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Why Fiscal Regimes Matter for Fiscal Sustainability: An Application to France

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Why Fiscal Regimes Matter for Fiscal Sustainability: An Application to France*

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Abstract

This paper introduces a Regime-Switching Model-Based Sustainability test allowing for periodic (or local) violations of Bohn (1998, QJE)'s sustainability condition. We assume a Markov-switching fiscal policy rule whose parameters stochastically switch between sustainable and unsustainable regimes. We demonstrate that long-run fiscal sustainability not only depends on regime-specific feedback coefficients of the fiscal policy rule but also on the average durations of fiscal regimes. Evidence on French data suggests that both the No-Ponzi Game condition and the Debt-stabilizing condition hold in the long-run, when accounting for fiscal regimes, contrary to standard MBS tests.

JEL: E6, H6
Keywords: Fiscal rules, Fiscal regimes, Public debt sustainability, Time-varying parameters, Markov-switching models

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1 Introduction

Fiscal policy rules describing the reaction of primary balance to the initial level of public debt have long been used to analyze fiscal sustainability. According to Bohn (1998)'s seminal contribution, primary public balance must increase after an increase of the public debt-to-GDP ratio to ensure sustainability, as defined by the respect of the government intertemporal budget constraint. This paper is motivated by the empirical evidence of fiscal episodes during which public debt-to-GDP is non-stationary and generates no improvement in primary public balance. Under these episodes,fiscal policy *periodically* violates Bohn's sustainability condition and thus raises critical questions on the long-run fiscal sustainability: is a periodically unsustainable fiscal policy a threat to long-run sustainability of public finance? How long can fiscal policy be periodically unsustainable without violating its sustainability constraints in the long-run?

To our knowledge, only a few papers have addressed a regime-switching (or time-varying) fiscal policy rule while also proposing a testing framework for long-run sustainability. In their seminal contribution Canzoneri et al. (2001) consider a time-varying fiscal policy rule and derive a necessary and sufficient condition such that the government intertemporal budget constraint holds in the long-run. Davig (2005) extends Wilcox (1989)'s unit-root testing procedure to a Markov-switching framework in which discounted debt can be periodically expanding. Finally, there is a literature on regime-switching monetary and fiscal policy rules that has successfully identified *local* equilibria in the data where fiscal policy (or monetary policy) is either "active" or "passive", following Leeper (1991). Still, these papers do not test whether fiscal policy *globally* satisfies the intertemporal budget constraint or the debt-stabilizing criterion in the long-run. Based on a Markov-switching monetary policy rule, Davig and Leeper (2007b) have proposed a long-run Taylor principle such that the price-level is *globally* determined despite periodic violations of the short-run Taylor principle; but there is no equivalent proposition for a *globally* sustainable fiscal policy. In contrast, we derive a formal test of global fiscal sustainability which depends on fiscal regimes' transition probabilities and on their respective durations.

The paper introduces a Regime-Switching Model-Based Sustainability (RS-MBS) test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes. We define unsustainable regimes by periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violating Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes. We demonstrate how fiscal regimes matter for global (in opposition with local) fiscal sustainability analysis.

The paper addresses the two usual concepts of long-run fiscal sustainability: the No-Ponzi game condition (related to the transversality condition) and the debt-stabilizing condition (related to the stationarity of the debt-to-GDP ratio). For each concept of fiscal sustainability, we derive the necessary and sufficient conditions for long-run (or global) fiscal sustainability which depend on regime-specific feedback coefficients of the Markov-switching fiscal policy rule and on expected durations (or persistence) of fiscal regimes. We show that fiscal policy can be *locally* unsustainable, with a periodically explosive public-debt-to-GDP ratio, and still be *globally* sustainable[1].

We apply the formal test to France. As a Euro Area member state, France has neither a domestic monetary policy nor a lender of last resort. Both features make the issue of fiscal sustainability very acute. First, the French government cannot expect a domestic accommodative monetary policy when or after

[1] Episodes of a locally-explosive debt which does not lead to global unsustainability or default are theoretically investigated in Blot et al. (2016).

it implements a non-Ricardian fiscal policy. Second, sustainability issues cannot be disregarded and left to the management of the lender of last resort. As a result, we focus exclusively on Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price-level.[2]

Our results are threefold. First, we estimate different specifications of Bohn's constant-parameters fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. We reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio.

The rest of the paper is organised as follows. Section 2 reviews the literature on fiscal sustainability. Section 3 presents the extension of the Model-based approach of sustainability to regime switches and develops a new condition for fiscal sustainability. Section 4 deals with an application of the empirical methodology to French data. Section 5 concludes.

2 Related literature

Bohn (1998) builds a Model-Based Sustainability (MBS) framework to analyze fiscal sustainability through the lens of fiscal policy rules (or fiscal reaction functions) in a simple general equilibrium model, as an alternative to the econometric analyses *à la* Hamilton-Flavin[3]. Basically, Bohn assumes the following framework composed of a linear fiscal rule (1)

$$s_t = \gamma b_{t-1} + \mu_t \tag{1}$$

where s_t is the primary surplus-to-GDP ratio, b_t is the end-of-period public debt-to-GDP ratio and finally μ_t is a vector including all cyclical components of primary surplus (e.g. output gap, temporary public spending), plus a constant and an error term. Thus, Bohn finds that a strictly positive feedback effect $\gamma > 0$ satisfies the No-Ponzi Game (NPG) condition[4].

Under a stricter sustainability condition[5], like a debt-stabilizing fiscal policy rule, the feedback effect should be larger than the growth-adjusted real average interest rate on public debt, that is $\gamma > r - y$[6].

[2] Another consequence of the first feature is methodological: the Leeper (1991) and Davig and Leeper (2011)'s policy interaction framework is not applicable to France.

[3] Seminal empirical investigations on fiscal sustainability proposed a testing framework based on the present-value budget constraint and the transversality condition, drawing on stationarity or cointegration properties of fiscal data (Hamilton and Flavin, 1986; Trehan and Walsh, 1988, 1991; Wilcox, 1989; Wickens and Uctum, 1993; Quintos, 1995). Still, the econometric analysis of fiscal sustainability has raised a number of issues and led to important criticisms by Bohn (1995, 1998, 2007).

[4] The Non-Ponzi Game condition states that the present-value of public debt tends to zero in the long-run, which means that the government must pay back at least a part of the interest charges.

[5] Bohn (2007) acknowledges that an upper bound on primary surplus, i.e a fiscal limit, requires a stationary public debt-to-GDP for fiscal sustainability to hold. Research about the upper-bound of primary surplus has been recently explored by Bi (2012); Bi and Traum (2012); Davig et al. (2011); Daniel and Shiamptanis (2013). Daniel and Shiamptanis show that stationarity and cointegration restrictions are necessary for fiscal sustainability when assuming existence of a fiscal limit. Existence of a fiscal limit (i.e. an upper bound on primary balance-to-GDP and on public debt-to-GDP) requires a sustainability criterion ensuring that public debt must be stable around a long-run value compatible with fiscal limit.

[6] If one considers a fiscal rule with variables in absolute level rather than as share of GDP, then this feedback should be larger than the real average interest rate on public debt. This is basically what Leeper (1991) finds when describing the stability conditions of an active monetary/passive fiscal regime.

MBS analysis has been shown to be empirically powerful in the case of US fiscal policy on long-run data (Bohn, 1998, 2008). On international panel data, Mendoza and Ostry (2008) find evidence that fiscal policy is "responsible" (i.e. there is evidence of a strictly positive feedback rule).

Two types of nonlinear specifications of the fiscal rules exist. On the one hand, fiscal rules are polynomial functions of public debt-to-GDP ratio, i.e. include quadratic and cubic terms (Bohn, 1998). This specification is motivated by the idea that primary surplus may either react more to lagged public debt or on the contrary may become "flatter" at higher public debt levels. This approach has been followed by Ghosh et al. (2013a,b) to account for "fiscal fatigue" where they derive debt limits as the maximum level of public debt beyond which primary balance can no longer adjust to stabilize debt. On the other hand, fiscal rules are time-varying. The assumption that simple linear policy rules (either monetary or fiscal) are constant over time is not convincing regarding multiple evidence of "structural breaks" or "regime changes".In particular, empirical literature on regime-switching fiscal rules has produced evidence that fiscal rules may be better described by "fiscal regimes", see Favero and Monacelli (2005); Chung et al. (2007); Davig and Leeper (2007a, 2011); Bianchi (2012); Burger and Marinkov (2012); Afonso and Toffano (2013)[7]. This literature generally identifies sub-periods during which fiscal policy does not stabilize public debt, and sometimes even displays a negative feedback effect of initial public debt on primary surplus.

The literature on regime-switching monetary and fiscal rules builds on Leeper (1991)'s seminal contribution, which developed a set of formal conditions for *local* equilibrium determinacy stemming from the properties of the monetary and fiscal rules. Fiscal policy is *passive* under the debt-stabilizing condition, *active* otherwise[8]. Recent research on fiscal policy (Bi, 2012; Bi and Leeper, 2013) explores regime-switching fiscal policies to derive an endogenous and stochastic fiscal limit. This literature analyzes fiscal sustainability as the sovereign default probability, computed from the fiscal limit distribution, rather than as generalized conditions on the regime-switching fiscal rule[9]. Davig and Leeper (2007b) define the long-run Taylor monetary principle, based on a Markov-switching Taylor rule, allowing for periodic (or local) violations of the short-run Taylor principle. But, to our knowledge, none has proposed and tested analogous conditions on a regime-switching fiscal rule such that NPG and debt-stabilizing conditions hold in the long-run. In this respect, this paper's motivation is similar to Davig and Leeper (2007b) but applied to fiscal policy.

Finally, this paper is also responding to two important contributions in the field of fiscal sustainability analysis. Canzoneri et al. (2001) investigate theoretically a particular time-varying fiscal policy rule in which public debt feedback effect on primary surplus is positive or null. They show primary surplus only has to react positively to public debt on an infrequent basis but "infinitely often" in order to satisfy the government intertemporal budget constraint. This analysis is restrictive in at least two respects. First, assuming primary surplus does not react negatively to initial public debt is a critical assumption, at odds with some empirical evidence on regime-switching policy rules (Favero and Monacelli, 2005; Davig and Leeper, 2007a, 2011; Afonso and Toffano, 2013). Second, the sustainability condition does not ensure a stationary public debt-to-GDP ratio, which is probably the relevant fiscal sustainability condition when the economy faces a fiscal limit. Alternatively, Davig (2005) proposes a unit-root testing framework using a Markov-switching model which accounts for episodes of periodically expanding discounted public debt. This approach is inherently subject to the criticisms addressed by Bohn (1995,

[7]For monetary policy, see Clarida et al. (2000); Auerbach (2002); Lubik and Schorfheide (2004), among others.

[8]Condition on monetary policy is the Taylor Principle: monetary policy is labeled "active" (A) when it reacts agressively to inflation (i.e. the Taylor principle holds) and "passive" (P) otherwise. From these two conditions, Leeper (1991) identify four local regimes: Monetary regime (AM/PF), Fiscal regime (PM/AF), Indeterminacy regime (PM/PF) and Explosive regime (AM/AF).

[9]Fiscal limit distributions are obtained by numerical approximation of the decision rule in calibrated or sometimes estimated Real business cycle models.

2007) to the econometric analysis of fiscal sustainability. In particular, unit-root testing does not provide any information about fiscal policy behavior since it does not involve an explicit model of fiscal policy.

3 Theory: Regime-Switching Model-Based Sustainability

We assume a stochastic real endowment and cashless economy composed of a representative rational household and a government. By assuming a real cashless economy, we implicitly assume that monetary policy has full control over the price-level and inflation dynamics. Using the terminology of the Fiscal Theory of Price-Level (Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2005), we only consider Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price level. Thus we assume that fiscal policy is the only game in town, and we study the worst-case scenario in which fiscal authorities are left without monetary support to ensure public debt sustainability: what are then the fiscal sustainability requirements and can we reject the null hypothesis of unsustainability (i.e. violation of these requirements)? Rejection of unsustainability in this "worst-case" scenario may be interpreted as credible evidence of sustainability.

3.1 Model

Stochastic real endowment. Total output Y_t is following a unit-root with drift:

$$Y_t = Y_{t-1}(1 + y + \varepsilon_t^y) \tag{2}$$

where $y > 0$ is the long-run growth rate of output and ε_t^y is an i.i.d random shock to the growth rate.

Representative household. Representative household's preferences are represented by the utility function $u(.)$ which is strictly increasing ($u'(.) > 0$) and concave ($u''(.) < 0$) and a subjective discount factor β. At each period, consumer chooses consumption C_t and buys public bond B_t at a price $(1 + r_t)^{-1}$ in order to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the following budget constraint:

$$C_t + (1 + r_t)^{-1} B_t = B_{t-1} + Y_t - T_t$$

and transversality condition:

$$\lim_{T \to +\infty} \mathbb{E}_t \frac{B_{t+T}}{1 + r_{t,T+1}} \geq 0$$

with $(1+r_{t,T+1})$ being the $T+1$-period ahead real interest rate. First order conditions of the representative consumer's maximization program yield the standard Euler equation:

$$(1 + r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \tag{3}$$

Equation (3) evaluates the stochastic discount factor $Q_{t,1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$ at the optimal solution of the representative consumer's program, which is the common pricing kernel of any asset in the economy. Hence, a $j-$period public bond has a price $(1 + r_{t,j})^{-1} = \mathbb{E}_t Q_{t,j}$ with $Q_{t,j} = \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$.

Government. Government spends G_t and collects lump-sum taxes T_t. At each start of period t, government carries one-period public bonds B_{t-1} and it will issue B_t at a price $(1+r_t)^{-1}$ at end of period. Thus, government faces the following one-period budget constraint:

$$(1+r_t)^{-1}B_t = (G_t - T_t) + B_{t-1} \tag{4}$$

with $S_t \equiv T_t - G_t$ representing the primary budget balance. Under balanced growth, all variables in level grow at rate y_t, thus we rewrite the government budget constraint in terms of output ratios:

$$b_t = \frac{1+r_t}{1+y_t} b_{t-1} - (1+r_t)s_t \tag{5}$$

where b_t is the end-of-period debt-output ratio, s_t is the primary surplus-output ratio, r_t and y_t are respectively the real interest rate and the growth rate of real output.

Preventing government from running a Ponzi scheme against its creditor implies the following Present-Value Budget Constraint (PVBC). Following Bohn (1995), we write the PVBC using the stochastic discount factor in order to account for uncertainty and consumer's risk-aversion:

$$B_{t-1} = \sum_{i=0}^{+\infty} \mathbb{E}_t\Big[Q_{t,i} S_{t+i}\Big] \tag{6}$$

which is equivalent to the following transversality condition (TC):

$$\lim_{T\to+\infty} \mathbb{E}_t\Big[Q_{t,T+1} B_{t+T}\Big] = 0 \tag{7}$$

Both the PVBC and TC must hold with equality since the representative consumer cannot run a Ponzi Scheme against government (Bohn, 1995).

We assume the following Markov-switching fiscal policy rule:

$$s_t = \gamma(z_t) b_{t-1} + \mu_t(z_t) \tag{8}$$

Regime-switching parameter $\gamma(z_t)$ represents the feedback effect of the initial public debt-output ratio b_{t-1} on primary surplus-output ratio *conditional on* fiscal regime z_t. Fiscal regimes are then defined as:

$$\gamma(z_t) = \begin{cases} \gamma_S > 0 & \text{if} \quad z_t = 1 \quad \text{(Sustainable Regime)} \\ \gamma_{NS} \leq 0 & \text{if} \quad z_t = 0 \quad \text{(Unsustainable Regime)} \end{cases} \tag{9}$$

During sustainable regimes ($\gamma_S > 0$) primary balance improves following a debt increase while it does not improve or even worsens during unsustainable regimes ($\gamma_{NS} \leq 0$)[10]. Finally, we define $\mu_t(z_t)$ by:

$$\mu_t(z_t) = \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon_t^s \tag{10}$$

where $\hat{y}_t$ is the output gap, $\hat{g}_t$ is temporary public spending, $\alpha(z_t)$ is a regime-switching constant, $\sigma(z_t)$ is the regime switching standard-error associated to an i.i.d distributed shock $\varepsilon_t^s \sim \mathcal{N}(0,1)$. We assume

[10] Canzoneri et al. (2010, p.959) discuss empirical results of Davig and Leeper (2007a, 2011) and note that a negative coefficient on lagged debt in the fiscal rule may be difficult to interpret since "regardless of whether the fiscal rule is Ricardian or non-Ricardian, we would expect a positive estimated coefficient". Indeed, Cochrane (2001) shows there exists a positive correlation between primary surplus and initial debt at *equilibrium*, even when fiscal policy is active (with primary surplus following an AR(1) process). Still, empirical research on regime-switching fiscal policy rules provides some evidence of periodic *negative* feedback effect, see Davig and Leeper (2011) and Afonso and Toffano (2013) for instance; these empirical results motivate our specification of unsustainable fiscal regimes by $\gamma_{NS} \leq 0$.

regime-switching to be stochastic and exogenous, following a hidden two-state Markov process z_t describing fiscal regimes. The use of a Markov-switching model rather than endogenous or threshold-switching models represents an agnostic way of modelling regime changes of fiscal policy without making any critical assumption about what drives fiscal regime shifts. In addition, given our economy is purely Ricardian, we also assume that fiscal regime z_t is independent of real output's growth rate.

Define $\gamma = (\gamma_S \ \gamma_{NS})$ a row-vector containing regime-specific parameters and $Z_t = (z_t \ 1-z_t)^T$ a column-vector associated to the Markov process z_t. Hence, we can define the scalar $\gamma(z_t)$ by:

$$\gamma(z_t) \equiv \gamma Z_t = \begin{pmatrix} \gamma_S & \gamma_{NS} \end{pmatrix} \times \begin{pmatrix} z_t \\ 1-z_t \end{pmatrix} \tag{11}$$

Markov process z_t is associated to a transition matrix P whose elements are $p_{ij} \equiv \mathbb{P}(z_t = i | z_{t-1} = j)$ for all $(i,j) \in \{0,1\}$ such that:

$$Z_t = PZ_{t-1} + v_t \quad \text{with} \quad v_t \equiv Z_t - \mathbb{E}_{t-1}[Z_t] \tag{12}$$

We assume z_t to be an ergodic Markov process[11] implying that $\mathbb{E}_t Z_{t+j} = P^j Z_t$ converges to a unique ergodic distribution π:

$$P^j Z_t \underset{j\to+\infty}{\longrightarrow} \pi \tag{13}$$

where $\pi = (\pi_S \ \pi_{NS})^T$ is the column-vector of ergodic probabilities associated to each fiscal regime. Ergodic probabilities are defined by:

$$\pi_i = \frac{1-p_{jj}}{(1-p_{ii}) + (1-p_{jj})} \tag{14}$$

for all $(i,j) \in \{0,1\}$. Hence, using equations (11) and (13), the conditional expectation at time t of feedback parameter $\gamma(z_t)$ converges toward its *unconditional* expectation, i.e. ergodic (or long-run) value:

$$\mathbb{E}_t[\gamma(z_{t+j})] = \gamma P^j Z_t \underset{j\to+\infty}{\longrightarrow} \gamma\pi \tag{15}$$

3.2 No-Ponzi Game condition

Following Bohn (1998), we derive sufficient condition on the sequence $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$ such that Present-Value Budget Constraint (6) and Transversality condition (7) hold. Denoting the j-periods growth-adjusted stochastic discount factor by

$$\tilde{Q}_{t,j} \equiv Q_{t,j} \prod_{i=0}^{j-1} (1 + y_{t+i}) \tag{16}$$

allows us to rewrite Transversality condition (7) in terms of debt-output ratio by:

$$\lim_{T\to+\infty} \mathbb{E}_t\Big[\tilde{Q}_{t,T+1} b_{t+T}\Big] = 0 \tag{17}$$

Then, using the regime-switching fiscal policy rule (8) and iterating on the flow budget constraint of government (5) up to date $t+T$, we obtain an expression for expected present-value debt-output ratio $\mathbb{E}_t\Big[\tilde{Q}_{t,T+1} b_{t+T}\Big]$ which explicitly depends on $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$. Finally, we find a sufficient condition on the

[11] Any Markov process is ergodic as long as $p_{ii} < 1$ and $p_{ii} + p_{jj} > 0$ for all $(i,j) \in \{0,1\}$ (Hamilton, 1994, Chap. 22), meaning there is no absorbing state.

regime-switching fiscal policy rule to satisfy the No-Ponzi Game condition, that allows us to conclude to the following proposition.

Proposition 1 (No-Ponzi Game) *In a dynamically efficient economy, and provided that $\mu_t(z_t)$ is bounded, a sufficient condition that transversality condition (17) holds is*

$$\gamma\pi > 0 \tag{18}$$

with $\gamma\pi \equiv \gamma_S\pi_S + \gamma_{NS}\pi_{NS}$ being the unconditional expectation of $\gamma(z_t)$. Using the definition of ergodic probabilities (14) and denoting expected duration of regimes by $d_i = \frac{1}{1-p_{ii}}$, we can express condition (18) by

$$\gamma_S > |\gamma_{NS}|\frac{d_{NS}}{d_S} \tag{19}$$

Proof 1 *See appendix A.1.*

To understand this condition, let us consider the following approximation of tranversality condition when $T \to +\infty$:

$$\mathbb{E}_t\left[\tilde{Q}_{t,T+1} b_T\right] \approx (1-(1+y)\gamma\pi)^T b_t \tag{20}$$

Following Bohn (2008), consider a Ponzi Scheme such that $\{s_t\}_{t=0}^{\infty} = 0$. This Ponzi Scheme implies debt-output ratio growing at a rate $\frac{r_t - y_t}{1+y_t}$. As a consequence the limit value of future discounted debt-output ratio is equal to initial debt-output ratio (which violates the transversality condition):

$$\mathbb{E}_t\left[\tilde{Q}_{t,T+1} b_{t+T}\right] = b_t \tag{21}$$

Thus, $\gamma\pi > 0$ implies the reduction of $\mathbb{E}_t\left[\tilde{Q}_{t,T+1} b_{t+T}\right]$ by a factor $(1-(1+y)\gamma\pi)^T$ relative to a Ponzi Scheme. Saying it differently: the average growth rate of debt-output ratio is reduced by a factor $(1-(1+y)\gamma\pi) > 0$.

Condition (18) states that a regime-switching fiscal policy has to satisfy the NPG condition *on average*, that is, sustainable regimes have to be frequent enough to balance unsustainable regimes in the long-run. Ruling out a Ponzi Scheme means that the longer unsustainable regimes *vis-à-vis* duration of sustainable regimes, the larger primary deficits during unsustainable regimes, then the larger the required reaction of primary surplus to debt during sustainable regimes. Still, provided (19) holds, fiscal policy can be periodically unsustainable *while* satisfying its present-value budget constraint (PVBC).

3.3 Debt-stabilizing condition

A stronger constraint on fiscal policy would require that debt-output ratio must be stationary at a sufficiently low level, below a "fiscal limit" defined as follows. Assume an exogenous upper-bound on the primary surplus-output ratio such that $s_t \leq s^{max}$. This assumption can be justified by tax evasion, following Daniel (2014) or more generally by the political inability and/or unwillingness to reduce public spending and increase taxes, following Daniel and Shiamptanis (2013)[12]. This directly implies the existence of a maximum level of debt-output ratio, i.e. a fiscal limit, such that:

$$b^{max} = s^{max} \sum_{i=0}^{+\infty} \mathbb{E}_t\left[\tilde{Q}_{t,i}\right] \tag{22}$$

[12] In a framework with distortionary taxation, the fiscal limit would arise endogenously from the existence of a dynamic Laffer curve, see Bi (2012); Bi and Leeper (2013).

Thus, for $b_t > b^{max}$ fiscal policy would be necessarily running a Ponzi Scheme against creditors. Since Proposition 1 does not rule out explosive path for the debt-output ratio, a necessary and sufficient condition for fiscal sustainability, in presence of a fiscal limit on the debt-output ratio, would be a debt-stabilizing fiscal rule around a steady-state level below the fiscal limit.

Therefore, a regime-switching fiscal rule implies that debt-output ratio follows a Markov-switching autoregressive process, defined by equations (5) and (8):

$$b_t = \phi(z_t)b_{t-1} + u_t(z_t) \tag{23}$$

where

$$\phi(z_t) = \frac{1+r_t}{1+y_t}\Big(1-(1+y_t)\gamma(z_t)\Big) \quad \text{and} \quad u_t(z_t) = -(1+r_t)\mu_t(z_t).$$

A sufficient condition for (strict) stationarity of stochastic processes like (23) is given by Kesten (1973), from which we deduce the following proposition.

Proposition 2 (Debt-stabilizing condition) *A sufficient condition for a (strictly) stationary debt-output ratio is*

$$\gamma\pi > \frac{r-y}{1+y} \tag{24}$$

which can be expressed in terms of expected durations

$$\gamma_S > \left|\gamma_{NS}\right|\frac{d_{NS}}{d_S} + \frac{r-y}{1+y}\frac{d_S+d_{NS}}{d_S} \tag{25}$$

Proof 2 *See appendix A.2.*

Provided conditions (24) or (25) hold, then public debt-output has an ergodic mean:

$$\mathbb{E}[b_t] = \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \mathrm{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1-\phi(z_t)]} \tag{26}$$

where $\mathbb{E}[\alpha(z_t)] < 0$ is the ergodic value of $\alpha(z_t)$.

As long as the growth-adjusted real interest rate is positive, a debt-stabilizing condition is stricter than the NPG condition. During sustainable regimes, the required reaction of primary surplus to initial debt must be large enough to compensate for both primary deficits during unsustainable regimes, weighted by the ratio of expected durations, and the growth-adjusted real interest rate, weighted by the inverse fraction of (expected) time spent in sustainable regimes. On the contrary, when $r < y$, condition (25) could eventually imply government is violating NPG condition (19) which is the minimum requirement for fiscal sustainability. Since history provides numerous examples of $r < y$, this illustrates why testing stationarity of debt-output ratio may sometimes be misleading as a test of fiscal sustainability. As a result, NPG condition and debt-stabilizing condition would be complements rather than substitutes: a stationary public debt-output ratio does not always rule out Ponzi Schemes.

The assumption of the existence of different fiscal regimes may, in general, imply that public debt-output ratio can periodically follow an explosive path. To see why, let us consider the example of Canzoneri et al. (2001) and assume $\gamma_{NS} = 0$. We find exactly the same proposition they made: based on equation (19), any infrequent $\gamma_S > 0$ would be sufficient to rule out Ponzi Schemes. Yet this equilibrium does not ensure a stable debt-output ratio, that is public debt is $I(1)$. For a stable debt-output ratio, assuming $r - y > 0$ and $\gamma_{NS} = 0$, a regime-switching fiscal policy must satisfy the following condition, from equation (25):

$$\gamma_S > \frac{r-y}{1+y}\frac{d_S + d_{NS}}{d_S} \tag{27}$$

For $\gamma_{NS} < 0$ the condition on γ_S is stronger. Under a regime-switching debt-stabilizing fiscal policy, debt-output ratio becomes periodically explosive, and explosive regimes can be really frequent without necessarily implying debt-output is globally non-stationary.

Periodic explosive dynamics of public debt has critical consequences on regime-switching policy rules, not only on $\gamma(z_t)$ but also on the constant $\alpha(z_t)$. Rewriting equation (8) in terms of deviations of primary balance and public debt from their respective steady-state values $s^*(z_t) = (s^*_S, s^*_{NS})$ and $b^*(z_t) = (b^*_S, b^*_{NS})$ yields:

$$s_t - s^*(z_t) = \gamma(z_t)\Big(b_{t-1} - b^*(z_t)\Big) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon^s_t \tag{28}$$

from which we deduce that $\alpha(z_t)$ is equal to:

$$\alpha(z_t) = s^*(z_t) - \gamma(z_t)b^*(z_t) \tag{29}$$

In a sustainable regime, primary surplus-output ratio s_t and debt-output ratio b_t must admit steady-state values[13]. Provided condition (25) holds, we would expect s^*_S to be equal to the debt-stabilizing primary surplus ratio, for a stationary debt-output target ratio b^*_S:

$$s^*_S = \frac{r-y}{1+y}b^*_S \tag{30}$$

which implies:

$$\alpha_S = \Big(\frac{r-y}{1+y} - \gamma_S\Big)b^*_S < 0 \tag{31}$$

provided that condition (24) holds, which would account for negative estimates of α_S but also $\mathbb{E}[\alpha(z_t)] = \pi_S\alpha_S < 0$ if $\gamma_{NS} < 0$. As a consequence, insofar as $b_t < b^*_S$ fiscal policy can run primary deficits without necessarily jeopardizing fiscal sustainability.

4 Empirical analysis

We apply Regime-Switching MBS analysis to French data. The mere observation of French sovereign interest rates, which have been historically low during the European sovereign-debt crisis, conveys some information about lenders' seemingly expectations that France's fiscal policy is on a sustainable path. However empirical investigation has given rise to contradictory outcomes: (Afonso, 2005; Lamé et al., 2014; Schoder, 2014) did not find evidence of a sustainable fiscal policy in France whereas (Afonso and Jalles, 2016; Chen, 2014; Fincke and Greiner, 2012; Weichenrieder and Zimmer, 2014) reached mixed evidence; in contrast, (Greiner et al., 2007) found that fiscal policy was sustainable.

This contradiction may be attributed to the lack of account for regime-switching fiscal policies. To assess this argument, we develop a two-stage empirical strategy. First, we estimate fiscal rules following Bohn's MBS tests. From these tests, we conclude that French public debt is not sustainable. Second, we estimate a Markov-switching fiscal rule and perform a Regime-Switching MBS test. The latter outcomes challenge the former results obtained with standard techniques: the existence of a locally unsustainable

[13] Under the unsustainable regime and periodic explosive dynamics of public debt, the time series properties of s_t can be twofold, depending on the value of γ_{NS}. When $\gamma_{NS} < 0$ we expect α_{NS} to be equal to zero. Explosive debt-output ratio dynamics are not compatible with any steady-state debt-output level, hence $b^*_{NS} = 0$. Then, primary balance would be necessarily non-stationary since the two variables would be negatively cointegrated with $\{b_t\}$ being non-stationary, implying $s^*_{NS} = 0$. Otherwise, if $\gamma_{NS} = 0$, $\{s_t\}$ could be stationary and then s^*_{NS} would be eventually significantly different from zero.

regime cannot be automatically interpreted as global unsustainability. We conclude that omitting fiscal regime-switches may lead to reject mistakenly French sustainability. Another advantage with the RS-MBS approach is that it dates sub-periods of sustainability and unsustainability in France. Thus, it permits to check whether these sub-periods fit the history of French public finances.

4.1 Dataset

The choice of annual data is guided by two arguments: availability on a long time span and consistency with the fiscal institutional process. First, fiscal sustainability can only be appreciated in the long-run: PVBC or stationarity might only be satisfied in the long-run – over half a century, or more. Regarding data availability for France, we are forced to renounce using *true* quarterly data which are available from 1995-Q4 only for public debt[14]. Still, a second argument prevents us from using quarterly data: fiscal decisions are taken on an annual basis in the law of finance, despite some infra-annual adjustments. Using quarterly data may result in spurious results as it may add noise to the *true* response of primary balance to the initial stock of debt.

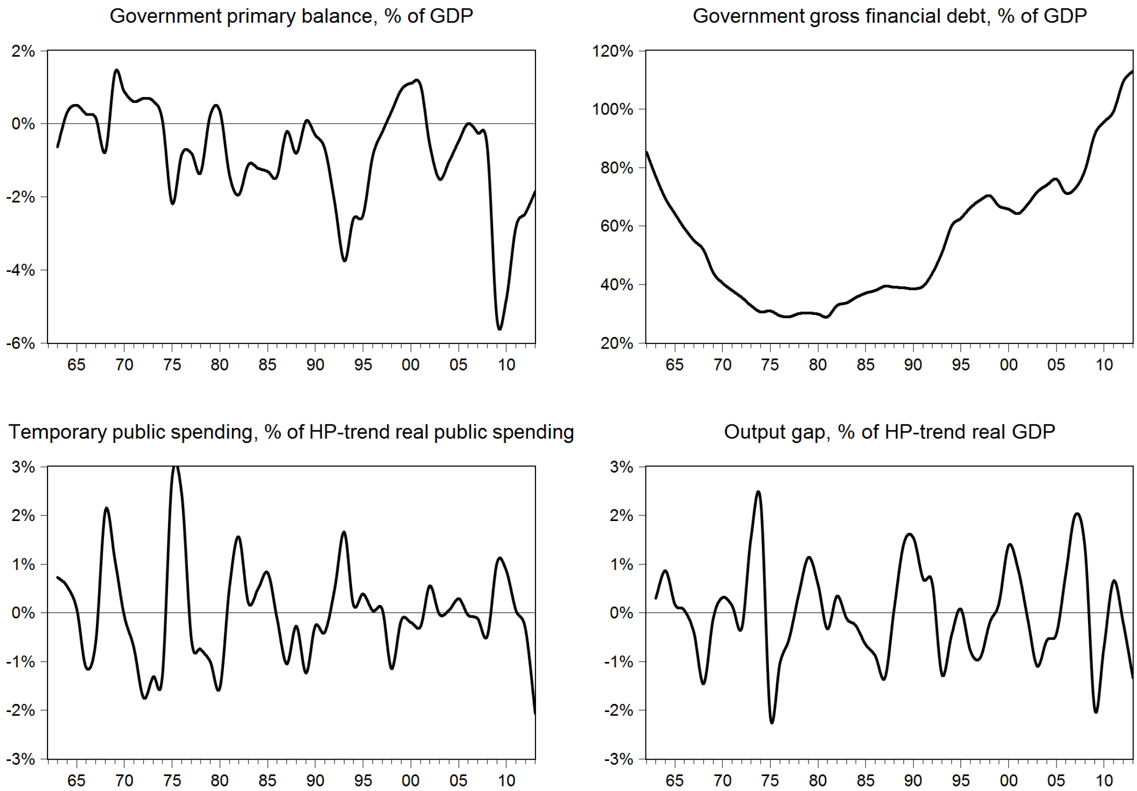
This paper uses the longest time series available for French public debt. Indeed, because of changes in national accounts systems, it is relatively hard to find historical data on French public debt. Most of available time series (in particular, those using Maastricht debt definitions) start by 1978. The IMF Historical Public Debt Database (HPDD) proposes a long-run time series for public debt, but still with missing observations for years 1978 and 1979, because of national accounting issues. Thus, regarding public debt, we use the OECD government total gross financial liabilities rather than the Maastricht definition of gross public debt since the OECD series goes back to 1969. We complete this series by backward interpolation between 1963 and 1968: for $t < 1969$, public debt at time t is equal to public debt at time $t+1$ minus the government overall budget balance at time $t+1$. This backward interpolation assumes that there were no stock-flow adjustments between 1963 and 1968. This is not a strong assumption on this period. Stock-flow adjustments are more important under large financialisation of public assets and liabilities and when public debts can be denominated in a foreign currency. Financialisation in France has started in the 1980s and public debt remains almost entirely denominated in the domestic currency. Regarding time convention in national accounts, public debt stock is the *end-of-period* stock of debt.

Overall budget balance and primary budget balance (budget balance *minus* interests paid) are taken from OECD database for years 1977-2013; observations for years 1963 to 1977 are completed using data collected by Creel and Le Bihan (2006), from French National Institute for Statistics and Economic Studies (INSEE). We build time series for output gap and temporary government spending by detrending and removing the cyclical component of real GDP and real government spending using the HP filter. Regarding the estimation of output gaps, many competing techniques are available and their relative strengths and weaknesses still discussed (see Cotis et al. (2005), for a survey of estimation methods). Our choice of the HP-filtered method has been motivated by its easiness, fastness and recent use by Fincke and Greiner (2012) and, with more sophistication, by Borio et al. (2014). To address the end point bias problem of the HP filter, we add univariate 3-year ahead forecasts for each series, using ARIMA models, prior to filtering and then dropping the last 3 observations[15]. Such a "mechanic" correction of the end point bias is applied, for instance, by the European Commission (Havik et al., 2014), when using the

[14] It is possible to build a quarterly measure for public debt using interpolation methods and quarterly government budget balance. Indeed Lamé et al. (2014) report the use of recalculated quarterly data of net French public debt, though on a shorter time span (1980Q1-2007Q4) than the one used in this paper.

[15] We also drop the first 3 observations at the beginning of filtered series which are affected by the end point bias.

Figure 1: Dataset overview, France (1962-2013)



HP filter. Finally, our dataset covers 51 years of annual data, from 1963 (1962, for gross public debt) to 2013. Data are shown in figure 1.

4.2 Model-Based Sustainability tests

We estimate different specifications of a standard fiscal policy rule and use constant-parameter estimates as a benchmark for comparison with Regime-Switching estimates. We specify the following fiscal rule, based on equation (1):

$$s_t = \gamma b_{t-1} + X_t' \beta + \varepsilon_t \tag{32}$$

where the dependent variable s_t is the primary balance-to-GDP ratio, b_{t-1} is the public debt-to-GDP ratio at end of period $t-1$ and X_t is a vector of control variables. It includes a constant, output gap $\hat{y}_t$, and cyclical government spending $\hat{g}_t$ as suggested by Bohn (1998). Then we include a dummy variable FinCrisis_t equal to one for years 2008–2013 in order to account for severe crisis years. To account for potential non-linearities regarding the level of debt, we also estimate fiscal rules as polynomial functions of debt-to-GDP ratio following Bohn (1998) and Mendoza and Ostry (2008). Finally, we account for a potential deterministic time trend, as suggested by unit-root and stationarity tests (available in the technical appendix). In presence of serial correlation in the residuals, we correct for serially correlated residuals of order one or two, depending on the model estimated.

Table 1 presents the results. Based on these estimates of constant-parameters fiscal policy rules, we find no evidence of fiscal sustainability[16]. Models (1)–(2) give no positive feedback effect, but rather

[16] This result contrasts with Fincke and Greiner (2012) who find a significant positive reaction of the primary surplus to debt. Two differences with our approach are worth mentioning. First, Fincke and Greiner do not strictly reproduce Bohn' fiscal rule:

Table 1: Constant-parameters Fiscal policy rules

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Initial Debt b_{t-1}	-0.0121	-0.0058	0.0283	0.0300*	0.0962	0.0547	0.0735
	(-0.71)	(-0.35)	(0.93)	(-1.72)	(1.50)	(0.86)	(1.38)
Quadratic debt b^2_{t-1}	.	.	.	.	-0.0555	-0.0429	-0.0367
					(-1.18)	(-0.87)	(-0.86)
Constant	-0.0025	-0.0052	0.0014	-0.0065	-0.0190	-0.0219	-0.0179
	(-0.24)	(-0.57)	(0.16)	(-0.96)	(-1.01)	(-1.15)	(-1.19)
Output gap $\hat{y}_t$	0.4190***	0.3807***	0.4800***	0.4527***	0.4565***	0.4163***	0.4360***
	(3 .38)	(3.23)	(3.91)	(3.56)	(3.64)	(3.21)	(3.38)
Temporary spending $\hat{g}_t$	-0.4053***	-0.3667***	-0.3448***	-0.3763***	-0.3754***	-0.4147***	-0.3982***
	(-3.18)	(-3.09)	(-2.73)	(-2.91)	(-2.85)	(-3.08)	(-2.98)
FinCrisis$_t$	.	-0.0179***	.	-0.0160**	.	-0.0112	-0.0131
		(-2.95)		(-2.25)		(-1.37)	(-1.70)
Trend	.	.	-0.0009	-0.0006**	-0.0008	.	-0.0006**
			(-1.55)	(-2.16)	(-1.61)		(-2.06)
DW	1.98	1.99	1.70	1.87	1.68	1.81	1.83
Adj. R^2	0.70	0.75	0.72	0.73	0.72	0.70	0.72
Observations	49	49	50	50	50	50	50

Notes: t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). Models (1)–(2) are controlling for second-order serial correlation in the residuals. Model (3)-(7) control for first-order serial correlation in the residuals.

negative though non-significant estimates for γ. We do not find evidence of a polynomial specification of the fiscal policy rule, since coefficients on debt b_{t-1} and quadratic debt b^2_{t-1} are never significant. Still, point estimates for polynomial specifications would imply a "flattening" of the fiscal policy rule for high debt-output ratio.

Unit-root and stationarity tests conclude to the potential presence of deterministic time trends respectively negative in s_t and positive in b_t. Thus we control for a deterministic trend in equation (32), in models (3)-(5) and (7) of Table 1. When estimating the fiscal rule with a time trend, the feedback coefficient on initial debt turns out to be positive, but never significant at 5% level. Only model (4) shows a positive but weakly significant (at 10% level) feedback response of primary surplus to initial debt. Moreover, deterministic trends enter negatively in all equations, which would imply $\lim_{t\to+\infty} s_t = -\infty$, thus obviously violating the PVBC.

4.3 Regime-Switching Model-Based Sustainability test

We estimate the following Markov-switching fiscal rule by direct maximisation of the log likelihood (Hamilton, 1989):

$$s_t = \gamma(z_t)b_{t-1} + \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + u_t \tag{33}$$

where except the autoregressive residuals and the error variance[17], all remaining parameters can periodically shift between two values, according to a hidden two-state Markov-process z_t.

Numerical optimization of the log likelihood function is raising identification issues, so we choose the following estimation strategy. We randomize the estimation algorithm by drawing 500 starting values and running initial ML estimations with 100 iterations on each draw, in order to reduce the dependence of the ML algorithm on starting values and thus the risk of reaching a local maximum of the log likelihood function; the main estimation algorithm begins using the starting values for which the maximization algorithm reached the highest value of the log likelihood function among the 500 initial

they limit cyclical public spending to spending related to the social insurance system though some of these expenditures may be structural; second, their sample is shorter (1970-2008) than ours.

[17] To account for first-order serial correlation in the data, we assume: $\left(1-\rho\right)u_t = \sigma\varepsilon_t$ with an i.i.d error term $\varepsilon_t \sim \mathcal{N}(0,1)$

random draws. Regarding model specification, we start estimating the most general model, allowing *all parameters, including error variance* to switch between regimes 1 and 2, thus being agnostic on the true structural form of the regime-dependent fiscal rule. At this stage, if the maximization algorithm converges, we can already appreciate how precise the resulting estimates are, both across regimes and in the long-run through the computation of the ergodic value of each parameter. This can be achieved through basic t-statistics and F-statistics analysis. We also look carefully at estimated regimes' properties: transition probabilities associated to the Markov process and filtered and smoothed regime probabilities. We check, in particular, if they are consistent with historical knowledge on fiscal policy shifts, and if they are sufficiently persistent, regarding the timing of fiscal policy.

If any subset of parameters were non-significantly different from zero in *both regimes* or if they were not taking significantly different values across regimes it would be a strong motivation to estimate a restricted model in which this subset of parameters would be regime-invariant. Thus, if any restricted model can be successfully estimated, that is, if the maximization algorithm successfully converges, then the same procedure as described before can be applied.

As a result of our estimation strategy, equation (33), without regime heteroskedasticity, seems to be the best specification of the Markov-switching fiscal policy rule[18].

Given the short length of the sample, we acknowledge that ML estimates must be considered with caution. Yet, given the potential presence of unit-root in the debt-to-GDP ratio, with stationary primary balance-to-GDP ratio, estimates of a *constant-parameters* fiscal policy rule would be equally dubious. But this paper builds on the idea that a non-linear fiscal policy behavior implies periodical explosive dynamics of public debt-to-GDP, without necessarily implying either instability of public debt-to-GDP ratio, or Ponzi schemes, in the *long run*.

Table 2 presents estimation results of equation (33). We report estimated parameters for each regime and we also compute implied long-run estimates of regime-switching parameters using ergodic probabilities. Standard deviations of long-run estimates are obtained using standard deviations and covariance of regime-specific parameters: for any regime-switching parameter $\alpha(z_t)$ which takes two values (α_1, α_2), with associated standard deviations $(\sigma_{\alpha_1}, \sigma_{\alpha_2})$ and covariance $\mathrm{Cov}(\alpha_1, \alpha_2)$, we compute the long-run (ergodic) estimate α using ergodic probabilities (π_1, π_2) by:

$$\alpha \equiv \alpha_1 \pi_1 + \alpha_2 \pi_2$$

and with standard deviation:

$$\sigma_\alpha \equiv \sqrt{(\pi_1 \sigma_{\alpha_1})^2 + (\pi_2 \sigma_{\alpha_2})^2 + 2\pi_1 \pi_2 \mathrm{Cov}(\alpha_1, \alpha_2)}$$

The results raise some comments. First, France's fiscal policy is well described by a two-state Markov-switching policy rule. One regime is sustainable with a strongly positive and significant correlation between primary balance s_t and initial debt b_{t-1}, implying a stable debt-to-GDP ratio, while the other one shows a non-significative positive correlation. As expected, the constant is significantly negative in the sustainable regime, which is consistent with a debt-stabilizing fiscal policy, while non-significant in the unsustainable regime, as explained in section 3. Second, both regimes appear to be strongly

[18] We have also estimated an alternative specification with regime heteroskedasticity. While the MLE successfully converged, our results appeared a posteriori to be highly dependent on initial values for estimation algorithm and they might be a local maximum of the log likelihood function. That is the main reason why we increased the number of random draws at the start of the estimation process. After having randomized the estimation algorithm, we no longer obtain successful convergent ML estimates of an equation with regime heteroskedasticity. We also estimated a model with a regime-invariant deterministic trend. We conclude to a non-significant (at 5% level) deterministic trend, so we choose to exclude the deterministic trend from our baseline specification.

Table 2: Estimated Markov-switching fiscal rule for France (1965–2013)

Regime-switching parameters	Regime 1	Regime 2	Long-run estimates
Debt b_{t-1}	0.0889***	0.0017	0.0370
	(3.08)	(0.07)	(1.51)
Constant	-0.0608*	-0.0256	-0.0399
	(-1.90)	(-0.84)	(-1.30)
Output gap $\hat{y}_t$	0.4214***	0.2894**	0.3429***
	(3.30)	(2.39)	(4.08)
Temporary spending $\hat{g}_t$	-0.0637	-0.5491***	-0.3524***
	(-0.50)	(-5.33)	(-4.20)
Regime-invariant parameters			
AR(1)	0.9443***	0.9443***	-
	(13.10)	(13.10)	
Standard-error σ	0.0046***	0.0046***	-
	(8.34)	(8.34)	
Regimes properties	Transition probabilities $p_i i$	Ergodic probabilities π_i	Expected durations (years) d_i
i=1	0.8770	0.4051	8.1
i=2	0.9162	0.5949	11.9
Durbin-Watson statistic	1.7724	Akaike info criterion	-6.8723
Log likelihood	180.3709	Schwarz criterion	-6.4090
Number of observations	49	Hannan-Quinn criter.	-6.6965

Notes: t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). We control for regime-invariant first-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log\hat{\sigma}$: consequently, standard erros and t-statistics are obtained applying the Delta method. For regime-switching parameters we compute "long-run estimates" as defined earlier. We report estimates for regime-invariant parameters twice in columns "Regime 1" and "Regime 2", for clarity purposes since they are constant in each regime-specific equation.

persistent with respective expected durations of 8.1 and 11.9 years, respectively for sustainable and unsustainable regimes. This would explain why OLS estimates were inconclusive about the long-run correlation between primary surplus and initial debt in table ??.

Figure 2: Estimated sustainable regime, France (1965-2013)

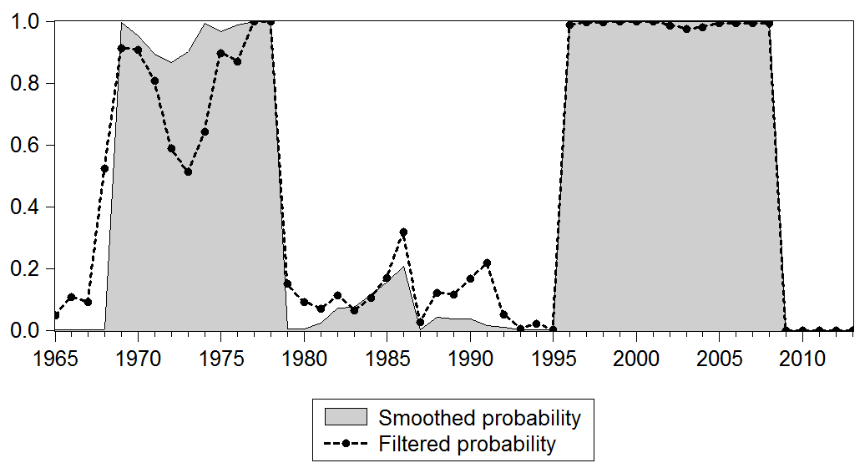


Figure 2 represents estimated smoothed and filtered probabilities for regime 1 which we label as sustainable. Results show a succession of periods of unsustainable or sustainable fiscal policies with marked decades. Public finances in the 1970s were sustainable over the most part. In sharp contrast, France's fiscal policy has been mostly unsustainable during the 1980s. Still, filtered probabilities show a small and transitory increase in the probability of being in a sustainable regime during the so-called "Tournant de la rigueur" of 1983-1986 when the Socialist government turned to disinflation and deficit-reduction

Table 3: Regime-Switching MBS: unilateral versus bilateral tests

Student tests for...	t-stat	Bilateral test p-value	Unilateral test p-value
No-Ponzi Game condition (19)	3.0841	0.0039	0.0020
Stable long-run debt-to-GDP ratio (25) $\frac{r-y}{1+y}=0.33\%$	2.8008	0.00801	0.0041

Notes: theses Student tests assume γ_{NS} is virtually equal to 0. Real interest rate is the ex-post real 10-year yield on French public bonds, obtained using the implicit GDP deflator from OECD Economic Outlook database.

policies. Overall, results are consistent with a comprehensive and historical analysis of France's fiscal policy. In the 1990s, results report that France's fiscal policy became gradually sustainable (or passive to use Leeper's terminology) and actually so by 1996, until 2008 and the advent of the Great Recession. This finding supports the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, despite it being under an Excessive Deficit Procedure from 2003 to 2007. In contrast with Weichenrieder and Zimmer (2014) who show that Euro membership of France has reduced the responsiveness of the primary surplus to debt, our results show that the 1999-2011 period (Euro membership years in Weichenrieder and Zimmer) was heterogeneous as regards fiscal responsiveness: it was positive until 2008 and then negative.

The long-term estimate of $\gamma\pi$ is positive, equal to 0.037 but non-significant (with a p-value equal to 0.1394). Still, this result raises two comments. First, the long-run estimate of $\gamma\pi$ appears non-significant mainly from the fact that the estimate of γ_{NS} is strongly non-significant (i.e. with a large estimated standard-error), and thus might be considered as virtually equal to 0. Second, significance tests are not appropriate to test for NPG and debt-stabilizing conditions on $\gamma\pi$ since they are *bilateral* tests. On the contrary, Propositions 1 and 2 call for *unilateral* tests for which critical values are lower with respect to bilateral tests[19].

Assuming that γ_{NS} is virtually equal to 0, we find significant evidence that France's fiscal policy not only satisfies the No-Ponzi Game condition (Proposition 1) but also the Debt-stabilizing condition (Proposition 2). In other words, given past history of French fiscal policy and fiscal regimes, we find significant evidence that France's fiscal policy has been sustainable overall the period 1965-2013, despite a prolonged period of unsustainability from 1979 to 1995.

[19]For instance, a bilateral test of the NPG condition on the parameter $\gamma\pi$ is build upon the null hypothesis $\gamma\pi=0$ against the alternative $\gamma\pi\neq 0$, while the unilateral test is build upon the null hypothesis $\gamma\pi=0$ against the alternative $\gamma\pi>0$ which is a more adequate testing hypothesis in the sustainability context.

Table 4: Expected regime durations and Debt-GDP ratios using market long-term interest rate

Scenario 1: Increasing expected duration of sustainable regime						
d_S	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2	0.14	0.86	1.27%	Satisfied	Yes	313%
4	0.25	0.75	2.23%	Satisfied	Yes	178%
7	0.37	0.63	3.28%	Satisfied	Yes	129%
8.1	0.40	0.60	3.59%	Satisfied	Yes	121%
15	0.56	0.44	4.94%	Satisfied	Yes	97%
30	0.72	0.28	6.35%	Satisfied	Yes	84%
60	0.83	0.17	7.41%	Satisfied	Yes	77%
∞	1.00	0.00	8.88%	Satisfied	Yes	71%
Scenario 2: Decreasing expected duration of unsustainable regime						
d_{NS}	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
50	0.14	0.86	1.24%	Satisfied	Yes	322%
30	0.21	0.79	1.89%	Satisfied	Yes	207%
15	0.35	0.65	3.12%	Satisfied	Yes	134%
11.9	0.41	0.59	3.60%	Satisfied	Yes	121%
6	0.58	0.42	5.11%	Satisfied	Yes	95%
3	0.73	0.27	6.49%	Satisfied	Yes	83%
1	0.89	0.11	7.91%	Satisfied	Yes	75%
0	1.00	0.00	8.88%	Satisfied	Yes	71%

Notes: Debt-output ratios are computed from equation (26) neglecting covariance terms. For scenarios 1 and 2, we use average market long-term interest rate $r = 3\%$, average real growth rate $y = 2.68\%$ and $r - y = 0.32\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_S and for $d_{NS} = 11.9$. In scenario 2, we compute expected debt-output ratios under various values of d_{NS} and for $d_S = 8.1$. All others parameters are constant and equal to point estimates obtained in table 2, except γ_{NS} which is set to 0.

Using point estimates reported in table 2 and historical average for the real interest rate and real GDP growth rate, table 4 reports the expected debt-to-GDP ratios, neglecting the covariance terms, under two alternative scenarios. In scenario 1, we suppose sustainable regimes last longer and we increase their expected duration (or persistence) while keeping the expected duration of unsustainable regimes constant and equal to their estimated value. In scenario 2, we suppose unsustainable regimes are shorter and we decrease their expected duration while keeping the expected duration of sustainable regimes constant and equal to their estimated value. Our computations indicate France's gross public debt-to-GDP ratio would reach an average value of 121% across fiscal regimes, which may be interpreted as too high to prevent sovereign default. First, this approach does not pretend sovereign default would be ruled out with certainty by a debt-stabilizing fiscal policy rule[20]. Using regime-switching models, this paper proposes a new non-linear test to discriminate between obviously unsustainable fiscal policies and most probable sustainable ones, given taking into account fiscal policy can periodically deviates from sustainability requirements. But we do not propose any measure of "fiscal space" or "fiscal vulnerability". Second, this expected debt-to-GDP ratio cannot be interpreted as a long-run steady-state ratio, in the usual sense. It represents a long-run average between a regime where public debt follows stable dynamics and a regime with explosive public debt. In particular, assuming $d_S \to +\infty$ or equivalently $d_{NS} = 0$, we obtain the underlying debt-to-GDP target ratio $b_S^* = 71\%$ towards which public debt converges during sustainable regimes[21].

[20] We agree with Daniel and Shiamptanis (2013, p.2308) who argue that "*a country following a responsible fiscal rule could still encounter solvency problems due to negative shocks or due to future plans which are insolvent. However, a country following a fiscal rule which is not responsible will encounter solvency problems with certainty.*"

[21] This level cannot be compared to Maastricht criterion of 60% of gross public debt. Indeed, we used the OECD's gross government financial liabilities in our estimates rather than Maastricht gross public debt, for data availability reasons. These two measures of gross public debt differ in terms of debt instruments and valuation methods. As a result, Maastricht debt is generally much lower than gross governement financial liabilities.

Table 5: Growth-adjusted real rates and Debt-GDP ratios

$\frac{r-y}{1+y}$	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2.5%	Yes	334%
2.0%	Yes	235%
1.5%	Yes	183%
1.0%	Yes	150%
0.5%	Yes	127%
0.0%	Yes	111%
-0.5%	Yes	98%
-1.0%	Yes	89%
-1.4%	Yes	81%
-1.9%	Yes	74%
-2.4%	Yes	69%
-2.8%	Yes	64%

Notes: Debt-output ratios are computed from equation (26) neglecting covariance terms. We use point estimates of γ_S, α_S, α_{NS}, except for γ_{NS} which is set to 0, and we use expected durations of regime d_S and d_{NS} from table 2. Then, we set $r = 3\%$ and compute expected debt-output ratios for various real GDP growth rate.

Finally, we show how the debt-to-GDP ratio vary with the level of the growth-adjusted real interest rate given our point estimates, in table 5. Our results indicate that a modest increase (resp. decrease) in the growth-adjusted real interest rate would result in a significant increase (resp. decrease) of the long-run average public debt-to-GDP ratio.

5 Conclusions

This paper introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes, where by unsustainable regime we mean a periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violation of Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, and fiscal regimes thus matter for fiscal sustainability analysis.

We prove formally that global fiscal sustainability differs from local sustainability. The former depends on the relative sensitiveness of the fiscal rule to the debt-to-GDP ratio from one regime to another, and also on the relative duration and persistence of both regimes.

The Regime-Switching MBS test is then applied to French data over a 51-year horizon and compared to standard MBS tests. Our results are threefold. First, we estimate different specifications of Bohn's constant-parameters fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, and notably, despite being under an

Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. They reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio.

Future research may now move towards the analysis of the interactions between monetary policy and fiscal policy, in presence of regime-switching policy rules, and their consequences on fiscal sustainability. In contrast with early attempts (see Davig and Leeper (2011) for example), a euro-area country like France cannot be described by a domestic monetary policy. Theoretical research is thus required to match domestic fiscal policy with a federal monetary policy. Beyond that, the question of fiscal sustainability in a Regime-switching MBS framework could be embedded in an open-economy framework. It would introduce another determinant of fiscal sustainability, namely cooperative or non-cooperative fiscal behaviours.

A Appendix

A.1 Proof of Proposition 1 (No-Ponzi Game)

We show that a strictly positive long-run feedback effect, i.e. (18)

$$\gamma\pi > 0$$

is a *sufficient* condition for the NPG (17) to hold, in a dynamically efficient economy and a bounded innovation process $\mu(z_t)$, following Bohn (1998, see online appendix). Using (8) and iterating of (5) yields:

$$\begin{aligned} b_{t+T} = & \prod_{i=0}^{T} \frac{1+r_{t+i}}{1+y_{t+i}} \Big(1-(1+y_{t+i})\gamma(z_{t+i})\Big) b_{t-1} \\ & - \sum_{k=0}^{T} (1+r_{t+k}) \left(\prod_{j=k+1}^{T} \frac{1+r_{t+j}}{1+y_{t+j}} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big) \right) \mu_{t+k}(z_{t+k}) \end{aligned} \tag{34}$$

Then, multiplying by (16), one gets an expression for the discounted debt-output ratio at time $t+T$:

$$\mathbb{E}_t\, \tilde{Q}_{t,T+1} b_{t+T} = \mathbb{E}_t \prod_{i=0}^{T} \Big(1-(1+y_{t+i})\gamma(z_{t+i})\Big) b_{t-1} - \mathbb{E}_t \sum_{k=0}^{T} \left(\prod_{j=k+1}^{T} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big) \right) a_{t,k} \tag{35}$$

with $a_{t,k} = (1+y_{t+k})\tilde{Q}_{t,k}\mu_{t+k}(z_{t+k})$. Taking the absolute value[22] of (35) and using triangle inequality yields:

$$\left|\mathbb{E}_t\, \tilde{Q}_{t,T+1} b_{t+T}\right| \leq \mathbb{E}_t \left| \prod_{i=0}^{T} \Big(1-(1+y_{t+i})\gamma(z_{t+i})\Big) b_{t-1} \right| + \underbrace{\mathbb{E}_t \left| \sum_{k=0}^{T} \left(\prod_{j=k+1}^{T} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big) \right) a_{t,k} \right|}_{W_t} \tag{36}$$

and applying the triangle inequality on W_t allow us to give an upper bound to the absolute value of (35):

$$\left|\mathbb{E}_t\, \tilde{Q}_{t,T+1} b_{t+T}\right| \leq \mathbb{E}_t \prod_{i=0}^{T} \Big|1-(1+y_{t+i})\gamma(z_{t+i})\Big| |b_{t-1}| + \mathbb{E}_t \sum_{k=0}^{T} \left| \prod_{j=k+1}^{T} \Big(1-(1+y_{t+j})\gamma(z_{t+j})\Big| \right| |a_{t,k}| \tag{37}$$

An important step is to give a tractable expression for

$$\mathbb{E}_t \prod_{i=0}^{T} \Big|1-(1+y_{t+i})\gamma(z_{t+i})\Big| \tag{38}$$

in order to study the limit property of equation (35). Thus remark that:

$$\begin{aligned} \mathbb{E}_t \textstyle\prod_{i=0}^{T} \Big|1-(1+y_{t+i})\gamma(z_{t+i})\Big| &= \mathbb{E}_t \left[\exp\left(\ln \textstyle\prod_{i=0}^{T} \Big|1-(1+y_{t+i})\gamma(z_{t+i})\Big| \right) \right] \\ &= \mathbb{E}_t \left[\exp\left(T \times \frac{1}{T} \textstyle\sum_{i=0}^{T} \ln \Big|1-(1+y_{t+i})\gamma(z_{t+i})\Big| \right) \right] \end{aligned} \tag{39}$$

[22]Note that $f(x) = |x|$ is convex, then Jensen inequality yields for any random variable X:

$$|\mathbb{E}[X]| \leq \mathbb{E}[|X|]$$

where $\frac{1}{T}\sum_{i=0}^{T}\ln\left|1-(1+y_{t+i})\gamma(z_{t+i})\right|$ is the Lyapunov exponent associated to the present-value debt-output ratio. Since both $(1+y_t)$ and z_t are stationary-ergodic, then we know that:

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{i=0}^{T}\ln\left|1-(1+y_{t+i})\gamma(z_{t+i})\right| = \mathbb{E}\left[\ln\left|1-(1+y_t)\gamma(z_t)\right|\right] \tag{40}$$

which is measurable at time t. If one assumes $(1+y_t)\gamma(z_t)<1$[23] then it yields

$$\ln\left|1-(1+y_t)\gamma(z_t)\right| = \ln(1-(1+y_t)\gamma(z_t))$$

. Applying Jensen's inequality on the logarithm function and the expectation operator yields an upper-bound for

$$\mathbb{E}\ln\Big(1-(1+y_t)\gamma(z_t)\Big) \le \ln\Big(1-\mathbb{E}(1+y_t)\gamma(z_t)\Big) \tag{41}$$

From what precedes[24], we deduce it exists an arbitrarily high $N\in\mathbb{N}$ such that:

$$\forall T\ge N,\quad \mathbb{E}_t\prod_{i=0}^{T}\left|1-(1+y_{t+i})\gamma(z_{t+i})\right| \le \exp\left[\ln\Big(1-\mathbb{E}(1+y_t)\gamma(z_t)\Big)^T\right] \tag{42}$$

which allows us to conclude

$$\mathbb{E}_t\prod_{i=0}^{T}\left|1-(1+y_{t+i})\gamma(z_{t+i})\right| \le \Big(1-\mathbb{E}(1+y_t)\gamma(z_t)\Big)^T \tag{43}$$

Finally, we define the following upper bound for equation (38):

$$\mathbb{E}_t\prod_{i=0}^{T}\left|1-(1+y_{t+i})\gamma(z_{t+i})\right| \le \Big(1-(1+y)\gamma\pi-(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)\Big)^T \tag{44}$$

where $\mathrm{Cov}(y_t,z_t)$ is the unconditional covariance of y_t and z_t.

At this stage, we need two assumptions to proceed further.

Assumption 1 *Following Bohn (1998), we assume dynamic efficiency which implies present-value of income is finite:*

$$\lim_{T\to+\infty} Y_t\sum_{i=1}^{T}\mathbb{E}_t\,\tilde{Q}_{t,i} = \overline{Y}$$

implying $\lim_{T\to+\infty}\mathbb{E}_t\,\tilde{Q}_{t,T}=0$, *by convergence of the serie* $\sum_{i=1}^{T}\mathbb{E}_t\,\tilde{Q}_{t,i}$.

Assumption 2 *Following Bohn (1998), we assume the innovation process* $\mu_t(z_t)$ *is bounded* $\left|\mu_t(z_t)\right|\le M$.

Assumptions 1-2 jointly imply $\lim_{T\to+\infty}\mathbb{E}_t a_{t,k}=0$[25] that is:

$$\forall\delta>0,\quad \exists K\in\mathbb{N}\quad/\quad \forall k>K,\quad \left|\mathbb{E}_t a_{t,k}\right|\le\delta \tag{45}$$

[23] This assumption is actually purely technical, since it mainly relies on the assumption $\left|\gamma(z_t)\right|$ is close to zero, about the size of a small interest rate and $(1+y_t)$ is close to 1.

[24] In particular, Jensen inequality implies that:

$$\frac{1}{T}\sum_{i=0}^{T}\ln\left|1-(1+y_{t+i})\gamma(z_{t+i})\right| \le \ln\Big(\frac{1}{T}\sum_{i=0}^{T}\left|1-(1+y_{t+i})\gamma(z_{t+i})\right|\Big)$$

and allows to define an upper-bound for $\mathbb{E}_t\prod_{i=0}^{T}\left|1-(1+y_{t+i})\gamma(z_{t+i})\right|$.

[25] Given that $\lim_{T\to+\infty}\mathbb{E}_t\,\tilde{Q}_{t,T}=0$ also implies $\lim_{T\to+\infty}\mathbb{E}_t(1+y_T)\tilde{Q}_{t,T}=0$

Then, using assumptions 1-2 along with equation (44) yields:

$$
\begin{aligned}
\left|\mathbb{E}_t \, \tilde{Q}_{t,T+1} b_{t+T}\right| \leq \Big(1-(1+y)\gamma\pi-(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)\Big)^T |b_{t-1}| \\
+\Omega\Big(1-(1+y)\gamma\pi-(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)\Big)^{T-K} \\
+\sum_{k=K}^{T}\Big(1-(1+y)\gamma\pi-(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)\Big)^{T-k}\delta
\end{aligned} \tag{46}
$$

where $\Omega = \sum_{k=0}^{K-1} E_t \prod_{j=k+1}^{K-1}\left|1-(1+y_{t+i})\gamma(z_{t+i})\right|\left|\mathbb{E}_t \, a_{t,k}\right|$ is finite. Finally, rearranging the last expression allows us to write:

$$
\begin{aligned}
\left|\mathbb{E}_t \, \tilde{Q}_{t,T+1} b_{t+T}\right| \leq \Big(1-(1+y)\gamma\pi-(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)\Big)^T |b_{t-1}| \\
+\Omega\Big(1-(1+y)\gamma\pi-(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)\Big)^{T-K} \\
+\frac{\delta}{(1+y)\gamma\pi+(\gamma_S-\gamma_{NS})\mathrm{Cov}(y_t,z_t)}
\end{aligned} \tag{47}
$$

Assumption 3 *In a purely Ricardian economy, we assume the fiscal regime z_t is independent of the real growth rate of the economy y_t, i.e.* $\mathrm{Cov}(y_t,z_t)=0$.

Therefore, under assumption 3, a sufficient condition for the NPG condition only requires:

$$
\gamma\pi > 0 \tag{48}
$$

which implies $(1+y)\gamma\pi > 0$. Therefore, we find that

$$
\forall\hat{\epsilon}>0\,, \quad \exists K\in\mathbb{N} \quad / \quad \forall T\geq K \qquad \left|\mathbb{E}_t\tilde{Q}_{t,T+1}b_T\right| < \hat{\epsilon}
$$

provided one sets $\hat{\epsilon} = \frac{\delta}{|(1+y)\gamma\pi|}$, from which we conclude that:

$$
\lim_{T\to+\infty} \mathbb{E}_t\tilde{Q}_{t,T+1}b_{t+T} = 0 \tag{49}
$$

Discussion. In a more general framework with $\mathrm{Cov}(y_t,z_t)\neq 0$, a sufficient condition to rule out Ponzi schemes, given a Markov-switching fiscal rule such as (8), would be:

$$
\gamma\pi > -\frac{(\gamma_S-\gamma_{NS})}{1+y}\mathrm{Cov}(y_t,z_t) \tag{50}
$$

and would critically depends on the covariance term $\mathrm{Cov}(y_t,z_t)$. If positive (i.e. if sustainable regimes are positively correlated to higher growth), it implies that a strictly positive $\gamma\pi$ would not be required to rule out Ponzi schemes; if negative, on the contrary, it would not be sufficient. Still, our empirical results provide an *ex post* validation for assuming $\mathrm{Cov}(y_t,z_t)=0$, since the estimated unconditional covariance between smoothed probabilities of a sustainable regime (i.e. the empirical counterpart of z_t) and the growth rate of real GDP is non-significantly different from zero, with a positive point estimate.

A.2 Proof of Proposition 2 (Debt-stabilizing condition)

Using the sufficient condition for a *strictly* stationary Markov-switching autoregressive process of order one, we show a strictly larger feedback effect than the average growth-adjusted real interest rate, i.e.

(24), is a sufficient condition for the debt-output ratio process (23) to be strictly stationary and fluctuate around its ergodic mean (26).

Considering stochastic processes $\{x_t\}$ described by:

$$x_t = \phi_0 + \phi(z_t)x_{t-1} + \varepsilon_t \tag{51}$$

where z_t is a discrete-time Markov process, defined on the state-space $z(\Omega)$. We know from Kesten (1973) that a sufficient condition for *strict stationarity* is:

$$\mathbb{E}\Big[\ln\big|\phi(z_t)\big|\Big] \equiv \sum_{i\in z(\Omega)} \ln\big|\phi(i)\big|\pi(i) < 0 \tag{52}$$

which means that a globally stationary process $\{x_t\}$ can be locally (or periodically) non-stationary. This condition ensures that $\{x_t\}$ is strictly (or strongly) stationary implying its joint-probability distribution does not change over time. Strict stationarity only implies $\{x_t\}$ has a finite mean but does not imply necessarily a finite variance. Since weak stationarity requires finite variance, this condition is not sufficient for weak stationarity. For a finite variance, this process must verify a stronger condition. Define $\Phi \equiv \mathrm{diag}\Big(\phi(i), \forall i \in z(\Omega)\Big)$ and $\rho(M)$ the spectral radius of any square-matrix M. Then, for this stricly stationary process to admit a unique stationary solution at second-order, it must satisfy the following condition:

$$\rho(\Phi^2 P) < 1 \tag{53}$$

where P is the transition matrix of the underlying Markov-chain.

Applying condition (52) to equation (23) yields the following condition:

$$\mathbb{E}[\ln\big|\phi(z_t)\big|] = \mathbb{E}\left[\ln\left|\frac{1+r_t}{1+y_t}\right| + \ln\big|1-(1+y_t)\gamma(z_t)\big|\right] < 0 \tag{54}$$

Hence, using usual approximation $\ln(1+x) \sim x$ when $x \to 0$ and taking unconditional expectations of r_t, y_t and $\gamma(z_t)$, we find a sufficient condition for strict stationarity of process $\{b_t\}$ is:

$$\gamma\pi > \frac{r-y}{1+y} \tag{55}$$

assuming that $\mathrm{Cov}(y_t, z_t) = 0$.

Therefore, process $\{b_t\}$ has an ergodic mean equal to

$$\begin{aligned} \mathbb{E}[b_t] &= \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \mathrm{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1-\Phi(z_t)]} \\ &= \frac{-(1+r)\,\mathbb{E}\,\alpha(z_t) - (\alpha_S - \alpha_{NS})\mathrm{Cov}(r_t, z_t) + \mathrm{Cov}(\phi(z_t), b_{t-1})}{(1+r)\gamma\pi + (\gamma_S - \gamma_{NS})\mathrm{Cov}(r_t, z_t) - \frac{r-y}{1+y}} \end{aligned} \tag{56}$$

which we approximate by

$$\mathbb{E}[b_t] \simeq \frac{-(1+r)\,\mathbb{E}\,\alpha(z_t)}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \tag{57}$$

neglecting covariance terms, following Bohn (1998, 2008) and Mendoza and Ostry (2008).

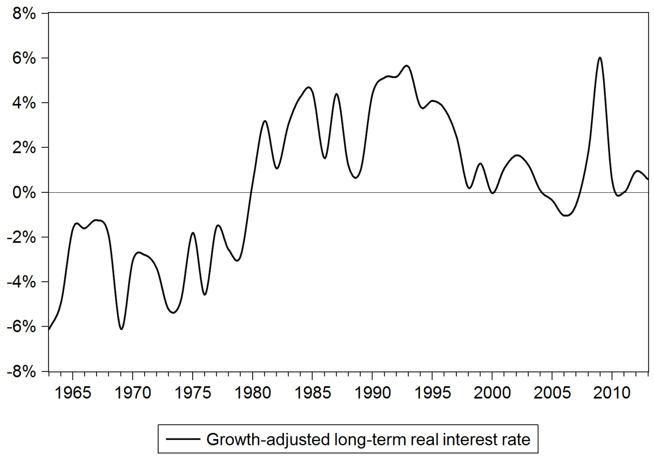
A.3 Data on real interest rates and real GDP growth rate

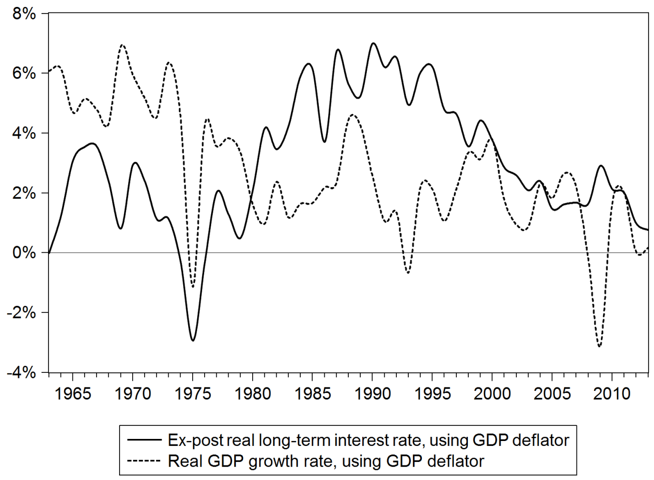
Table 6 presents descriptive statistics on long-run ex-post real interest rate (using the yield on 10-year public bonds) and real GDP growth. Figure 3 plots the growth-adjusted real interest rate and each time series separately.

Table 6: Descriptive statistics on real interest rates and real GDP growth, 1963-2013

	Long-term real rate	Real GDP growth rate
Mean	3.00%	2.68%
Median	2.86%	2.31%
Maximum	6.99%	6.91%
Minimum	-2.94%	-3.11%
Std. Dev.	2.2%	2.1%
Skewness	-0.060	-0.125
Kurtosis	2.759	2.994
Jarque-Bera normality test	0.154	0.132
p-value	0.926	0.936
Sum	1.530	1.366
Sum Sq. Dev.	0.023	0.022
Observations	51	51

Figure 3: Growth-adjusted real interest rate, real interest rates and real GDP growth rate





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