum_{i=1}^{N} w_{it} r_{it}, \qquad with \qquad w_{it} = \frac{B_{i,t-1}}{\sum\limits_{i=1}^{N} B_{i,t-1}}, \tag{1}$$

where m stands for the market, r_{it} represents the bond yield of country i at time t, w_{it} the weight of country i in the market index and N the number of countries in the sample. The weights used in the construction of the index are calculated as the public debt of country i, $B_{i,t-1}$, divided by the total public debt of the eleven Eurozone countries considered in the analysis. We consider that at each date t the only information available on public debt is the one from date $t-1$. This implies that weights for date t are known in $t-1$, *i.e.* $\mathrm{E}_{t-1}(w_{it}) = w_{it}$. This index can be viewed as the yield that would apply to Eurozone bonds if such an instrument existed. The idea underlying its construction is that the interest paid on the mutualized debt of all countries should be equal to the sum of interest paid on the N individual debts:

$$r_{mt} \sum_{i=1}^{N} B_{i,t-1} = \sum_{i=1}^{N} r_{it} B_{i,t-1}. \tag{2}$$

The choice of the weights is essential in our analysis as they have two major roles: they are included in our bond index (equation 3) and they are also used to construct the CES measure (equation 10). By analogy with the literature devoted to backtesting and to risk model validation, two strategies can be employed in the selection of countries' weights. First, the weights can be defined as the historical relative share of a country in the total debt. This type of weighting scheme implies calculating the weights at specific time moments (*i.e.* quarterly in our case). This approach is used by Brownlees and Engle (2012), Acharya et al. (2012) and Banulescu and Dumitrescu (2014) to measure

[6] For comparison reasons, we choose to keep the expression "Marginal Expected *Shortfall*" when talking about sovereigns, although this concept was initially constructed for financial returns. However, in our framework, the MES is not a measure of expected loss for an equity investor in a financial firm if the overall market declined substantially, but a measure of the expected financing conditions for a country in crisis times.

the systemic risk of financial institutions. The second strategy, derived from the financial literature on risk management analysis, consists in keeping the weights constant during the analyzed period. We choose to adopt the former approach in our empirical analysis and to consider the latter for robustness check only.

Another important issue is the choice of the macroeconomic variable used in the weighting scheme. Given that we analyze sovereign systemic risk, public debt seems the more intuitive variable for the construction of weights. Moreover, we opt for this variable as it is a good indicator not only of the size of countries, but also of the difficulties they may encounter in repaying their debt. Additionally, the weights based on public debt verify a condition that is fundamental in the construction of our index:

$$\sum_{i=1}^{N} w_{it} = 1 \tag{3}$$

Expected Shortfall (ES)

We consider that aggregate risk is measured by the conditional Expected Shortfall (ES)[7] defined as the expected market bond yield conditional on this yield exceeding a given threshold, C. The main change with respect to the traditional ES, constructed on financial returns, is the nature of the distribution. We are no longer in a profit and loss distribution, where extreme losses in the financial market represent the systemic event. The conditioning event is, in the case of sovereigns, a situation where the whole area faces a difficulty to raise funds, *i.e.* high bond rates, needed to finance the mutualized debt. The ES is defined by the following relation:

$$ES_{m,t-1}(C) = \mathrm{E}_{t-1}(r_{mt}|r_{mt} > C). \tag{4}$$

Due to the fact that systemic events are situated in the right tail of the bond yield distribution, the value we choose for the threshold, C, is the historical Value-at-Risk at a 10% confidence level computed for the yields of the market. We consider the realization of the condition $r_{mt} > C$ as being the systemic event, or more exactly, an area-wide event that influences all countries at the same time. Consequently, a sole country being in trouble does not represent a systemic event for our analysis. This can only be the case if the bond yields for this country are so high that they trigger an increase in the market index as well. We argue that during the Eurozone debt crisis it was not a single country that caused the crisis, but several countries showed signs of weaknesses for various reasons (see footnote 1).

[7] This choice is based on risk measures properties. According to the criteria defined by Artzner et al. (1999), the ES is a coherent risk measure as opposed to the Value-at-Risk, for example.

Marginal Expected Shortfall (MES)

The MES is defined as the partial derivative[8] of the system's ES with respect to the weight of country i in the Eurozone:[9]

$$MES_{it}(C) = \frac{\partial ES_{m,t-1}(C)}{\partial w_{it}} = \frac{\partial \sum_{i=1}^{N} w_{it}\mathrm{E}_{t-1}(r_{it}|r_{mt} > C)}{\partial w_{it}} = \mathrm{E}_{t-1}(r_{it}|r_{mt} > C). \quad (5)$$

Two interpretations can be given to the MES at this stage. First, if we consider the partial derivative expression, we can define the MES as the change in the ES (risk) of the system when the weight of a given country, i, increases by one unit. In other words, the MES shows how the risk of the whole market evolves when a country increases its public debt. Second, the MES can also be viewed as the bond yield that would be applied to country i when the system is under stress, *i.e.* the bond yield index exceeds the threshold value. In particular, the MES gives the rate that would be demanded for country's i bonds if the Eurozone experienced a systemic event.

To compute this measure, we use a bivariate process of bond yields for each country i and the system. This allows us to consider not only individual volatility of bond yields, but also the correlations of each bond yield with the market. These correlations are particularly important when studying systemic risk, as they account for the interconnections between countries' yields. Stationarity tests for our data reveal that all series are integrated of order 1, therefore data is considered as a random walk:

$$r_{jt} = r_{j,t-1} + z_{jt}, \qquad with \quad j = \{i, m\}, \quad (6)$$

where z_{jt} is an innovation process such that $\mathbb{E}_{t-1}(z_{jt}|r_{j,t-1}) = 0$ and $\mathbb{E}(z_{jt}) = 0$. The bivariate Garch takes the form:

$$\begin{aligned} z_{it} &= \sigma_{it}\varepsilon_{it} \\ z_{mt} &= \sigma_{mt}\varepsilon_{mt}, \end{aligned} \quad (7)$$

where σ_{mt} and σ_{it} represent the volatilities of innovations for the bond yield index and the country's bond yield, respectively. For the computation of the MES, we express the

[8] Details are provided in Appendix A.

[9] To simplify notations, we use MES_{it} instead of $MES_{it|t-1}$, but it should be understood as the MES computed at time t given the information available at time $t-1$.

innovation as[10]:

$$\varepsilon_{it} = (\rho_{imt}\varepsilon_{mt} + \sqrt{1-\rho_{imt}^2}\xi_{it}) \tag{8}$$

where $\varepsilon_{jt} = (\varepsilon_{mt} \quad \xi_{it})'$ is the vector of independently and identically distributed shocks with zero mean and identity covariance matrix and ρ_{imt} represents the conditional correlation of the bond yield index with country i. The MES is hence computed as:

$$\begin{aligned} MES_{it}(C) &= \mathrm{E}_{t-1}(r_{it} \mid r_{mt} > C) \\ &= r_{i,t-1} + \mathrm{E}_{t-1}(z_{it} \mid z_{mt} > C - r_{m,t-1}) \\ &= r_{i,t-1} + \sigma_{it}\rho_{imt}\mathrm{E}_{t-1}(\varepsilon_{mt} \mid \varepsilon_{mt} > \tfrac{C-r_{m,t-1}}{\sigma_{mt}}) + \sigma_{it}\sqrt{1-\rho_{imt}^2}\mathrm{E}_{t-1}(\xi_{it} \mid \varepsilon_{mt} > \tfrac{C-r_{m,t-1}}{\sigma_{mt}}), \end{aligned} \tag{9}$$

Component Expected Shortfall (CES)

The second systemic risk measure that we use in our analysis is the percentage Component Expected Shortfall (CES%) recently proposed by Banulescu and Dumitrescu (2014). We use this measure to evaluate the absolute contribution of each country (in percentage) to the overall risk at time t and compute it as follows:

$$CES\%_{it}(C) = \frac{\partial ES_{t-1}(C)}{\partial w_{it}} \frac{w_{it}}{ES_{t-1}(C)} = \frac{w_{it}MES_{it}(C)}{\sum_{i}^{N} w_{it}MES_{it}(C)}. \tag{10}$$

One attractive feature of this measure is that the sum of CES% of all countries in the sample adds up to 100%. This facilitates the interpretation of results. The larger the CES% of a country, the more systemically important this country is. The CES% actually represents a decomposition of the system's risk (measured by the ES) while taking into account the debt weight of each country, thus the country's size. The higher the public debt of a country, the higher the impact on the system if the country is left to default: the "Too Big To Fail" concept used for financial institutions can therefore be easily transposed to countries. All together, the CES% captures both size and correlation effects.

3.2 Estimation framework

A bivariate DCC is used to model the volatility and the correlations between the innovations of the bond yield index, z_{mt}, and of the yields of each country, z_{it}. According to

[10] For more details on this writing, see appendix B.

this model, the conditional covariance matrix is decomposed as follows:

$$H_t = D_t R_t D_t, \tag{11}$$

where D_t represents a 2-by-2 diagonal matrix of volatilities and R_t denotes the time-varying correlation matrix. As usual, a two step approach is used to estimate this model.

In the first stage, a univariate asymmetric Garch, namely the GJR Garch(1,1), is considered for each of the eleven countries and for the market index. The asymmetric model allows us to take into account the leverage effect, *i.e.* an increase in yields, due for example to macroeconomic announcements about employment or inflation levels, tends to increase yields volatility more than a drop of the same magnitude. Our choice of an asymmetric Garch model is also motivated by the recent literature underlying that bond returns are characterized by asymmetric volatility (Goeij and Marquering, 2006). The conditional variance is given by:

$$\sigma_{it}^2 = \omega_i + \alpha_i z_{i,t-1}^2 + \gamma_i z_{i,t-1}^2 \mathbb{I}_{(z_{i,t-1}>0)} + \beta_i \sigma_{i,t-1}^2, \tag{12}$$

$$\sigma_{mt}^2 = \omega_m + \alpha_m z_{m,t-1}^2 + \gamma_m z_{m,t-1}^2 \mathbb{I}_{(z_{m,t-1}>0)} + \beta_m \sigma_{m,t-1}^2, \tag{13}$$

where $\mathbb{I}_{(\cdot)}$ denotes the indicator function. For simplicity, in the estimation process, we suppose that $\omega_i = (1 - \alpha_i - \beta_i - 0.5\gamma_i)\bar{\sigma}_{it}^2$, with $\bar{\sigma}_{it}^2$ the non conditional variance of the series.[11] The model parameters, α, γ and β, are estimated using the Quasi Maximum Likelihood approach (QML).

In the second stage, we express the dynamic conditional correlation matrix as:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}, \tag{14}$$

where Q_t is the pseudo conditional correlation matrix of the innovations standardized by their conditional standard deviation obtained previously, $\varepsilon_t^* = \frac{z_{it}}{\sigma_{it}}$. To compute Q_t, we introduce these standardized innovations into a DCC(1,1) model:

$$Q_t = (1 - \alpha_C - \beta_C)S + \alpha_C \varepsilon_{t-1}^* \varepsilon_{t-1}^{*'} + \beta_C Q_{t-1}, \tag{15}$$

where ε_t^* is the vector of standardized innovations, S is the unconditional covariance matrix of ε_t^* and α_C and β_C the DCC parameters to be estimated by QML. The conditions for the positive definiteness of H_t can be found in Engle and Sheppard (2001). Consistency and asymptotic normality conditions based on the work of Newey and McFadden (1994)

[11] Symmetrically, we impose the same specification for the market, that is, $\omega_m = (1 - \alpha_m - \beta_m - 0.5\gamma_m)\bar{\sigma}_{mt}^2$, where $\bar{\sigma}_{mt}^2$ represents the non conditional variance.

can also be found in Engle and Sheppard (2001). To obtain robust standard errors, the covariance matrix is computed as $A^{-1}BA^{-1}T^{-1}$, where A represents the analytic hessian and B the covariance of the scores.

The final elements needed to compute the MES are tail expectations. For this, we use a non parametric kernel estimation. The methodology that we apply follows Scaillet (2005), with two notable differences. First, in our case, the conditioning with respect to past information is not necessary since we apply the formula on standardized residuals that are independent and identically distributed. Second, the sign in the conditioning event is reversed. The tail expectations are:

$$\widehat{E}_h(\varepsilon_{mt}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \varepsilon_{mt} \, \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}, \tag{16}$$

$$\widehat{E}_h(\xi_{it}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \xi_{mt} \, \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}, \tag{17}$$

with $\Phi(x)$, the normal cumulative distribution function (Gaussian Kernel function) and h, a positive bandwidth. κ is the cutoff point given by $(C - r_{m,t-1})/\sigma_{mt}$.[12] For the determination of the bandwidth, we follow Scaillet (2005) and fix its value at $T^{-1/5}$ times the empirical standard deviation, equal to 1 in our case.

Conditional variances and correlations from the DCC model together with tail expectations are inserted in equation (9) to compute the MES. The latter is used in the construction of the CES. The results obtained using this methodology applied on European data are presented and analyzed in the next section.

4 Data and Results

In this section, we first provide the descriptive statistics related to our data, then we analyze the estimation results of the DCC model and finally, we study our systemic risk measures, namely the CES and the MES.

4.1 Descriptive statistics and estimation results

The empirical analysis focuses on the contribution of Eurozone countries to the systemic risk generated in the ongoing debt crisis. It is conducted using daily data on 10-year nominal sovereign bond yields for the period January, 2001 − December, 2013 (closing

[12] For more details on the formulas, see appendix C.

prices)[13] and quarterly data on public debt for the same time span. Eleven Eurozone countries[14] are analyzed in order to determine the evolution in time of their systemic risk contribution.

Table 1 summarizes the descriptive statistics of the market index, country bond yields and country weights. If we compare the minimum, maximum or average yields of all countries, the Greek yields are always the highest. The average yield for the market index is equal to 4.14%, being lower in value than the one of any periphery country (Greece, Ireland, Italy, Portugal and Spain).[15]. Furthermore, there is a rather small volatility[16] in the series of yields, Greece being the most volatile in the panel. At the beginning of the analyzed period, all countries' bond yields were extremely close, around 5%, proving that a European convergence of the long-term government bond yields was achieved, encouraged by the EMU creation. Despite this convergence in interest rates, divergences in competitiveness between countries were still present.[17] With the emergence of the debt crisis at the end of 2009, the divergences deepened as the sovereign yields of several periphery countries soared. Countries responded to the financial crisis by taking strict budgetary measures and offering rescue funds to the financial sector, measures that had the downside of increasing public debt. In addition, the problems at the euro area level, related to the balance of payment and to competitiveness divergences, aggravated the overall situation. As investors lost confidence in some countries ability to repay their debt, high levels of debt were sanctioned by higher yields. For example, during the debt crisis, the bond yield applied to Greece reached a maximum of 34.33%, whereas for Portugal and Ireland the maximum yield attained 16.21% and 14.10%, respectively.

The last row of Table 1 shows the weight of each country in the market index as of 2013Q3.[18] Germany, France and Italy stand for almost 70% of the total debt of the zone, while those of the PIGS countries – Portugal, Ireland, Greece and Spain – account for less than 20% (*i.e.* less than the weights of the three biggest countries taken individually). For example, Italy has almost a quarter of the Eurozone debt, whereas Greece less than 5% of the total debt. Thus, the weights of these two countries

[13] The main source for the data is the Macrobond database.

[14] The countries considered are: Germany, France, Italy, Spain, Ireland, Greece, Portugal, Belgium, Netherlands, Austria and Finland. These countries correspond to ten out of the eleven countries that initially formed the Eurozone. The eleventh country, Luxembourg, was omitted from our sample due to missing data. Greece, who joined the area in January 2001, before the introduction of notes and coins, was added to the sample.

[15] As measured by their GDP in 2013.

[16] The volatility is computed here as the standard deviation of each series.

[17] In Southern-European countries, as the cost of borrowing diminished, capital inflows lead essentially to consumption and housing booms, making these countries less competitive. On the contrary, Germany became more competitive, exporting its products to the periphery and undertaking reforms to restructure its economy after the reunification (Battistini et al., 2013).

[18] Recall that we use a time-varying weighting scheme. For the evolution in time of these weights refer to Figure $A1$ in Appendix D.

would suggest that Greece does not have the size to be considered a major threat for the other economies, whereas Italy can be viewed as a TBTF country. Based on this, we can infer that the funds needed to rescue Italy would be a lot greater that the rescue package already offered to Greece. Nevertheless, this preliminary reasoning should be further refined and improved by examining the results of systemic risk measures.

Table 1: Descriptive statistics of the series

	Market index	Belgium	Germany	Ireland	Greece	Spain	France	Italy	Netherlands	Austria	Portugal	Finland
Avg Yield	4.14	3.96	3.43	4.98	7.60	4.48	3.74	4.55	3.64	3.77	5.50	3.65
Max	5.45	5.80	5.27	14.10	34.33	7.49	5.34	7.64	5.39	5.47	16.21	5.47
Min	2.72	1.91	1.02	2.98	3.21	3.02	1.67	3.20	1.49	1.51	3.10	1.38
Vol	0.58	0.78	1.07	1.68	5.78	0.74	0.83	0.66	0.97	0.91	2.38	1.01
Corr	-	0.86	0.57	0.45	0.12	0.59	0.73	0.75	0.62	0.71	0.29	0.63
Skw	0.08	-0.43	-0.57	2.10	2.08	0.74	-0.35	1.01	-0.49	-0.49	1.95	-0.49
Kur	2.36	2.98	2.35	7.80	6.84	3.54	2.56	4.85	2.44	2.72	6.09	2.49
Weights 2013	-	4.44	23.70	2.28	3.53	10.63	21.28	23.02	4.92	2.68	2.35	1.17

Note: The table presents descriptive statistics for the bond yield index and for each country in our sample. The results are based on daily data on bond yields over the period January, 2001 - December, 2013. The last row of the table presents the weights of each country in the market index, in 2013, 3rd quarter. These weights are computed as the debt of each country divided by the total debt of all countries.

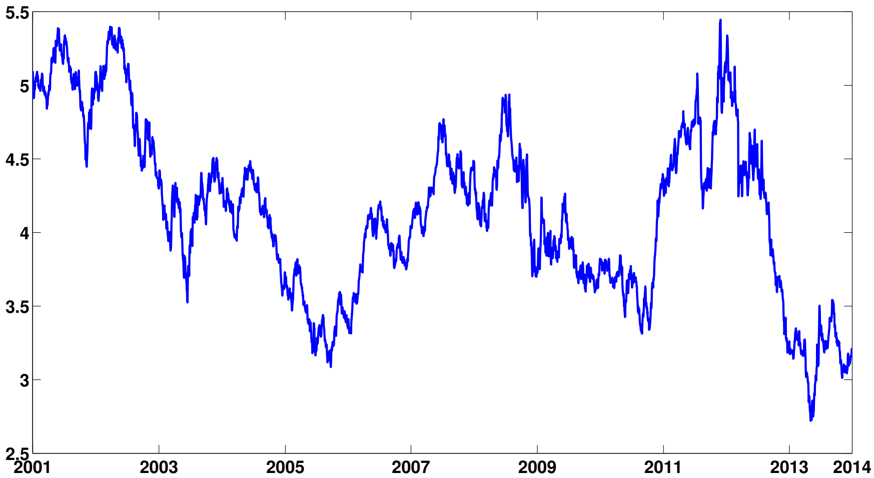


Figure 1: Government Bond Yield Index

Figure 1 provides the evolution of the market index. The bond yield for the Eurozone started increasing in the second half of 2010 and reached a peak at the end of 2011. This captures the economic turmoil at that time, when uncertainty about a Greek and Italian default was fostering and Ireland and Portugal were downgraded to junk by Moody's rating agency. At the end of 2012, the situation on the bond market improved and the index reached in 2013 a historical low.

Further on, we proceed with the estimation of our econometric model. Volatilities and correlations are estimated using the methodology outlined in Section 2. Table 2 reports the results of the GJR-GARCH and DCC models. The α parameter, which gives the impact of new information on the volatility of the series, and β, which determines the degree of memory of the process, are always significant. The asymmetric effect given by γ is significant for 7 out of the 11 countries considered, proving therefore the need to account for such an effect on the conditional variance. The DCC parameters are also always significant and have the expected values.

Table 2: GJR-GARCH and DCC estimated parameters

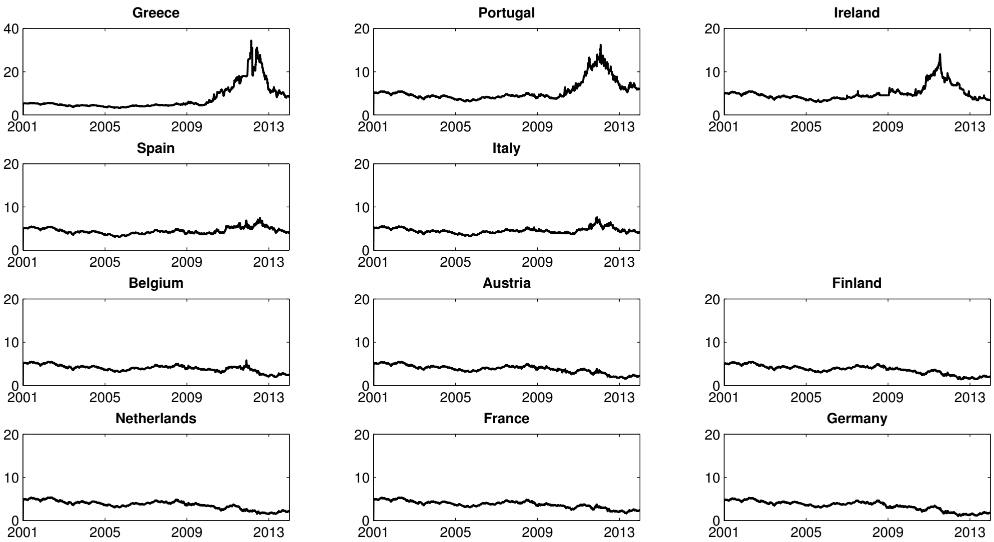
Parameters	α	γ	β	α_C	β_C
Market Index	0.028***	0.012***	0.961***	-	-
Belgium	0.051***	0.011***	0.930***	0.046***	0.949***
Germany	0.038***	0.000	0.954***	0.043***	0.955***
Ireland	0.150***	0.134***	0.715***	0.007***	0.993***
Greece	0.038***	0.077***	0.923***	0.026***	0.974***
Spain	0.050***	0.061***	0.914***	0.062***	0.933***
France	0.032***	0.000	0.960***	0.016***	0.982***
Italy	0.036***	0.061***	0.928***	0.039***	0.960***
Netherlands	0.033***	0.000	0.957***	0.022***	0.975***
Austria	0.128***	0.000	0.819***	0.021***	0.978***
Portugal	0.047***	0.042***	0.931***	0.023***	0.975***
Finland	0.048***	0.000	0.938***	0.048***	0.947***

Note: This table presents the estimated parameters of the GJR-GARCH and DCC models. *** denotes significance at the 99% level.

4.2 Systemic risk analysis: the European case

The evolution of the MES for each country is plotted in Figure 2. The MES shows what the bond yields would be for a particular country, if the entire zone was facing a crisis. At the same time, the MES allows us to identify the systemic riskiness of the analyzed countries. The upper panel of the figure shows the most systemically important countries according to this measure. These countries are the periphery countries,the ones that experienced the biggest problems during the crisis. By contrast, the lower panel shows the Eurozone members that have not been at the core of the debt crisis. Overall, while the marginal contribution to systemic risk of periphery countries increased in crisis times with respect to calm periods, the one of each core country, except Belgium, had a pronounced declining trend over the whole period. Greece proves to be the most systemically risky country on the analyzed time span, with a sharp increase in 2011. During this year, financial markets became increasingly worried about a possible exit of Greece from the Eurozone, the Greek parliament voted drastic austerity measures and the European Union (EU) agreed on helping the country with several billion euros. In the same year, Greece was downgraded several times by credit rating agencies and reached the lowest investment-grade rating. All these events contributed to an abrupt increase in Greek yields and consequently to a raise in its marginal contribution to systemic risk. The following two MES values are the ones of Portugal and Ireland, countries that also required bailout funds in order to keep the commitments towards repaying their debt. The smallest MES value is associated to Germany, country whose bonds have been seen as a refuge value in Europe and have always displayed the lowest long-term yields.

Figure 2: Marginal Expected Shortfall (MES) by country



Note: This figure presents the evolution of the MES for each country in our sample. The scale is the same for all countries, excepting Greece.

Table 3 displays the value of the MES for all countries in the sample at two selected dates: June 2006 (corresponding to a pre-crisis period) and June 2012 (considered as a crisis period). In 2006, the MES already highlights three of the periphery countries, Greece, Italy and Portugal, as being the most systemically important countries. It is worth underlining, thus, that the MES is able to identify as systemically important countries, in calm periods, the countries that would have difficulties further on in terms of public debt and whose sovereign debt situation will be at the source of their crisis. However, in 2006 we do not notice an important difference between the periphery and the core countries, as the MES values are rather clustered together. Consequently, Spain and Ireland are mixed with core countries and find themselves towards the bottom of the classification(*i.e.* Spain and Ireland experienced private debt problems but the MES can not capture this in calm periods and in the *in sample* construction that we develop).

In 2012, the rankings differ for some of the countries, the most notable change being for Ireland and Spain that find themselves at the top of the classification, due to the increase in their marginal contribution to systemic risk. At this date there is a strong difference between the MES of core and periphery countries as the average MES of the latter $(12, 01)$ is more than 5 times higher than the average of the former countries $(2, 27)$. At the two considered dates, Greece has the highest marginal contribution to systemic risk and Germany has the lowest one. We interpret the Greek MES of 29.04 (June 2012) as follows: if the Greek weight in the market index increases by 1%, than the Expected Shortfall of the market is expected to increase by 29.04%. We can also interpret this result as the average bond yield that would apply to new bonds issued by the Greek government, if the Eurozone was subject to a systemic event. Finally, we can remark that the marginal contribution to systemic risk for all core countries declined between the two dates and increased for periphery countries (for Greece, Ireland and Portugal the marginal contribution more than doubled).

Table 3: MES based ranking

15/06/2006		**15/06/2012**	
Greece	4.19	**Greece**	29.04
Italy	4.18	**Portugal**	10.62
Portugal	4.00	**Ireland**	7.27
Finland	3.92	**Spain**	6.91
Austria	3.92	**Italy**	6.21
Belgium	3.91	**Belgium**	3.34
France	3.91	**France**	2.69
Spain	3.87	**Austria**	2.36
Ireland	3.87	**Netherlands**	2.01
Netherlands	3.87	**Finland**	1.84
Germany	3.84	**Germany**	1.37

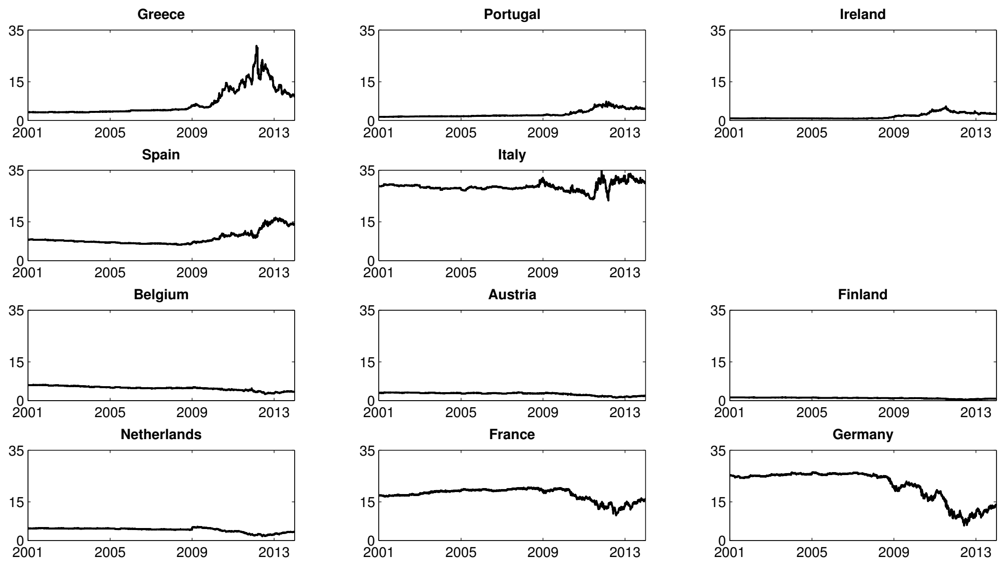
Note: This table provides the countries' rankings based on the MES measure at two different dates.

Based on the MES and countries' weights we compute the CES. We evaluate the overall contribution of each country to systemic risk using the percentage form of the CES. While the MES captures mainly a bond yield effect, the CES% combines the MES and a size effect. The larger the CES%, the greater the contribution of the country to systemic risk, with all the countries CES% summing up to 100%.

Figure 3 shows the dynamics of this measure from the beginning of 2001 until the end of 2013. For comparison reasons, the scale for the subplots is kept the same. Before 2010, the evolution of the CES is on average stable for all the countries, the top 3 countries, Germany, Italy and France, being distanced from the others. A turning point in this evolution is the period $2010 - 2012$, when contributions to systemic risk changed. Sharp increases have been registered for risky countries (Greece, Italy, Spain, Ireland and Portugal), whereas the other Eurozone members have experienced a decrease in their absolute contribution to systemic risk. This is in line with previous findings obtained based on the MES. If in calm periods, the CES% captures mainly the size effect, in crisis times this measure is able to account for the combination of the two effects: yield and debt weight.

Table 4 supports the above results. The rankings obtained based on the CES%

Figure 3: Percentage Component Expected Shortfall (CES%) by country



Note: This figure presents the evolution of the CES% for each country in our sample. The scale is the same for all countries.

identify the countries that effectively suffered major transformations and became the most systemically important. More precisely, Greece, Spain, Portugal, Italy and Ireland experienced increases in their contribution to systemic risk. If in calm periods, their contribution to systemic risk was about 42%, in crisis times it raises to approximately 75%. This is an interesting feature as it shows that countries that registered an increase in their absolute contribution to systemic risk (in other words, a rise in their CES%) are the same as the ones identified by the MES.

In general, according to this measure, risk is concentrated: the top three countries (Italy, Germany and France in 2006 and Italy, Greece and Spain in 2012) account for around $60 - 70\%$ of the global risk.

Greece is the country that experienced the highest increase in its percentage contribution to systemic risk (from 3.87% in 2006 to 20.63% in 2012), mainly to the detriment of Germany who was one of the few save havens left in the Eurozone and whose contribution to systemic risk was divided by more than three. Italy has the highest CES% both before and during the debt crisis. This result can be explained by the fact that in 2006 the size effect has certainly dominated, also justifying the presence of Germany and France in top-3, whereas with the outbreak of the debt crisis, the yield effect added to the first effect (and thus pushed Greece and Spain in top-3).[19]

To consolidate our results, we perform different robustness check studies.

Table 4: $CES\%$ based ranking

15/06/2006		**15/06/2012**	
Italy	28.37	**Italy**	30.76
Germany	25.69	**Greece**	20.63
France	19.62	**Spain**	13.59
Spain	6.59	**France**	12.25
Belgium	4.95	**Germany**	7.37
Netherlands	4.45	**Portugal**	5.14
Greece	3.87	**Belgium**	3.24
Austria	2.83	**Ireland**	3.21
Portugal	1.83	**Netherlands**	2.04
Finland	1.03	**Austria**	1.34
Ireland	0.78	**Finland**	0.43

Note: This table provides the countries' rankings based on the CES measure at two different dates.

First, we compare our bond yield index with the S&P Eurozone Sovereign Bond Index $7-10Y$ proposed by McGraw Hill Financial. The S&P Index is, as in our case, a market-value-weighted index. However, the weights are based on outstanding par amounts for bonds.[20] We observe the same dynamics and levels for both indexes. Furthermore, the results obtained with the S&P index are similar to the ones obtained in this section.[21] Second, we also performed an analysis using constant weights (as of 2011) in the construction of our index and for the CES% measure. The results do not change. Finally, we estimate an econometric model using this time a Garch(1,1) and find that the countries' ranking remain the same. We used this model because there might be a low level of asymmetry in some series as underlined by several of the GJR Garch(1,1) coefficients.

A large part of the systemic risk in the debt crisis originated from periphery countries. Our findings reveal the importance of undertaking a careful screening process when

[19] For a more general view of our results in terms of CES% at different moments in time, refer to Figure $A2$ in Appendix D.

[20] For more information, see http://us.spindices.com/indices/fixed-income/sp-eurozone-sovereign-bond-index.

[21] These results are available upon request.

further enlarging the Eurozone. If in calm periods, the heterogeneity among countries does not seem to be a major problem, in crisis times macroeconomic divergences get deepened and worsen the already critical situation.

5 Conclusions

In this paper, we used two systemic risk measures usually used in the financial literature, the Marginal Expected Shortfall (MES) and the Component Expected Shortfall (CES%), and transposed them to the case of sovereign debt in the Eurozone. More precisely, we constructed these measures based on Eurozone members' bond yields and debts, in order to evaluate countries' contribution to systemic risk. The MES and the CES% rank countries according to their riskiness and identify those that contribute the most to the overall risk. In a pre-crisis period, the MES can identify some of the countries that might experience difficulties, when the whole system would be in distress. Our findings underline the fact that countries with deteriorated public finances, sanctioned by investors with high yields, have experienced in recent years an important increase in their (marginal and absolute) contribution to systemic risk. Thus the two measures allow us to identify the countries that are the greatest systemic risk contributors

If the MES relies more on bond yields, the CES% can be viewed as a hybrid systemic risk measure which combines both yields and debt size effects. These measures give information about which countries need more monitoring and therefore can represent a useful tool for authorities. However, their forecasting power is limited and the measures would benefit from including into the analysis other macroeconomic factors. However, in this paper, we choose to concentrate more on aggregate risk than country specific risk. Nevertheless, the analysis can be expended to integrate country specific factors such as inflation, growth rates and public deficit, and to take into account the budgetary constraint. This would bring further insights to the measurement of systemic sovereign risk.

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Appendix A: Details on the calculation of MES (1)[22]

We use as a starting point the expression of the expected shortfall of the Eurozone system at time t,

$$ES_{m,t-1}(C) = E_{t-1}(r_{mt}|r_{mt} > C). \tag{18}$$

Then, following Scaillet (2004), we intend to prove that that the first order derivative with respect the the weight of the i^{th} country, *i.e.* MES, is given by:

$$\frac{\partial ES_{m,t-1}(C)}{\partial w_{it}} = E_{t-1}(r_{it}|r_m > C). \tag{19}$$

In order to prove this, we denote by $\breve{r}_{mt}$ the yield applied to the system except for the contribution of the i^{th} country, where $\breve{r}_{mt} = \sum_{j=1\ j\neq i}^{n} w_{jt}r_{jt}$ and $r_{mt} = \breve{r}_{mt} + w_{it}r_{it}$. In our proof, the threshold C is not restricted to a scalar. Rather, C is assumed to depend on the distribution of the market yield and hence on the weights and the probability p to be in the distribution tale, as in the case of the VaR. We therefore provide a general proof for equation (19). First, the ES is expressed as follows:

$$\begin{aligned} ES_{m,t-1}(C) &= E_{t-1}(\breve{r}_{mt} + w_{it}r_{it}|\breve{r}_{mt} + w_{it}r_{it} > C(w_{it},p)) \\ &= \frac{1}{p}\int_{-\infty}^{\infty}\left(\int_{C(w_{it},p)-w_{it}r_{it}}^{\infty}(\breve{r}_{mt} + w_{it}r_{it})f(\breve{r}_{mt},r_{it})\ d\breve{r}_{mt}\right)dr_{it}, \end{aligned} \tag{20}$$

where $f(\breve{r}_{mt}, r_{it})$ is the joint probability density function of the two series of yields. Second, the first order derivative with respect to the weight of country i is:

$$\begin{aligned} \frac{\partial ES_{m,t-1}(C)}{\partial w_{it}} &= \frac{1}{p}\int_{-\infty}^{\infty}\left(\int_{C(w_{it},p)-w_{it}r_{it}}^{\infty}(r_{it})f(\breve{r}_{mt},r_{it})\ d\breve{r}_{mt}\right)dr_{it} \\ &- \frac{1}{p}\int_{-\infty}^{\infty}\left(\frac{\partial C(w_{it},p)}{\partial w_{it}} - r_{it}\right)C(w_{it},p)f\left(C(w_{it},p) - w_{it}r_{it}, r_{it}\right)dr_{it}. \end{aligned} \tag{21}$$

However, the probability of being in the right tail of the distribution of the market yields is constant, *i.e.* $\Pr\left(\breve{r}_{mt} + w_{it}r_{it} > C\right) = p$. Consequently, the first order derivative of this probability is null. It follows that:

$$\left(\frac{\partial C(w_{it},p)}{\partial w_{it}} - r_{it}\right)f\left(C(w_{it},p) - w_{it}r_{it}, r_{it}\right) = 0. \tag{22}$$

[22] The proof provided here is similar to the one found in Banulescu and Dumitrescu (2014). However, the conditioning event is reversed in our case, therefore results are different.

Hence, equation (21) becomes:

$$\begin{aligned}\frac{\partial ES_{m,t-1}(C)}{\partial w_{it}} &= \frac{1}{p}\int_{-\infty}^{\infty}\left(\int_{C(w_{it},p)-w_{it}r_{it}}^{\infty}(r_{it})f(\breve{r}_{mt},r_{it})\ d\breve{r}_{mt}\right)dr_{it} \\ &= E_{t-1}(r_{it}|\breve{r}_{mt}+w_{it}r_{it}>C(w_{it},p)) \\ &= E_{t-1}(r_{it}|r_{mt}>C).\end{aligned} \tag{23}$$

We proved thus that the MES is given by equation (23).

Appendix B: Details on the calculation of MES (2)

When computing the marginal expected shortfall, we make use of a detailed expression for z_{it}. The innovations for the bond yield of country i and for the yield of the market can be written as Garch processes:

$$\begin{aligned} z_{it} &= \sigma_{it}\varepsilon_{it} \\ z_{mt} &= \sigma_{mt}\varepsilon_{mt} \end{aligned} \tag{24}$$

To unfold this expression, we use some standard results of the Capital Asset Pricing Model (CAPM), written for the innovations, as our data is not stationary. Mainly, the yields for country i are such that:

$$z_{it} = \beta_i z_{mt} + \mu_{it}, \tag{25}$$

with μ_{it} an error term and β_i estimated as in any linear regression:

$$\beta_i = \frac{cov(z_i, z_m)}{var(z_m)} = \frac{\sigma_{imt}}{\sigma_{mt}^2}. \tag{26}$$

Moreover, the conditional correlation between the innovations for the yield of country i and the market index is given by:

$$\rho_{imt} = \frac{\sigma_{imt}}{\sigma_{it}\sigma_{mt}}, \tag{27}$$

which allows us to rewrite the equation of z_{it} as:

$$\begin{aligned} z_{it} &= \tfrac{\sigma_{imt}}{\sigma^2_{mt}}\sigma_{mt}\varepsilon_{mt} + \mu_{it} \\ &= \tfrac{\sigma_{imt}}{\sigma_{mt}}\varepsilon_{mt} + \mu_{it} \\ &= \tfrac{\sigma_{imt}}{\sigma_{mt}\sigma_{it}}\sigma_{it}\varepsilon_{mt} + \mu_{it} \\ &= \rho_{imt}\sigma_{it}\varepsilon_{mt} + \sigma_{\mu it}\xi_{it}, \end{aligned} \tag{28}$$

where μ_{it} is the residual of the linear regression and ξ_{it} is the standardized residual. Furthermore, the variance of z_{it}, $\sigma^2_{it} = \rho^2_{imt}\sigma^2_{it} + \sigma^2_{\mu it}$, gives us an expression for the variance of the residuals, μ:

$$\sigma^2_{\mu it} = \sigma^2_{it}(1 - \rho^2_{imt}), \tag{29}$$

and thus, their standard error:

$$\sigma_{\mu it} = \sigma_{it}\sqrt{(1 - \rho^2_{imt})}. \tag{30}$$

Finally, putting all these results together, we obtain the formula for the yield of country i:

$$\begin{aligned} z_{it} &= \rho_{imt}\sigma_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{(1-\rho^2_{imt})}\xi_{it} \\ &= \sigma_{it}\underbrace{\left(\rho_{imt}\varepsilon_{mt} + \sqrt{(1-\rho^2_{imt})}\xi_{it}\right)}_{\varepsilon_{it}} \end{aligned} \tag{31}$$

Appendix C: Tail Expectations[23]

We use a non-parametric kernel framework to estimate the tail expectations $E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > (C - r_{m,t-1})/\sigma_{mt})$ and $E_{t-1}(\xi_{it}|\varepsilon_{mt} > (C - r_{m,t-1})/\sigma_{mt})$ as in Scaillet (2005).

For simplicity, let us denote the systemic risk event $(C - r_{m,t-1})/\sigma_{mt}$ by κ. Consequently, the tail expectation on market yields becomes:

$$E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > (C - r_{m,t-1})/\sigma_{mt}) = E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > \kappa). \tag{32}$$

Based on the definition of the conditional mean, we can express equation (32), as a

[23] The proof provided here is following Banulescu and Dumitrescu (2014). However, the conditioning event is reversed in our case, therefore results are different.

function of the probability density function f:

$$E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > \kappa) = \int_{\kappa}^{\infty} \varepsilon_{mt} f(u|u > \kappa)\ du$$
$$= \int_{-\infty}^{\infty} \varepsilon_{mt} f(u|u > \kappa)\ du - \int_{-\infty}^{\kappa} \varepsilon_{mt} f(u|u > \kappa)\ du, \tag{33}$$

where the conditional density $f(u|u > \kappa)$ can be written as:

$$\frac{f(u)}{Pr(u > \kappa)}. \tag{34}$$

Next, we need to compute $f(u)$ and $Pr(u > \kappa)$. For this, we first consider the standard kernel density estimator of the density f at point u given by:

$$\hat{f}(u) = \frac{1}{Th} \sum_{1}^{T} \phi(\frac{u - \varepsilon_{mt}}{h}),$$

where h stands for the bandwidth parameter, and T is the sample size (Silverman, 1986; Wand and Jones, 1995; Simonoff, 1996). Second, the probability to be in the tail of the distribution can be defined as the integral of the probability density function over the domain of definition of the variable u, *i.e.* $p = Pr(u > \kappa) = \int_{\kappa}^{\infty} f(u)\ du$. Thus, we can replace $\hat{f}(u)$ with the kernel estimator and obtain:

$$\hat{p} = \frac{1}{Th} \sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right).$$

Consequently, equation (32) becomes:

$$\hat{E}_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \varepsilon_{mt} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}. \tag{35}$$

Similarly, it can be shown that:

$$\hat{E}_{t-1}(\xi_{it}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \xi_{it} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}. \tag{36}$$

Appendix D: Figures

Figure A1: Debt weights by country over the period 2000 – 2011

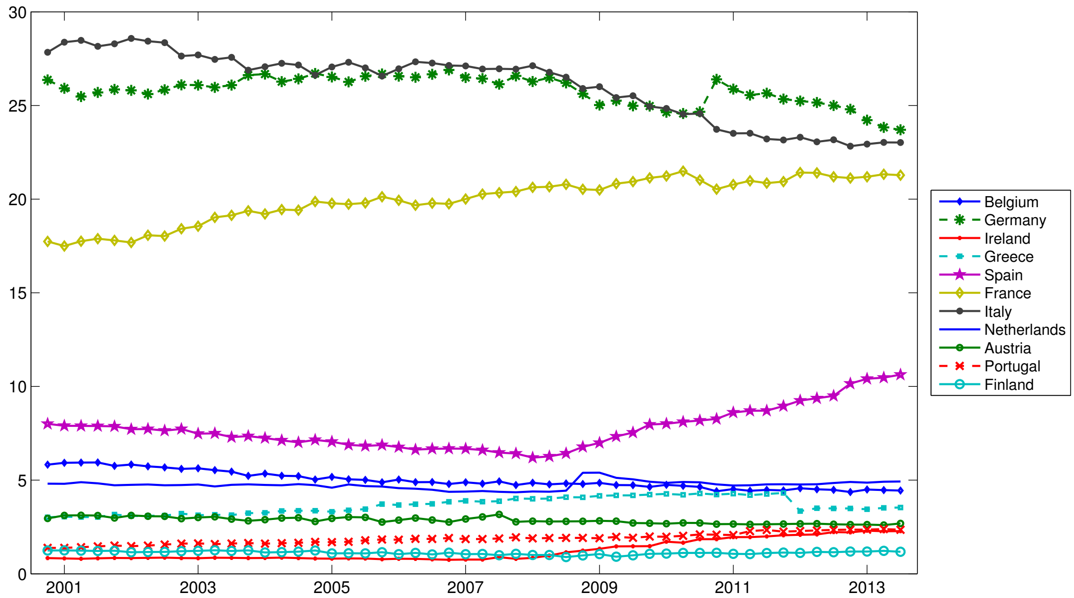

Figure A2: Percentage Component Expected Shortfall by country at different dates